Math 111, Monday, 4/11

Q's on 1.4

New material: 1.5 (part 1)

Mission 2 handed out – individual

Checkpoint 3 on Wed. (1.3 + 1.4)

1.5 Practice Homework: (re-structured)

Day 1 – Single transformations + 1 horiz + 1 vert

19–25 odd, 31–35 odd, 41, 43, 51, 55,

83, 85, 87, 91, 93

Day 2 – Order of Transformations

7–18 odd, 27, 29, 57, 59, 63, 95

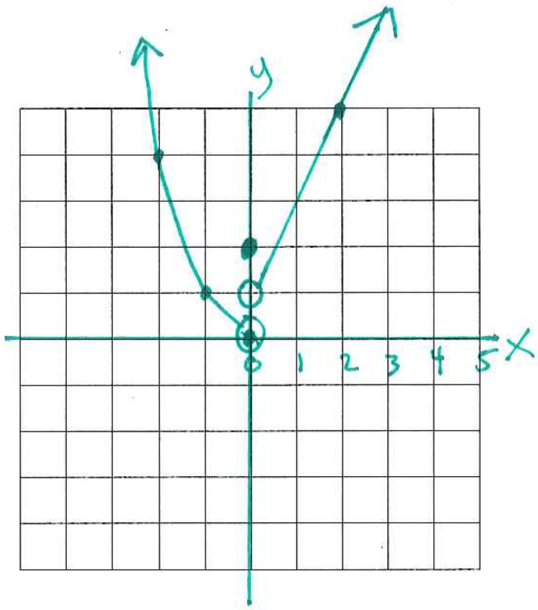
Completing the square (any time)

73–81 odd

+ supplement (day 2)

1.4 Questions

25. $f(x) = \begin{cases} x^2 \text{ if } x<0 \\ 2 \text{ if } x=0 \\ 2x+1 \text{ if } x>0 \end{cases}$



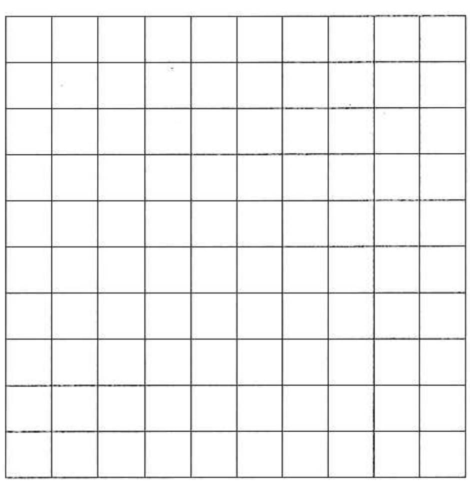
x^2

x	y
-3	9
-2	4
-1	1
0	0

$2x+1$

x	y
0	1
2	5

(0,2)



Math 111 Lecture Notes

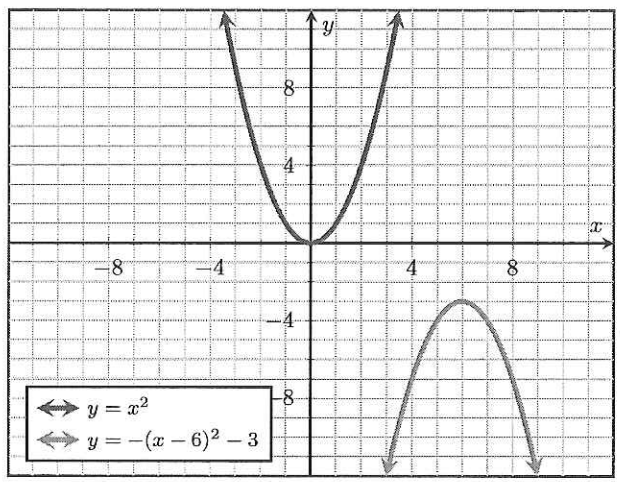
Section 1.5: Function Transformations

In this section, we will explore *function transformations.* We will explore these numerically (in tabular form), algebraically (as formulas), and graphically. When you studied the vertex form of a parabola, you were actually studying function transformations for a specific function—namely, $f(x) = x^2$. For example, when graphing $y = -(x-6)^2 - 3$, you know that the graph points downward and that the vertex is $(6, -3)$.

