## D1: Voting Methods - SOLUTIONS

Group Activity

1. Ranked Choice Voting Election. Our class will elect the best candy out of the three that are running for office: **R=Reese's Peanut Butter Cups, S=Starburst, K=KitKat** 

Please get ballot forms for your group and fill them out anonymously and turn them in. When all the results are tabulated, make a preference schedule.

Preference Schedule

Number of Voters			
1 <sup>st</sup> choice			
2 <sup>nd</sup> choice			
3 <sup>rd</sup> choice			

- a. How many voters voted in this election?
- b. How many votes are needed for a majority?
- c. How many votes are needed for a plurality win?
- d. Find the winner under the plurality method.
- e. Find the winner under the Instant Runoff Voting method.
- f. Find the winner under the Borda Count method.
- g. Find the winner under the Pairwise Comparisons method.
- h. Which method do you think is the most fair in this situation and why?

## **Chapter D: The Math of Democracy**

MTH 105

2. A homeowners' association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options have been proposed. The votes are:

Number of voters	8	9	11	7	7	5
1st choice	В	А	D	А	В	С
2nd choice	С	D	В	В	А	D
3rd choice	А	С	С	D	С	А
4th choice	D	В	А	С	D	В

a. How many voters voted in this election? 8+9+11+7+7+5=47

b. How many votes are needed for a majority?  $47 \div 2 = 23.5$ 

## 24 votes are needed for a majority

c. How many votes are needed for a plurality win?  $47 \div 4 = 11.75$ **12 votes are needed for plurality** 

- d. Find the winner under the plurality method.
  A=16 B=15 C=5 D=11
  A is the plurality winner.
- e. Find the winner under the Instant Runoff Voting method.

A=16	<u>B=15</u>	<b>C=5</b>	D=11
+8			<u>+5</u>
<u>+7</u>			<b>16</b>
<mark>31</mark>			
	the stand with		

A is the instant runoff winner.

f. Find the winner under the Borda Count method.  $A = 1 \cdot 11 + 2 \cdot 13 + 3 \cdot 7 + 4 \cdot 16 = 122$ 

 $B = 1 \cdot 14 + 2 \cdot 0 + 3 \cdot 18 + 4 \cdot 15 = 128$ 

 $C = 1 \cdot 7 + 2 \cdot 27 + 3 \cdot 8 + 4 \cdot 5 = 105$ 

 $D = 1 \cdot 15 + 2 \cdot 7 + 3 \cdot 14 + 4 \cdot 11 = 115$ 

B is the Borda Count winner. Note: The total is 470 which is (1+2+3+4)(47)

D 27

g. Find the winner under the Pairwise Comparisons method.

A 21,	<mark>B 26</mark>	<mark>B 33</mark> ,	C 14	C 20,
A 23,	<mark>C 24</mark>	B 22,	D 25	
<mark>A 31</mark> ,	D 16			

Both B and D have two pairwise wins, so it is not clear who would win. Who do you think should win?

h. Which method do you think is the most fair in this situation and why?

Your opinion<sup>©</sup>

3. In the election shown below under the plurality method, explain why voters in the third column may feel they cannot vote for their first choice. How could it affect the outcome of the election?

Number of voters	96	90	10
1st choice	А	В	С
2nd choice	В	А	В
3rd choice	С	С	А

- a. How many voters voted in this election? 96+90+10=196
- b. How many votes are needed for a majority?  $196 \div 2 = 98$ , 98 + 1 = 99**99 votes are needed for a majority**
- c. How many votes are needed for a plurality win?  $196 \div 3 = 65.33$ A minimum of 66 votes are needed for plurality
- d. Find the winner under the plurality method.
  A=96 B=90 C=10
  A is the plurality winner.
- e. Under the plurality method, explain why voters in the third column may feel they cannot vote for their first choice. How could that affect the outcome of the election?

The 10 voters prefer C, but their candidate C has no chance of winning. If they vote for C, then A will win, who is their last choice. They may feel like they have to vote for B, their second choice. This is called voting insincerely.

f. Find the winner under the Instant Runoff Voting method. A=96 B=90 C=10

A=96	<b>B=90</b>
	+10
	100

B is the winner with Instant Runoff Voting.

g. Find the winner under the Borda Count method.  $A = 1 \cdot 10 + 2 \cdot 90 + 3 \cdot 96 = 478$ 

 $B = 2 \cdot 96 + 2 \cdot 10 + 3 \cdot 90 = 482$  B is the winner with Borda Count.

C = 1.96 + 1.90 + 3.10 = 216

h. Find the winner under the Pairwise Comparisons method.

A 96, **B 100** A 186, C 10

**B** is the winner with the Condorcet Method

Which method do you think is the most fair in this situation and why?
 Every method except plurality seems fair because more people prefer B to A and C.

**B 186**, C 10