Math 111, Mon, 4/4

Happy Square root day! 4/4/16

Please turn in Team Mission 1

Hand back checkpoints – talk about formatting

Q's on 1.2

New Material: 1.3 + supplement

Checkpoint 2 on Wednesday (1.2 + beginning of 1.3 even/odd)

Questions on 1.2

9h.

25.c. $f(x) = \dfrac{x+2}{x-6}$

If $f(x) = 2$, what is x?

$y = 2$

x	y

$$\cancel{(x-6)}\frac{(x+2)}{\cancel{x-6}} = 2(x-6)$$

$$x + 2 = 2x - 12$$

$$14 = x$$

If $f(x) = 2$, $x = 14$ $\quad (14, 2)$

3a mission

$$\left(\frac{g}{h}\right)(x) = \frac{g(x)}{h(x)}$$

$2x - 1 \geq 0$

$\cancel{2x}$

$$= \frac{2x-10}{\sqrt{2x-1}} \cdot \frac{(\sqrt{2x-1})}{(\sqrt{2x-1})}$$

$$\frac{2x-10}{(\sqrt{2x}-1)} \cdot \frac{(\sqrt{2x}+1)}{(\sqrt{2x}+1)}$$

$$= \frac{(2x-10)(\sqrt{2x-1})}{2x-1}$$

$$\sqrt{(2x-1)^2}$$

$$2x - 1 \neq 0$$

$$2x - 1 > 0$$

$$2x > 1$$

$$x > \frac{1}{2}$$

$\{x \mid x > \frac{1}{2}\}$ or $(\frac{1}{2}, \infty)$

For each function find and simplify $f(-x)$. What patterns do you notice?

a. $f(x) = 3x^4 - 6x^3 - 10x^2 + x - 3$

$$f(-x) = 3(-x)^4 - 6(-x)^3 - 10(-x)^2 + (-x) - 3$$
$$= 3x^4 - 6(-x^3) - 10x^2 - x - 3$$
$$= 3x^4 + 6x^3 - 10x^2 - x - 3$$

This is neither even, nor odd

b. $f(x) = -2x^4 - 7x^2 - 3$

$$f(-x) = -2(-x)^4 - 7(-x)^2 - 3$$
$$= -2x^4 - 7x^2 - 3$$

$f(-x) = f(x)$

This is an even function

c. $f(x) = 5x^3 + 4x$

$$f(-x) = 5(-x)^3 + 4(-x)$$
$$= -5x^3 - 4x$$
$$= -(5x^3 + 4x)$$
$$= -f(x)$$

This is called an odd function

Math 111 Lecture Notes

Section 1.3: Properties of Functions

A function f is **even** if for every x in the domain of f it holds that $f(-x) = f(x)$. Visually, an even function is *symmetric about the y-axis.*

A function f is **odd** if for every x in the domain of f it holds that $f(-x) = -f(x)$. Visually, an odd function is *symmetric about the origin.*

Example 1. Two classic examples of even and odd functions are $f(x) = x^2$ and $g(x) = x^3$, respectively, as shown in Figures 1 and 2 below.

Figure 1. Graph of $y = f(x)$

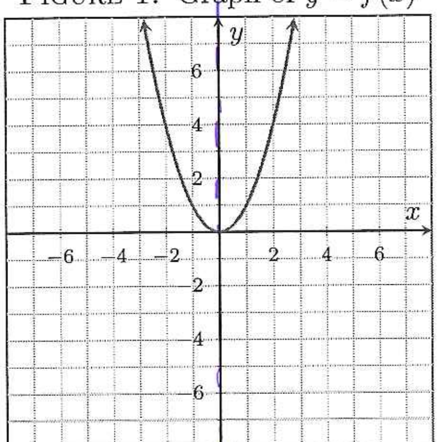
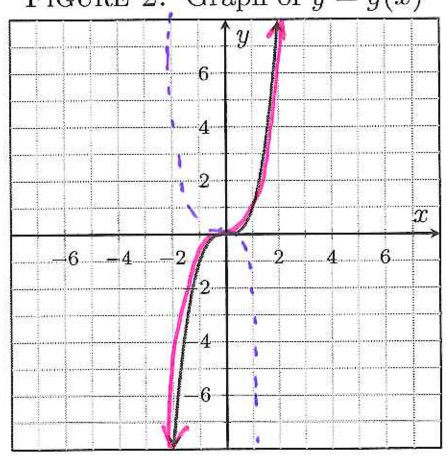


Figure 2. Graph of $y = g(x)$



Algebraically verify that f is an even function and that g is an odd function.

$$f(-x) = (-x)^2 = x^2 = f(x)$$

even

$$g(-x) = (-x)^3 = -x^3 = -f(x)$$

odd

Example 2. Algebraically determine if the following functions are even, odd or neither.