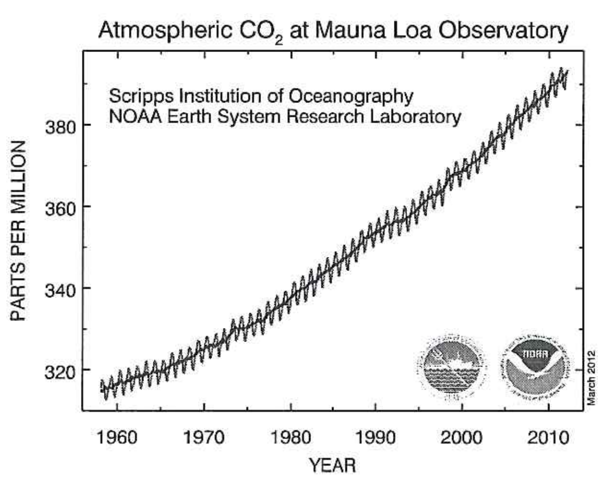
$(x - h)$ K

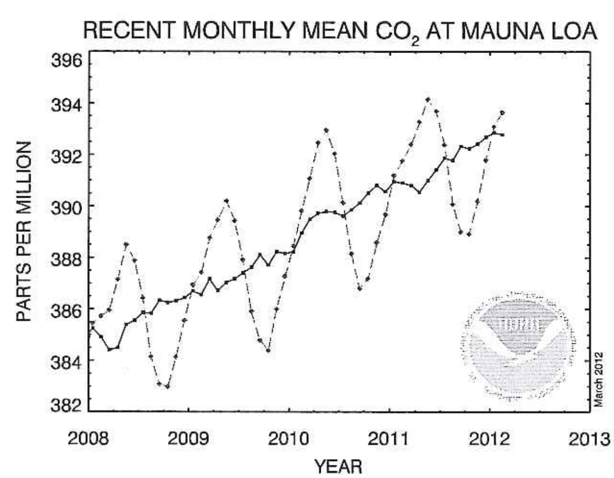
vertex $(6,-3)$

Figure 1



We could also say that the graph is reflected about the x-axis, shifted right 6 units, and then shifted down 3 units. In this course, we will be able to apply similar transformations to any function—not just parabolas! One such example is shown below. ☺





http://www.esrl.noaa.gov/gmd/ccgg/trends/

Let $y = f(x)$, where x is the number of months after January 1, 2011 and $f(x)$ is the amount of CO_2 in the atmosphere after x months. We will measure $f(x)$ in parts per million above 380 and restrict x to $-3 \leq x \leq 9$. The data for September 2010 through September 2011 is shown in Figure 2.

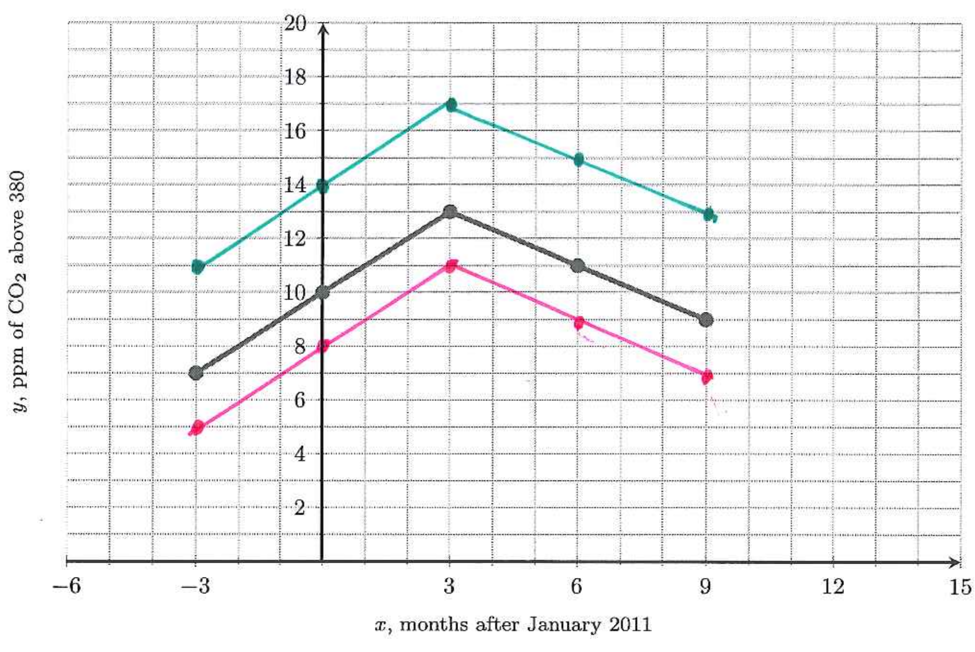
Vertical Shifts

Example 1. Complete Table 1 using the function values for f. What happens to the graph in each case? Sketch and label the graph of $y = f(x) + 4$ and the graph of $y = f(x) - 2$ in Figure 2.

