Welcome to Math 111!

Course Overview

Section 1.1

Team Mission 1 Handed out today

Why are you taking Math 111?

Transfer degree

- Nutrition, Accounting, Business, Civil Engineering

What do you want to get out of this class?

- proficiency + skills for later coursework

What kind of class do you want to come to?

calm
respectful
fun
supportive
encouraging

minimize distractions
phones on silent
no texting

Math 111 Lecture Notes

Section 1.1: Functions

A set containing ordered pairs (x, y) defines y as a function of x if and only if no two ordered pairs in the set have the same x-coordinate. In other words, every input maps to exactly one output. We write $y = f(x)$ and say "y is a function of x." For the function defined by $y = f(x)$,

- x is the independent variable (also known as the input)
- y is the dependent variable (also known as the output)
- f is the function name

$f(x) = y$

Example 1. Determine whether or not each of the following represents a function.

input : code x
C1
D2
D4
output : food y

(a) The set of ordered pairs of the form (code, food item).

(A0, Doritos)
(A2, Doritos)
⋮
This is a function because each input has exactly one output

(b) The set of ordered pairs of the form (food item, code).

(Doritos, A0)
(Doritos, A2)
This is not a function because the input of Doritos has 2 different outputs

vertical line test

Is a function

not a function

Example 2. Determine if y is a function of x for each of the following relations.

(a) The set of ordered pairs that satisfy the equation $x + y = 4$.

$y = -x + 4$

This is a function because each input will have a single output.

(b) The set of ordered pairs that satisfy the equation $x^2 + y = 4$.

This is a function because each input will have a single output

$(1, 3)$

$(-1, 3)$

$y = -x^2 + 4$

$y = -(1)^2 + 4$
$= -1 + 4$
$= 3$

$y = -(-1)^2 + 4$
$= -1 + 4$
$= 3$

(c) The set of ordered pairs that satisfy the equation $x + y^2 = 4$.

$y^2 = -x + 4$

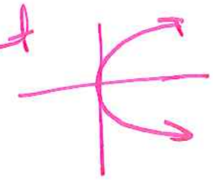
$y = \pm\sqrt{-x+4}$

$(1, \sqrt{3})$

$(1, -\sqrt{3})$

$y = \pm\sqrt{-1+4}$
$= \pm\sqrt{3}$

Not a function because one input can give 2 different outputs

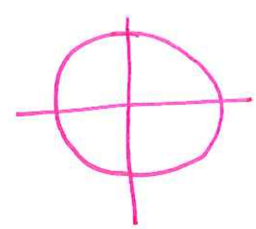


(d) The set of ordered pairs that satisfy the equation $x^2 + y^2 = 4$.

This is not a function because there are 2 possible outputs for the same input

$y^2 = -x^2 + 4$

$y = \pm\sqrt{-x^2+4}$



Domain and Range

- The domain of a function is the set of all possible inputs.
- The range of a function is the set of all possible outputs.

The largest possible set for each the domain and range is the set of all real numbers. So far, there are two primary possible circumstances that restrict the domain of a function. When a number causes either of the following to occur, it *must* be excluded from the domain of a function:

- Division by zero
- The square root (or any even root) of a negative number

Example 3. State the domain of each of the following functions.

(a) $f(x) = \sqrt{7 - 2x}$

$7 - 2x \geq 0$

$\frac{-2x}{-2} \geq \frac{-7}{-2}$

$x \leq \frac{7}{2}$

$\{x \mid x \leq \frac{7}{2}\}$ set-builder notation

$(-\infty, \frac{7}{2}]$ interval notation

(b) $\frac{n(t)}{m(t)}$ where $n(t) = t + 2$ and $m(t) = t^2 - 1$

$\frac{n(t)}{m(t)} = \frac{t+2}{t^2-1}$

$t^2 - 1 \neq 0$

$t^2 \neq 1$

$t \neq \pm\sqrt{1}$

$t \neq \pm 1$

$\{t \mid t \neq \pm 1\}$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(c) $s(t) = 5t^2 + 4t + 3$

All real numbers

$\{x \mid x \in \mathbb{R}\}$

$\mathbb{R}$

$(-\infty, \infty)$

(d) $g(x) = \dfrac{\sqrt{x-5}}{x-9}$

2 restrictions

$x - 9 \neq 0$ and $x - 5 \geq 0$

$x \neq 9$ and $x \geq 5$

$\{x \mid x \geq 5 \text{ and } x \neq 9\}$

$[5, 9) \cup (9, \infty)$

Group Work 1. State the domain of each of the following functions.