(a) $h(x) = x^3 - x^1$

$$h(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -h(x)$$

odd

(b) $g(t) = \frac{1}{2}t^4 - 1x^0$ — even

$$g(-t) = \tfrac{1}{2}(-t)^4 - 1$$
$$= \tfrac{1}{2}t^4 - 1$$
$$= g(t)$$

even

(c) $f(t) = t^3 + 1x^0$ ← even

$$f(-t) = (-t)^3 + 1$$
$$= -t^3 + 1$$

neither

$|-3|$
$= 3$

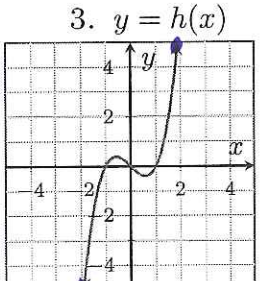
(d) $f(x) = |x| - 4$

$$f(-x) = |-x| - 4$$
$$= |x| - 4 \leftarrow$$
$$= f(x)$$

even

$f(-x) = f(x)$

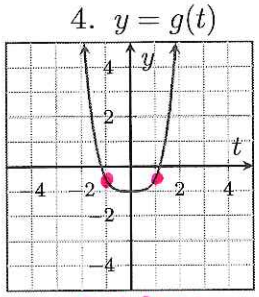
FIGURE 3. $y = h(x)$



odd

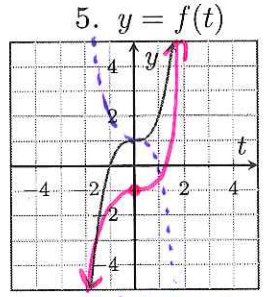
$f(-x) = f(x)$

FIGURE 4. $y = g(t)$



even

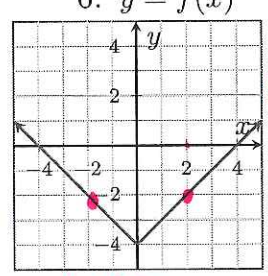
FIGURE 5. $y = f(t)$

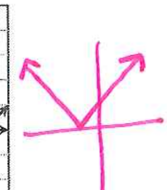


neither

$f(-x) = f(x)$

FIGURE 6. $y = f(x)$





even

Example 3. Algebraically determine if the function f defined by $f(x) = -\dfrac{2x^3 - x}{3x^4 + 5x^2}$ is even, odd or neither.

$$f(-x) = -\frac{2(-x)^3 - (-x)}{3(-x)^4 + 5(-x)^2}$$

$$= -\frac{-2x^3 + x}{3x^4 + 5x^2}$$

$$= \frac{2x^3 - x}{3x^4 + 5x^2}$$

$$= -f(x)$$

odd

Group Work 1. Determine if the following functions are even, odd or neither.

(a) $g(x) = \dfrac{x^2}{x^4 + 5}$

$$g(-x) = \frac{(-x)^2}{(-x)^4 + 5}$$

$$= \frac{x^2}{x^4 + 5}$$

$$= g(x)$$

even

(b) $f(x) = 5x^3 + 3x^2$

$$f(-x) = 5(-x)^3 + 3(-x)^2$$

$$= -5x^3 + 3x^2$$

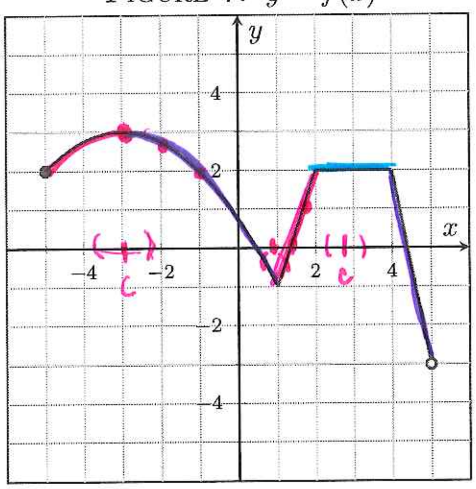
neither

A function f is **increasing** on an open interval I if for every x_1 and x_2 in I with $\boxed{x_1 < x_2}$ we have $f(\overset{y_2}{x_2}) > f(\overset{y_1}{x_1})$.
x_1 is to the left of x_2
A function f is **decreasing** on an open interval I if for every x_1 and x_2 in I with $x_1 < x_2$ we have $f(x_2) < f(x_1)$.
A function f is **constant** on an open interval I if for every x_1 and x_2 in I with $x_1 < x_2$ we have $f(x_2) = f(x_1)$.