Table 1

x	-3	0	3	6	9
$f(x)$	7	10	13	11	9
$f(x)+4$	11	14	17	15	13
$f(x)-2$	5	8	11	9	7

Figure 2



Summary of Vertical Shifts outside

The graph of $y = f(x) + k$ is transformation of the graph of $y = f(x)$.

- If $k > 0$, then the graph of the original function shifts up by k units.
- If $k < 0$, then the graph of the original function shifts down by k units.

Horizontal Shifts

Horizontal shifts are not quite as straightforward as vertical shifts. The primary reason is that in order to shift the graph horizontally, we need to add or subtract from x—*before* we evaluate the function. The end result is that horizontal transformations work a bit backwards from what you may expect, as we will discover in the example below.

Example 2. Complete Table 2 using the function values for f. What happens to the graph in each case? Sketch and label the graph of $y = f(x+3)$ and the graph of $y = f(x-6)$ in Figure 3.

Table 2

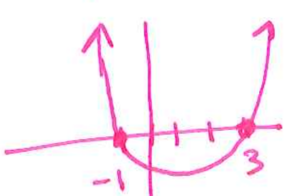
x	-6	-3	0	3	6	9	12	15
$f(x)$	und.	7	10	13	11	9	und.	und.
$f(x+3)$	$f(-6+3)$ $f(-3)=7$	$f(0)=$ 10	$f(3)$ $=13$	11	9	und	und	und.
$f(x-6)$	undef	undef	undef	7	10	13	11	9

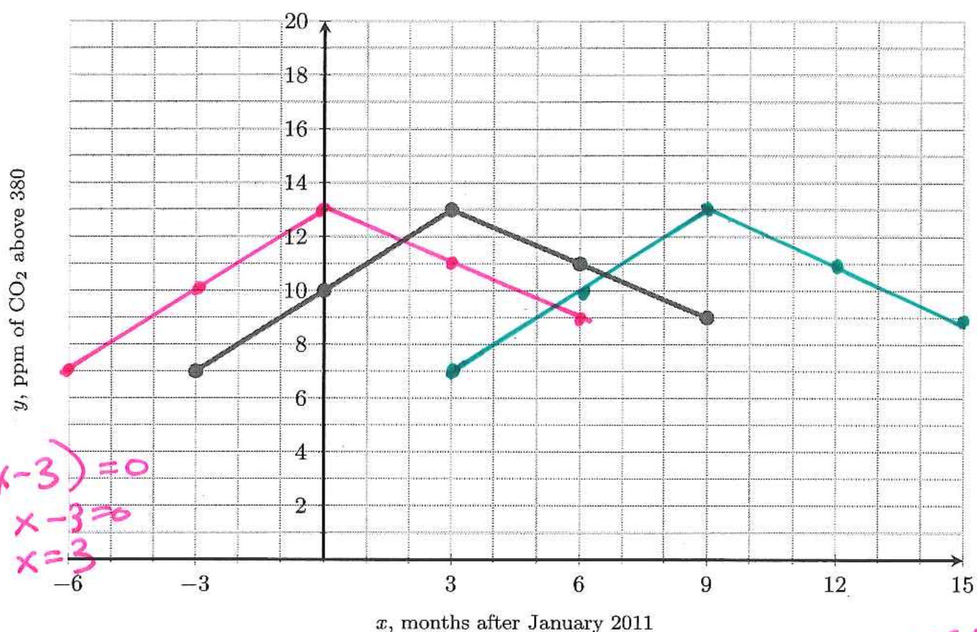
left 3

right 6

Figure 3

why is it opposite?





