Math 111 - Wed, 5/11

Please turn in your project

Questions on 4.4 + 4.5

Checkpoint 6

New material : 4.7 + 4.8 (Applications)

Boss #2 on Monday, 1:00 - 3:20

Part 1 - No calculator

Part 2 - Graphing calculator needed

Review packet handed out today

- Solutions will be posted in MyPCC x friday

Math + 3D Printing - this Saturday + 6/4

Chap 4 review

18
25
27
35
45
41.

18. $H(x) = \log_2(x^2-3x+2)$ Domain

$$x^2-3x+2 > 0$$
$$(x-2)(x-1) > 0$$

$(x-2)(x-1)=0$
$x=2$ or $x=1$

$$D: (-\infty, 1) \cup (2, \infty)$$

$$G(x) = \log_3(3x-1)$$

$$3x-1 > 0$$
$$+1 \quad +1$$
$$\frac{3x}{3} > \frac{1}{3}$$
$$x > \frac{1}{3}$$

25. $\ln\left(\frac{(2x+3)}{x^2-3x+2}\right)^2$

$$2\left[\ln\left(\frac{(2x+3)}{(x-1)(x-2)}\right)\right]$$

$$2\ln(2x+3) - 2\left[\ln((x-1)(x-2))\right]$$
$$-2\left[\ln(x+1) + \ln(x-2)\right]$$
$$2\ln(2x+3) - 2\ln(x-1) - 2\ln(x-2)$$

27. $\ln\left(\frac{x-1}{x}\right) + \ln\left(\frac{x}{x+1}\right) - \ln(x^2-1)$

$$= \ln\left(\frac{x-1}{\cancel{x}} \cdot \frac{\cancel{x}}{x+1}\right) - \ln\left(\frac{(x+1)(x-1)}{1}\right)$$

$$= \ln\left(\frac{\cancel{(x-1)}}{(x+1)}\cdot\frac{1}{(x+1)\cancel{(x-1)}}\right)$$

$$= \ln\left(\frac{1}{(x+1)^2}\right)$$

$$= \ln(x+1)^{-2}$$

$$= -2\ln(x+1)$$

25 backwards

$$2\ln(2x+3)-2\ln(x-1)-2\ln(x-2)$$

$$2\left[\ln(2x+3)-\ln(x-1)\right]-2\ln(x-2)$$

$$2\ln\left(\frac{2x+3}{x-1}\right)-2\ln(x-2)$$

$$2\left[\ln\left(\frac{2x+3}{x-1}\right)-\ln(x-2)\right]$$

$$2\ln\left(\frac{(2x+3)}{(x-1)(x-2)}\right)$$

35.

$$8^{6+3x}=4$$

$$(2^3)^{6+3x}=2^2$$

$$2^{18+9x}=2^2$$

$$18+9x=2$$

$$\log_2 8^{6+3x}=\log_2 4$$

$$(6+3x)\log_2 8=\log_2 4$$

$$3(6+3x)=2$$

$$18+9x=2$$

4.5 #47.

$$\ln(x^2\sqrt{1-x})$$

$$\ln x^2 + \ln\sqrt{1-x}$$

$$2\ln x + \frac{1}{2}\ln(1-x)$$

Math 111 Lecture Notes

Section 4.7: Compound Interest

This section has *a lot* of formulas. You do not have to memorize the formulas in this section—the two you will need to use are given below and will be provided on any exams.

Compound Interest Formula:
The amount A after t years due to a principal P invested at an $\boxed{\text{annual interest rate } r}$ compounded n times per year is

initial investment

$$A = P \cdot \underbrace{\left(1 + \frac{r}{n}\right)}_{\text{growth factor}}^{nt}$$

Continuous Interest Formula:
The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt}$$

Example 1. You invest $\boxed{\$3{,}000}$ P into a bank account. For each interest rate below, write the general formula and compute the value of the investment after 7 years.

- 5% compounded quarterly $n=4$

  $.05$

  $$A = 3000\left(1+\frac{.05}{4}\right)^{4\cdot 7}$$
  $$= \$4{,}247.98$$

- 5% compounded monthly $n=12$

  $$A = 3000\left(1+\frac{.05}{12}\right)^{12\cdot 7}$$
  $$= 3000\left(1+\frac{.05}{12}\right)^{84}$$
  $$= \$4{,}254.11$$

- 5% compounded daily $n=365$

  $$A = 3000\left(1+\frac{.05}{365}\right)^{365\cdot 7}$$
  $$= \$4{,}257.10$$

Example 2. Complete Table 1 using the previous examples.

TABLE 1

Compounding Frequency	Annual Growth Factor	Effective Annual Rate
Annual	1.05	5%
Quarterly	$\left(1+\frac{.05}{4}\right)^4 = 1.05095$ -1	5.095%
Monthly	$\left(1+\frac{.05}{12}\right)^{12} = 1.05116$ -1	5.116%
Daily	$\left(1+\frac{.05}{365}\right)^{365} = 1.05127$ 674	5.12674%
Continuously	$e^{.05} = 1.051271096$	5.1271096%

Example 3. Now assume that you have $1 and it earns 100% annual interest. Table 2 shows the growth factor for each of the compounding frequencies listed. (This is utterly silly in reality–but will show you exactly where e comes from!!)