Example 4. Determine the following for the function f graphed in Figure 7. State each using interval notation.

use open intervals for inc, dec constant.

FIGURE 7. $y = f(x)$



(a) Increasing: $(-5,-3) \cup (1,2)$

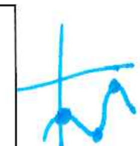
(b) Decreasing: $(-3,1) \cup (\cancel{3}\,4,5)$

(c) Constant: $(2,4)$

(d) Domain of f: $[-5,5)$

(e) Range of f: $(-3,3]$

A function has a **local maximum** at $\overset{x\text{-value}}{c}$ if there exists an open interval I containing c so that for all x not equal to c in I, it holds that $f(x) < f(c)$. The output $f(c)$ is referred to as the **local maximum** of f.
A function has a **local minimum** at c if there exists an open interval I containing c so that for all x not equal to c in I, it holds that $f(x) > f(c)$. The output $f(c)$ is referred to as the **local minimum** of f.



Example 5. Use Figure 7 to answer the following:

(a) Identify all local $\underbrace{\text{maximum values of } f}_{y}$ and state $\underbrace{\text{where they occur.}}_{x}$

$(-3,3)$
$f(-3)=3$

There is a local max of 3 at $x = -3$.

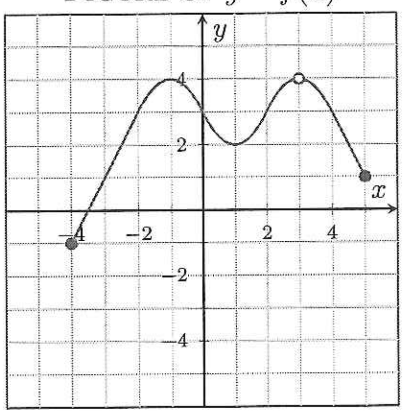
(b) Identify all local $\underbrace{\text{minimum values of } f}_{y}$ and state $\underbrace{\text{where they occur.}}_{x}$

There is a local minimum of -1 at $x = 1$.

Let f be a function defined on an closed interval I.
A function has an **absolute maximum** at u if it holds that $f(x) \leq f(u)$ for all x in the interval I. The output $f(u)$ is referred to as the **absolute maximum** of f.
A function has an **absolute minimum** at u if it holds that $f(x) \geq f(u)$ for all x in the interval I. The output $f(u)$ is referred to as the **absolute minimum** of f.

Example 6. Use Figure 8 to answer the following:

FIGURE 8. $y = f(x)$



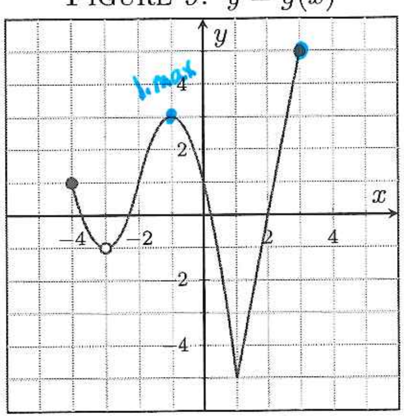
(a) Identify all absolute maximum values of f and state where they occur. There is an absolute max ~~at~~ of 4 at $x=-1$

(b) Identify all absolute minimum values of f and state where they occur. There is an absolute ~~max~~ minimum ~~at~~ of -1 at $x=-4$.

Group Work 2. Use Figure 9 to answer the following:

(a) Identify all local maximum values of g and state where they occur. local maximum of 3 at $x=-1$

FIGURE 9. $y = g(x)$



(b) Identify all local minimum values of g and state where they occur. local minimum of -5 at $x=1$

(c) Identify all global absolute maximum values of g and state where they occur.
absolute max of 5 at $x=3$.

(d) Identify all absolute minimum values of g and state where they occur.
absolute minimum of -5 at $x=1$.

Concavity

So far, we have looked at where a function is increasing and decreasing and where it attains maximum and minimum values. We will now study the concept of *concavity.* This concept involves looking at the rate at which a function increases or decreases.