$(x+1)(x-3)=0$

$x+1=0 \quad x-3=0$

$x=-1 \quad x=3$

inside change → opposite

Summary of Horizontal Shifts

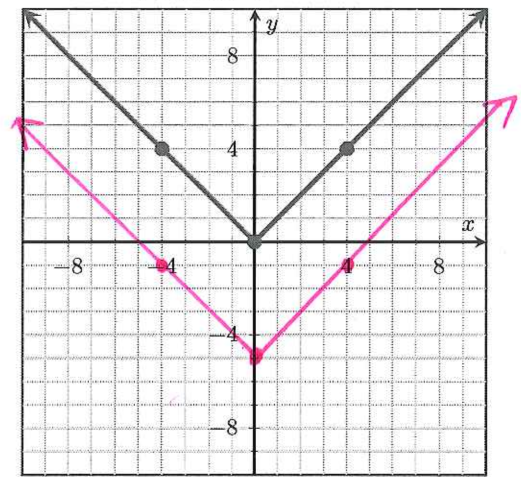
The graph of $y = f(x+h)$ is transformation of the graph of $y = f(x)$.

- If $h > 0$, then the graph of the original function shifts left by h units.
- If $h < 0$, then the graph of the original function shifts right by h units.

Example 3. For each function below, the "original" or "basic" function is $y = |x|$. Use the properties of horizontal and vertical shifts to graph the stated transformations. The full graph and 3 key points are given in each.

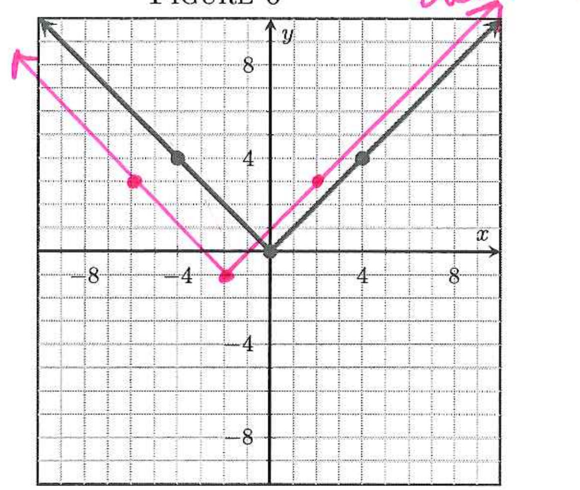
(a) Graph $y = |x| - 5$. outside vertical down 5

FIGURE 4



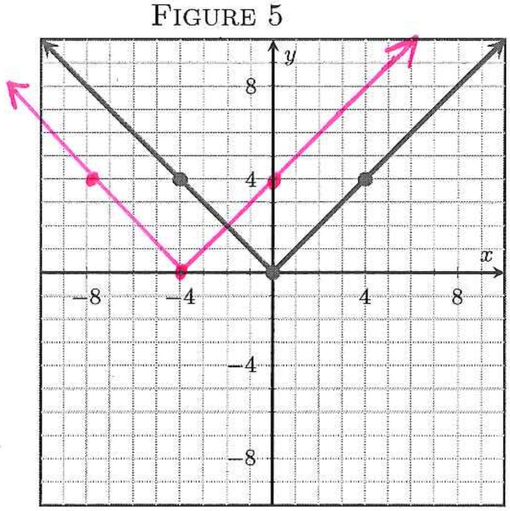
(c) Graph $y = |x + 2| - 1$. left 2 down 1

FIGURE 6



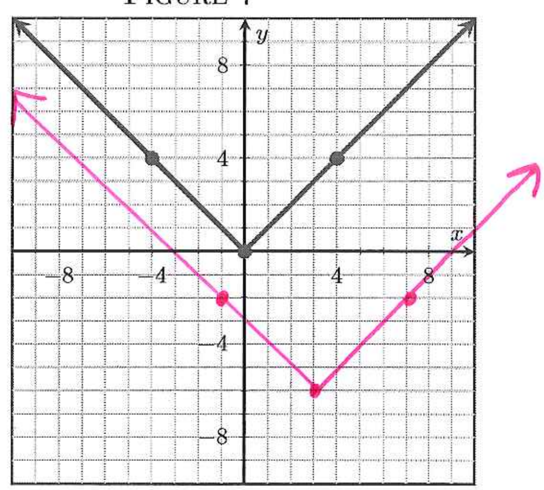
(b) Graph $y = |x + 4|$. inside shift left 4

FIGURE 5



(d) Graph $y = |x - 3| - 6$. right 3 down 6

FIGURE 7



Vertical Stretches and Compressions

Example 4. Assume the base temperature setting for the thermostat in a house is 64°F. Let $g(x)$ be the number of degrees above 64°F x hours after 6AM. Complete Table 3 using the function values for g. What happens to the graph in each case? Sketch and label the graph of $y = 2g(x)$ in Figure 8 and the graph of $y = \frac{1}{2}g(x)$ in Figure 9.