(a) $f(x) = \sqrt{3x-5}$

$3x-5 \geq 0$

$3x \geq 5$

$x \geq \frac{5}{3}$

$\{x \mid x \geq \frac{5}{3}\}$

$[5/3, \infty)$

(c) $\frac{h(t)}{k(t)}$ where $h(t) = \sqrt[3]{t-2}$ and $k(t) = 5t+40$

no restrictions on $\sqrt[3]{}$

$\frac{h(t)}{k(t)} = \frac{\sqrt[3]{t-2}}{5t+40}$

$5t+40 \neq 0$

$\frac{5t}{5} \neq \frac{-40}{5}$

$t \neq -8$

$\{t \mid t \neq -8\}$

$(-\infty, -8) \cup (-8, \infty)$

(b) $g(x) = \dfrac{x^2-7x+10}{x^2+x-6}$

$x^2+x-6 \neq 0$

$(x+3)(x-2) \neq 0$

$x+3 \neq 0$ or $x-2 \neq 0$

$x \neq -3$ or $x \neq 2$

$\{x \mid x \neq -3 \text{ or } 2\}$

$(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

(d) $f(t) = \dfrac{4t+1}{\sqrt{5-t}}$ ← square root and denominator

$5-t \geq 0$

$5-t > 0$

$\frac{-t}{-1} > \frac{-5}{-1}$

$t < 5$

$\{t \mid t < 5\}$

$(-\infty, 5)$

Function Operations

- The sum of f and g, $f+g$, is defined by $(f+g)(x) = f(x) + g(x)$.
- The difference of f and g, $f-g$, is defined by $(f-g)(x) = f(x) - g(x)$.
- The product of f and g, $f \cdot g$, is defined by $(f \cdot g)(x) = f(x) \cdot g(x)$.
- The quotient of f and g, $\frac{f}{g}$, is defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$.

Example 4. Let $f(x) = \dfrac{x}{x+3}$ and let $g(x) = \dfrac{3}{x+3}$. Find and fully simplify $(f+g)(x)$ and $\left(\dfrac{f}{g}\right)(x)$. State the domain of $f+g$ and $\dfrac{f}{g}$.

$$(f+g)(x) = f(x) + g(x)$$
$$= \frac{x}{x+3} + \frac{3}{x+3}$$
$$= \frac{x+3}{x+3}$$
$$= 1, \quad x \neq -3$$

D: $\{x \mid x \neq -3\}$

D: $(-\infty, -3) \cup (-3, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{x}{x+3}}{\frac{3}{x+3}}$$
$$= \frac{x}{\cancel{x+3}} \cdot \frac{\cancel{x+3}}{3}$$
$$= \frac{x}{3}, \quad x \neq -3$$

D: $\{x \mid x \neq -3\}$

$(-\infty, -3) \cup (-3, \infty)$

Example 5. Let $P(t)$ be the population of a country (in millions) t years after January 1, 2000. The function P is defined by $P(t) = 0.01t^2 + 2$. Let $N(t)$ be the number of people (in millions) that a country can feed t years after January 1, 2000. The function N is defined by $N(t) = 4 + 0.5t$. Use this to complete the following problems.

(a) One way of measuring prosperity is to measure the surplus. The surplus, $S(t)$, is defined to be $N(t) - P(t)$. Find and simplify $S(t)$.

$$\begin{aligned} S(t) &= N(t) - P(t) \\ &= 4 + .5t - (.01t^2 + 2) \\ &= 4 + .5t - .01t^2 - 2 \\ &= -.01t^2 + .5t + 2 \end{aligned}$$

(b) Evaluate and interpret $S(45)$.

$$\begin{aligned} S(45) &= -.01(45)^2 + .5(45) + 2 \\ &= -.01(2025) + 22.5 + 2 \\ &= -20.25 + 22.5 + 2 \\ &= 4.25 \text{ million people} \end{aligned}$$

years after 2000

In 2045, the county could feed 4.25 million extra people.

(c) Evaluate and interpret $S(55)$.

$$\begin{aligned} S(55) &= -.01(55)^2 + .5(55) + 2 \\ &= -30.25 + 27.5 + 2 \\ &= -.75 \end{aligned}$$

In 2055, the county has a food deficit of $-.75$ million people.