The graph of a function f whose rate of change increases (becomes less negative or more positive as you move left to right) over an interval is **concave up** on that interval. Visually, the graph "bends upward."

The graph of a function f whose rate of change decreases (becomes less positive or more negative as you move left to right) over an interval is **concave down** on that interval. Visually, the graph "bends downward."

FIGURE 10. Concave UP ☺

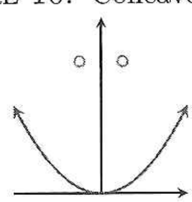
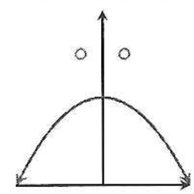


FIGURE 11. Concave DOWN ☹



Example 7. The function defined by $f(x) = x^2$ is concave up on its entire domain. Notice that it is **decreasing** on the interval $(-\infty, 0)$ and **increasing** on the interval $(0, \infty)$. The function defined by $f(x) = -x^2$ is concave down on its entire domain. Notice that it is **increasing** on the interval $(-\infty, 0)$ and **decreasing** on the interval $(0, \infty)$.

FIGURE 12. Graph of $y = x^2$

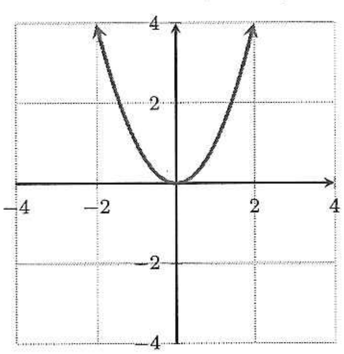
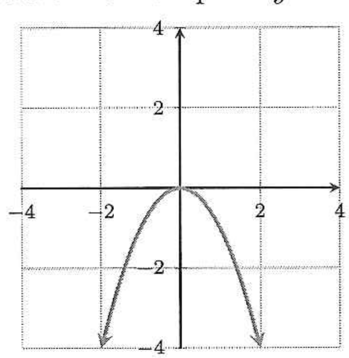


FIGURE 13. Graph of $y = -x^2$



EXAMPLE 3: Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

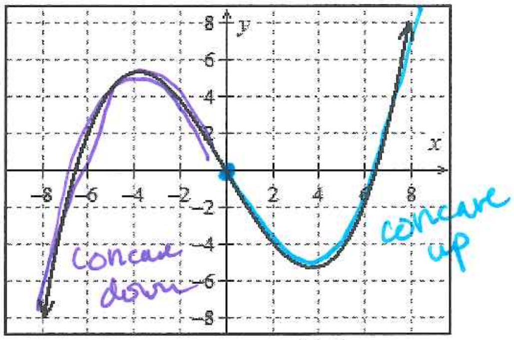


Figure 7: $y = f(x)$

b.

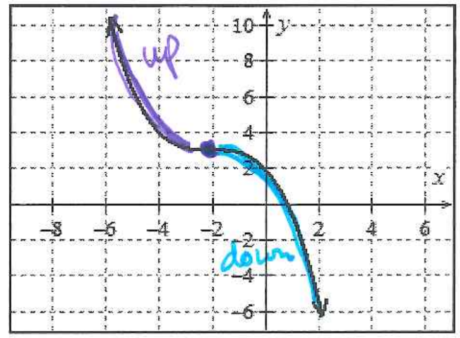


Figure 8: $y = g(x)$

open intervals

Solution: **a.** f is concave up on the interval $(0, \infty)$ and concave down on the interval $(-\infty, 0)$.

b. g is concave up on the interval $(-\infty, -2)$ and concave down on the interval $(-2, \infty)$.

EXERCISES:

1. Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

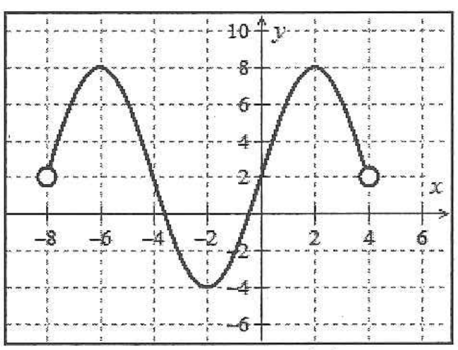


Figure 9: $y = r(x)$

b.