TABLE 3

x	-2	0	4	7	8
$g(x)$	-2	6	6	0	-2
$2g(x)$	-4	12	12	0	-4
$\frac{1}{2}g(x)$	-1	3	3	0	-1

y-value × 2 →

y-value × ½ →

FIGURE 8 2g(x)

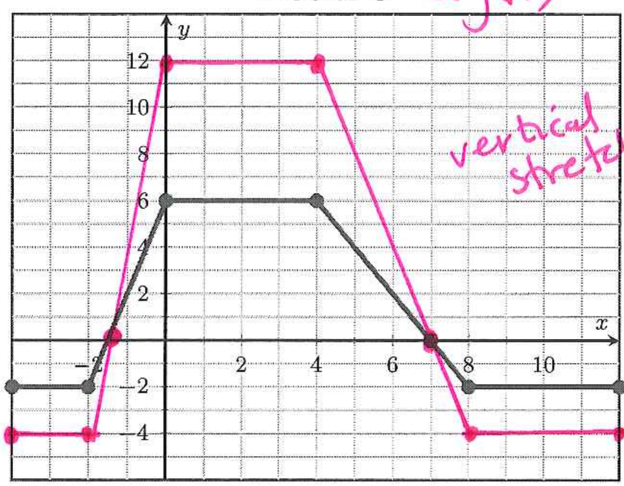
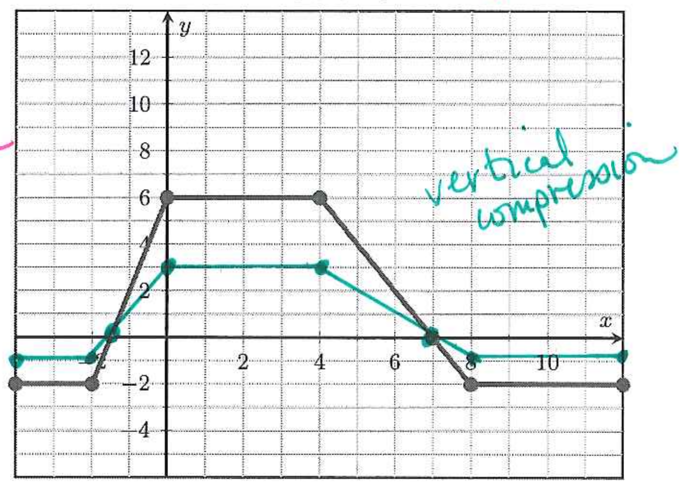


FIGURE 9 ½g(x)



outside

Summary of Vertical Stretches and Compressions

The graph of $y = Af(x)$ is transformation of the graph of $y = f(x)$. If

- If $|A| > 1$, then the graph of the original function Stretches vertically by a factor of $|A|$.
- If $0 < |A| < 1$, then the graph of the original function compress vertically by a factor of $|A|$.

Horizontal Stretches and Compressions

Horizontal stretches and compressions, much like horizontal shifts, work in a somewhat counterintuitive way. This again is a result of the fact that we will multiply x by a number *before* we evaluate the function.

Example 5. The graph of $y = h(x)$ is shown below. Complete Table 4 and then graph $y = h\left(\frac{1}{2}x\right)$ in Figure 10.

Table 4

x	-12	-8	-6	-4	0	2	4	8	16
$h(x)$	und.	und.	0	0	4	0	-4	6	und.
$h\left(\frac{1}{2}x\right)$	0	0	1	2	4	2	0	-4	6