TABLE 2

Compounding Frequency	Annual Growth Factor
Annual	$\left(1+\frac{1}{1}\right)^1 = 2$
Semi-annual	$\left(1+\frac{1}{2}\right)^2 \approx 2.25$
Quarterly	$\left(1+\frac{1}{4}\right)^4 \approx 2.441406$
Monthly	$\left(1+\frac{1}{12}\right)^{12} \approx 2.613035$
Daily	$\left(1+\frac{1}{365}\right)^{365} \approx 2.714567$
Hourly	$\left(1+\frac{1}{8760}\right)^{8760} \approx 2.718127$
Each minute	$\left(1+\frac{1}{525600}\right)^{525600} \approx 2.718279$
Each second	$\left(1+\frac{1}{31536000}\right)^{31536000} \approx 2.718282$
Continuously	$e^1 \approx 2.718282$

Example 4. You invest \$5,000 into an account that earns 2.25% interest compounded continuously.

(a) Write the formula that models the value of this investment after t years. $A = Pe^{rt}$

$$A = 5000e^{.0225t}$$

$$A(t) = 5000e^{.0225t}$$

(b) What will the value of the account be after 5 years?

$$A(5) = 5000e^{.0225(5)}$$

$$= \$5,595.36$$

(c) How long will it take for the account value to double?

$$A(t) = 5000e^{.0225t}$$

$$\frac{10,000}{5000} = \frac{5000e^{.0225t}}{5000}$$

$$2 = e^{.0225t}$$

$$\ln 2 = \ln e^{.0225t}$$

$$\frac{\ln 2}{.0225} = \frac{.0225t}{.0225}$$

$$30.81 = t$$

It would take about 30.81 years.

or

$$\ln 2 = .0225t$$

Effective Rate:
$r_{eff} = \left(1+\frac{r}{n}\right)^n - 1 \qquad r_{eff} = e^r - 1$

The **effective rate of interest** is the equivalent annual simple interest that would yield the same amount as compounding n times per year, or continuously, after 1 year.

Example 5. Determine which of the following interest rates for an investment is a better deal:

- 6% compounded monthly

$$r_{eff} = \left(1+\frac{.06}{12}\right)^{12} - 1$$
$$\approx .061678$$
$$\approx 6.1678\%$$

- 5.95% compounded continuously

$$r_{eff} = e^{.0595} - 1$$
$$\approx .061306$$
$$\approx 6.1306\%$$

Group Work 1. Determine which of the following interest rates for an investment is a better deal:

- 9% compounded quarterly

$$r_{eff} = \left(1+\frac{.09}{4}\right)^{4} - 1$$
$$\approx .093083$$
$$\approx 9.3083\%$$

- 8.95% compounded continuously e

$$r_{eff} = e^{.0895} - 1$$
$$\approx .093627$$
$$\approx 9.3627\%$$

Example 6. What interest rate (compounded continuously) is required for the value of an investment to double in $\underbrace{15 \text{ years}}_{=t}$?

$$A = Pe^{rt}$$

$$\frac{2P}{P} = \frac{Pe^{r\cdot 15}}{P}$$

$$2 = e^{15r}$$

$$\ln 2 = \ln e^{15r}$$

$$\frac{\ln 2}{15} = \frac{15r}{15}$$

$$r \approx .0462$$

To double in 15 years, the account needs a continuous annual rate of about 4.62%

Example 7. What interest rate (compounded annually) is required for the value of an investment to triple in 15 years?

1 time per year
$n=1$

$t=15$

$$A = P\left(1+\frac{r}{1}\right)^{1\cdot t}$$

$$\frac{3\cancel{P}}{P} = \frac{\cancel{P}(1+r)^{15}}{P}$$

triple our \$

$$\sqrt[15]{3} = \sqrt[15]{(1+r)^{15}}$$

$$\sqrt[15]{3} = 1+r$$

$$\sqrt[15]{3} - 1 = r$$

or

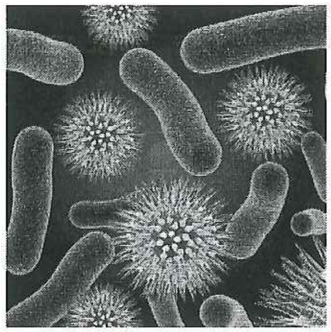
$$3^{1/15} - 1 = r$$

$$.07599 \approx r$$

$$\approx 7.599\%$$

Math 111 Lecture Notes

Section 4.8: Exponential Growth and Decay Models



Populations that obey **uninhibited growth** grow exponentially according to the formula

$$A(t) = A_0 e^{kt}$$

where k is the continuous growth rate and A_0 is the initial amount.

Substances that undergo **uninhibited radioactive decay** do so exponentially according to the formula

$$N(t) = N_0 e^{kt}$$

where k is the continuous decay rate and N_0 is the initial amount.

The **doubling time** for a population is the amount of time it takes a population growing exponentially to double in size.

The **half-life** for a radioactive substance is the amount of time it takes for the quantity of the substance to be one half its original amount.