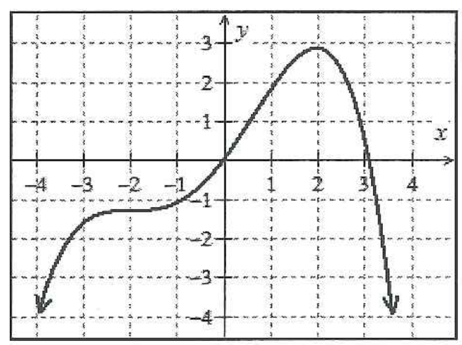


Figure 10: $y = s(x)$

c.

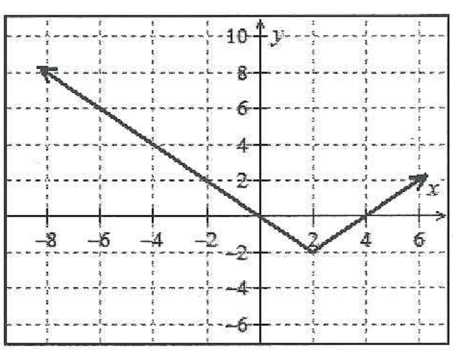


Figure 11: $y = t(x)$

d.

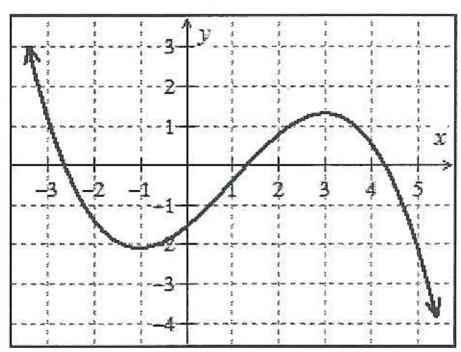
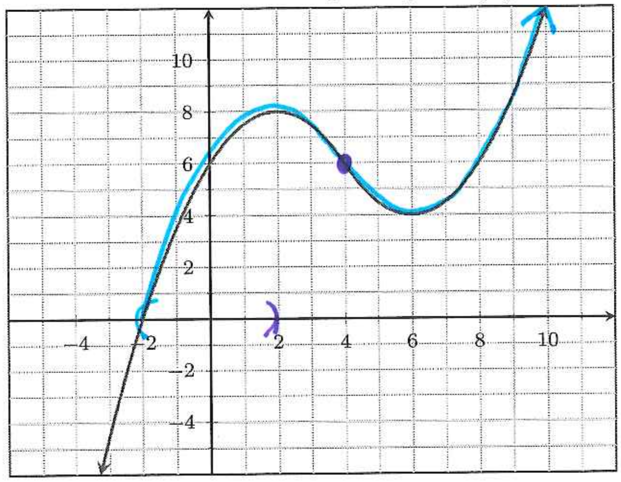


Figure 12: $y = w(x)$

Example 8. The graph of $y = h(x)$ is shown in Figure 14. Use this to answer the following.

FIGURE 14. Graph of $y = h(x)$



(a) State the interval(s) where h is positive.

↑ y-value $(-2, \infty)$ $\boxed{\text{x-intervals}}$

(b) State the interval(s) where h is negative.

$(-\infty, -2)$

(c) State the interval(s) where h is increasing.

$(-\infty, 2) \cup (6, \infty)$

(d) State the interval(s) where h is decreasing.

$(2, 6)$

(e) State the interval(s) where h is concave up.

$(4, \infty)$

(f) State the interval(s) where h is concave down.

$(-\infty, 4)$

(g) State any absolute maximum or absolute minimum values for h and where they occur.

none

absolute highest + lowest.

(h) State any local maximum or local minimum values for h and where they occur.

local max of 8 at $x=2$.

local peaks + valleys.

local min of 4 at 6.

Example 9. Graph the function defined by $k(x) = 2x^4 - 6x^3 - 6x^2 + 22x + 2$ on your calculator.

(a) Determine an appropriate window that shows the important features (such as the x-intercept(s), y-intercept, and any local maxima or minima).

xmin = −5 ymin = −20
xmax = 5 ymax = 20
xscl = 1 yscl = 5

(b) Use the MAXIMUM and MINIMUM features to find any local maxima and minima and where they occur.

local min of −18.612 at $x \approx -1.147$
local min of 3.651 at $x \approx 2.397$
local max of 14 at $x = 1$

(c) (Review) Use the ZERO feature and the VALUE feature to determine the x-intercepts and y-intercept.

x-intercepts ≈ (−1.817, 0) and (−.0889, 0)

y-intercept = (0, 2)