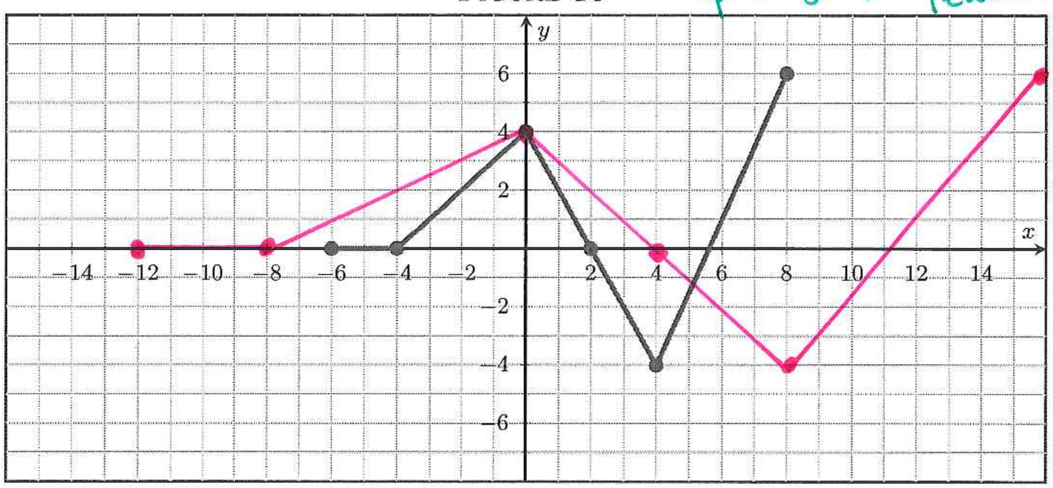
horizontal stretch × 2

h(-6)

from the graph

if no graph, leave blank.

Figure 10



Example 6. The graph of $y = h(x)$ is shown below. Complete Table 5 and then graph $y = h(4x)$ in Figure 11. An "X" is placed where the function is defined but difficult to evaluate.

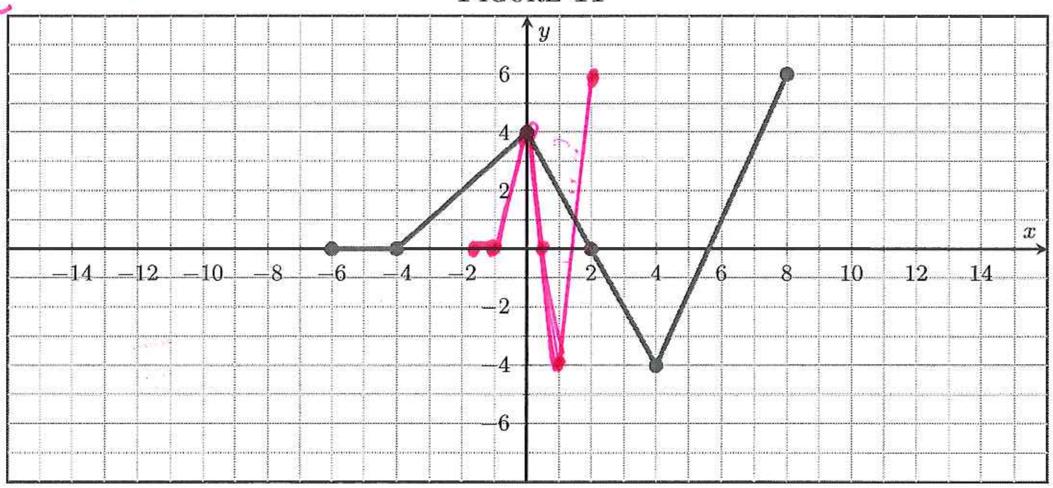
TABLE 5

x	-6	-4	-1.5	-1	0	0.5	1	2	4	8
$h(x)$	0	0	X	3	4	X	2	0	-4	6
$h(4x)$	undef	undef	0	0	4	0	-4	6	undef	undef

or blank

horizontal compression by $\frac{1}{4}$

FIGURE 11



inside

Summary of Horizontal Stretches and Compressions

The graph of $y = f(Bx)$ is transformation of the graph of $y = f(x)$.

- If $|B| > 1$, then the graph of the original function compresses horizontally by a factor of $\frac{1}{|B|}$.
- If $0 < |B| < 1$, then the graph of the original function stretches horizontally by a factor of $\frac{1}{|B|}$.

Horizontal and Vertical Reflections

Example 7. The graph of $y = h(x)$ is shown below. Complete Table 6 and then graph $y = -h(x)$ in Figure 12 and graph $y = h(-x)$ in Figure 13.