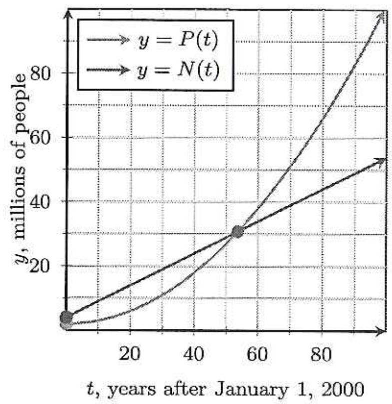
(d) Another way of measuring prosperity is to calculate the per capita food supply, $R(t)$, which is defined by $\dfrac{N(t)}{P(t)}$. Find and simplify $R(t)$.

(e) Evaluate and interpret $R(45)$.

(f) Evaluate and interpret $R(55)$.

(g) The two functions are graphed in Figure 1. Use your calculator to find this point of intersection. List each coordinate accurate to three decimal places. Then interpret their point of intersection.

FIGURE 1



Example 6. Let $f(x) = 5x^2 + 3x + 4$. Find and simplify each of the following:

(a) $f(x) + 2$
(b) $f(x + 2)$
(c) $f(3x)$
(d) $f(-x)$
(e) $-f(x)$
(f) $f(x + 2) - f(x)$

a) $f(x)+2 = 5x^2+3x+4+2$
$= 5x^2+3x+6$

b) $f(x+2) = 5(x+2)^2+3(x+2)+4$
$=5(x+2)(x+2)+3x+6+4$
$=5(x^2+2x+2x+4)+3x+10$

$=5(x^2+4x+4)+3x+10$
$=5x^2+20x+20+3x+10$
$=5x^2+23x+30$

c) $f(3x) = 5(3x)^2+3(3x)+4$
$=5(9x^2)+9x+4 = 45x^2+9x+4$

d) $f(-x) = 5(-x)^2+3(-x)+4$
$=5x^2-3x+4$

e) $-f(x) = -(5x^2+3x+4)$
$=-5x^2-3x-4$

f) $f(x+2)-f(x) = 5(x+2)^2+3(x+2)+4-(5x^2+3x+4)$
$=5(x+2)(x+2)+\cancel{3x}+6+\cancel{4}-5x^2-\cancel{3x}-\cancel{4}$
$=5(x^2+4x+4)+6-5x^2$
$=\cancel{5x^2}+20x+20+6-\cancel{5x^2}$
$=20x+26$

The difference quotient of a function f is defined by

$$\frac{f(x+h)-f(x)}{h}$$

Example 7. Find and completely simplify the difference quotient for $f(x) = -8x + 5$.

$$\frac{f(x+h)-f(x)}{h} = \frac{-8(x+h)+5-(-8x+5)}{h}$$

$$= \frac{\cancel{-8x}-8h+\cancel{5}+\cancel{8x}-\cancel{5}}{h}$$

$$= \frac{-8\cancel{h}}{\cancel{h}}$$

$$= -8, \quad h \neq 0$$

Example 8. Find and completely simplify the difference quotient for $f(x) = 3x^2 - 9x + 2$.

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-9(x+h)+2-(3x^2-9x+2)}{h}$$

$$= \frac{3(x+h)(x+h)-\cancel{9x}-9h+\cancel{2}-3x^2+\cancel{9x}-\cancel{2}}{h}$$

$$= \frac{3(x^2+xh+xh+h^2)-9h-3x^2}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2-9h-\cancel{3x^2}}{h}$$

$$= \frac{3h^2+6xh-9h}{h}$$

$$= 3h+6x-9, \quad h \neq 0$$

Group Work 2. Let $f(x) = 3x - 5$. Find and simplify each of the following:

(a) $f(x) - 15$
(b) $f(x - 15)$
(c) $f(4x)$
(d) $-f(x)$
(e) $f(-x)$
(f) $\frac{f(x+h)-f(x)}{h}$

a) $f(x) - 15 = 3x - 5 - 15$
$= 3x - 20$

b) $f(x-15) = 3(x-15) - 5$
$= 3x - 45 - 5$
$= 3x - 50$

c) $f(4x) = 3(4x) - 5$
$= 12x - 5$

d) $-f(x) = -(3x-5)$
$= -3x + 5$

e) $f(-x) = 3(-x) - 5$
$= -3x - 5$

f) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3(x+h) - 5 - (3x-5)}{h}$

$= \dfrac{\cancel{3x} + 3h \cancel{-5} \cancel{-3x} \cancel{+5}}{h}$

$= \dfrac{3\cancel{h}}{\cancel{h}}$

$= 3, \ h \neq 0$