$N=2$ $t=10$ N_0

Example 1. The fruit fly drosphilia have a doubling time of 10 days.[2] There are initially 8 fruit flies.

(a) The population of fruit flies is modeled by $N(t) = N_0e^{kt}$. Use the doubling time to find the value of k.

$$2 = e^{k\cdot 10}$$
$$\ln 2 = \ln e^{10k}$$
$$\frac{\ln 2}{10} = \frac{10k}{10}$$
$$.069315 \approx k$$

or

$$\frac{2\cancel{N_0}}{\cancel{N_0}} = \frac{\cancel{N_0}e^{k\cdot 10}}{\cancel{N_0}}$$
$$2 = e^{k\cdot 10}$$

or

$$\frac{16}{8} = \frac{\cancel{8}e^{k\cdot 10}}{\cancel{8}}$$
$$2 = e^{k\cdot 10}$$

(b) What is the continuous growth rate?

$$k = 6.93\%$$

(c) Write the full formula for $N(t)$.

$$N(t) = 8e^{.0693t}$$

(d) How many fruit flies will there be after 30 days?

$$N(30) = 8e^{.0693(30)}$$
$$\approx 63.97$$
≈ 64 fruit flies

(e) When will there be 1000 fruit flies?
$t?$

$$\frac{1000}{8} = \frac{8e^{.0693(t)}}{8}$$
$$125 = e^{.0693t}$$
$$\frac{\ln 125}{.0693} = \frac{.0693t}{.0693}$$
$$69.67 \approx t$$

or

$$\ln 125 = \ln e^{.0693t}$$
$$\ln 125 = .0693t$$

It will take about 69.67 days to reach 1000 fruit flies.

[2] https://www.lscore.ucla.edu/hhmi/performance/VickiHahmFinal.pdf

$\frac{1}{2} =$ t

Example 2. The half-life of carbon-14 is $\boxed{\text{5600 years}}$. Write the percentage of carbon-14, $A(t)$, remaining after t years of decay. Round the value you find for k accurate to six decimal places.

solving

$$A(t) = A_0 e^{kt}$$

$$\frac{\frac{1}{2}\cancel{A_0}}{\cancel{A_0}} = \frac{\cancel{A_0}}{\cancel{A_0}} e^{k(5600)}$$

$$\frac{1}{2} = e^{5600k}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln e^{5600k}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{5600} = \frac{5600k}{5600}$$

$$-.000124 \approx k \qquad A(t) = A_0 e^{-.000124t}$$

Example 3. In 1991, two hikers discovered a historic iceman in the Ötztal Alps in Italy.[3] Assuming 46% of his carbon-14 was found remaining in the sample, how many years ago did the iceman die? Use the formula you found in the previous example.

$$A(t) = A_0 e^{-.000124t}$$

$$\frac{.46\cancel{A_0}}{\cancel{A_0}} = \frac{\cancel{A_0}}{\cancel{A_0}} e^{-.000124t}$$

$$.46 = e^{-.000124t}$$

$$\ln .46 = \ln e^{-.000124t}$$

$$\frac{\ln .46}{-.000124} = \frac{-.000124t}{-.000124}$$

$$6{,}262.33 \approx t$$

The iceman died about 6262 years before he was found.

[3] http://www.nupecc.org/iai2001/report/B44.pdf

Example 4. The radioisotope Sodium-24 decays at a continuous rate of about 4.5% per hour. What is the half-life of this radioactive substance?[4]

$k = -.045$

t

$$A(t) = A_0 e^{kt}$$

$$\frac{1}{2} = e^{-.045t}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{-.045} = \frac{-.045t}{-.045}$$

$$15.4 \approx t$$

~~years~~ hours

Example 5. The radioisotope Barium-139 has a half-life of 86 minutes. Find the continuous rate of decay.

$$N(t) = N_0 e^{kt}$$

$$\frac{\frac{1}{2}\cancel{N_0}}{\cancel{N_0}} = \frac{\cancel{N_0}}{\cancel{N_0}} e^{k(86)}$$

$$\frac{1}{2} = e^{86k}$$

$$\frac{\ln \frac{1}{2}}{86} = \frac{86k}{86}$$

Should be negative for decay $\rightarrow$ $-.0081 \approx k$

The continuous rate of decay is about 0.81% per minute.

[4] http://www.ndt-ed.org/EducationResources/HighSchool/Radiography/halflife2.htm

Example 6. The half-life of Cobalt-60 is 5.27 years.[5]. If 15 grams are present now, how many grams will be present in 100 years?
N_0

2 Steps: (1) use the half-life to find k and write the formula
(2) use the formula with k to answer the question

(1) $\frac{1}{2} = e^{k(5.27)}$

$\frac{\ln \frac{1}{2}}{5.27} = \frac{5.27k}{5.27}$

$-.1315 \approx k$

$N(t) = 15e^{-.1315t}$

(2) $N(100) = 15e^{-.1315(100)}$

$\approx .000029$

After 100 years, only .000029 grams remain.

[5] http://www.bt.cdc.gov/radiation/isotopes/cobalt.asp