Table 6

x	-8	-6	-4	-2	0	2	4	8
$h(x)$	und.	0	0	2	4	0	-4	6
$-h(x)$	und	0	0	-2	-4	0	4	-6
$h(-x)$	6		-4	0	4	2	0	undefined

outside vertical

inside horizontal

vertical flip over the x-axis

horizontal flip over the y-axis

Figure 12

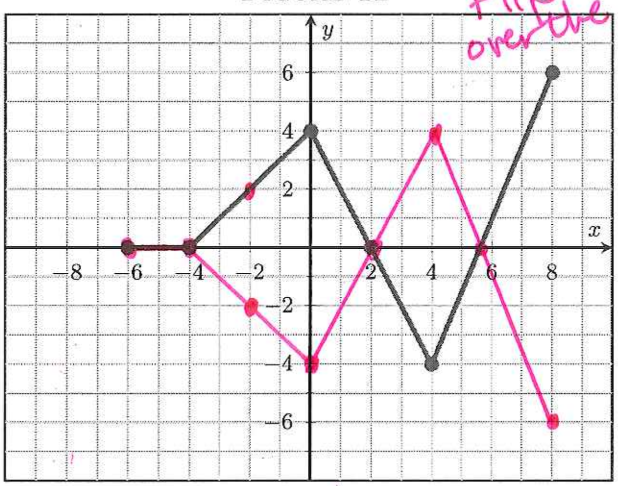
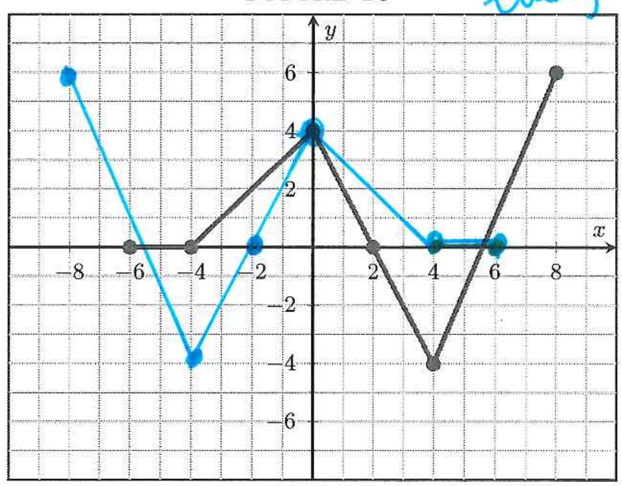


Figure 13



outside vertical

Summary of Horizontal and Vertical Reflections

- The graph of $y = -f(x)$ is transformation of the graph of $y = f(x)$. It reflects the graph of the original function across the ___x-___ axis.
- The graph of $y = f(-x)$ is transformation of the graph of $y = f(x)$. It reflects the graph of the original function across the ___y-___ axis.

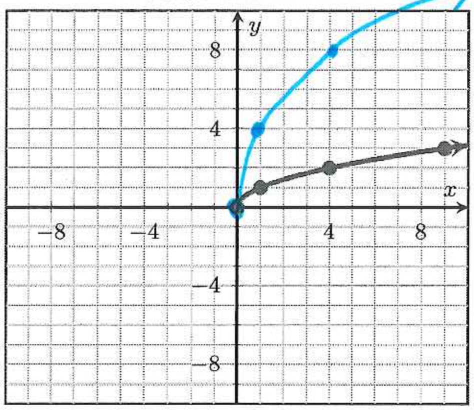
inside horizontal

vertical shift up or down	$f(x)+h$, $f(x)-h$
horizontal shift ~~up~~ or ~~down~~ left or right	$f(x+h)$, $f(x-h)$
vertical stretch or compression	$3f(x)$, $\frac{1}{3}f(x)$
horizontal stretch or compression	$f(\frac{1}{3}x)$, $f(3x)$
vertical flip	$-f(x)$
horizontal flip	$f(-x)$

Example 8. For each function below, the "original" or "basic" function is $y = \sqrt{x}$. Use the properties of horizontal and vertical stretches and compressions to graph the stated transformations. The full graph and 4 key points are given in each.

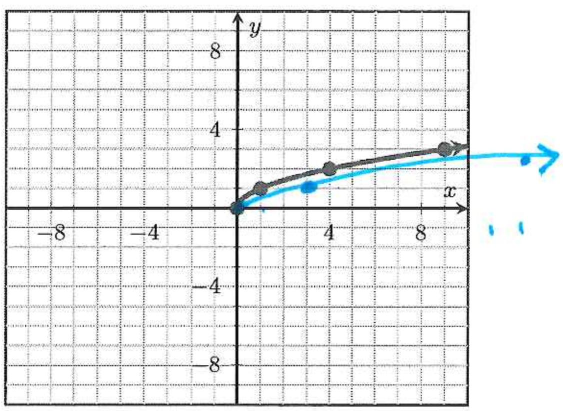
(a) Graph $y = 4\sqrt{x}$. $4f(x)$ vertical stretch by 4

FIGURE 14



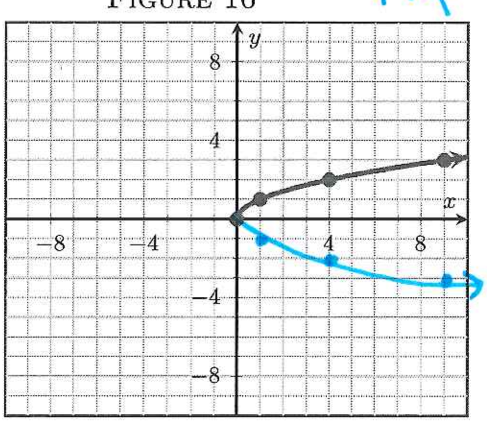
(b) Graph $y = \sqrt{\frac{1}{3}x}$. horizontal stretch by 3

FIGURE 15



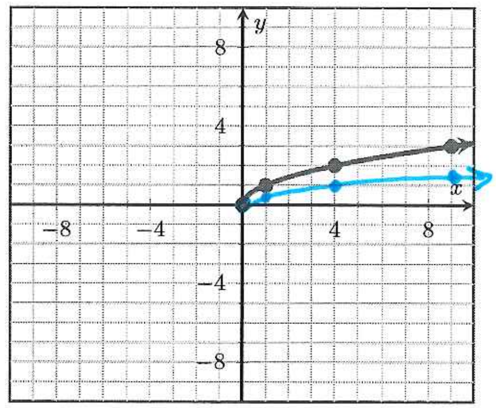
(c) Graph $y = -\sqrt{x}$. vertical flip

FIGURE 16



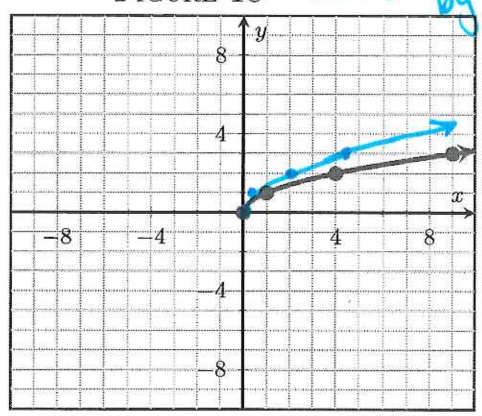
(d) Graph $y = \frac{1}{2}\sqrt{x}$. $\frac{1}{2}f(x)$ vertical compression by $\frac{1}{2}$

FIGURE 17



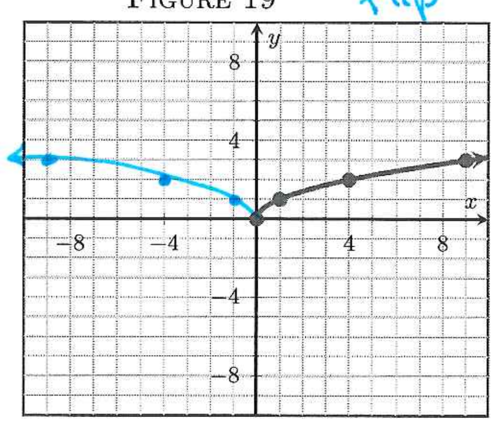
(e) Graph $y = \sqrt{2x}$. horizontal compression by $\frac{1}{2}$

FIGURE 18



(f) Graph $y = \sqrt{-x}$. horizontal flip

FIGURE 19



$\sqrt{x}$

Example 9. The point $(4,12)$ is on the graph of $y=f(x)$. Determine the point on the graph of...

(a) $y=f(x+2)-1$

(4,12)

shift left 2 (2,12)

shift down 1 (2,11)

(d) $y=f\left(\frac{1}{3}x\right)$

(4,12)

horizontal stretch by 3 (12,12)

(b) $y=5f(x)$

(4,12)

vertical stretch by 5 (4,60)

y-values multiplied by 5

(e) $y=f(-x)-5$

(4,12)

horizontal flip (-4,12)

down 5 (-4,7)

(c) $y=-f(x-5)+4$

(4,12)

vertical flip (4,-12)

Right 5 (9,-12)

up 4 (9,-8)

(f) $y=2f(4(x+1))-3$

vertical stretch by 2

Horizontal compression by $\frac{1}{4}$

Left 1

Down 3