Math 111 – Tues, 5/24

Return tests + go over

Section 3.1

Checkpoint 7 on Thursday (on 3.1)

Mission 4 handed out ~~on Thursday~~ today

+ due Thurs, June 2 (Team)

Math 111 Lecture Notes

Section 3.1: Polynomial Functions

standard form: $f(x) = 3x^2 + 2x - 1$

factored form $g(x) = (x-3)(x+1)$

A **power function** is of the form $f(x) = a_n x^n$ where a_n is a real number and n is a non-negative integer.

A **polynomial function** is of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a non-negative integer.

The **leading term** is $a_n x^n$. This determines the long-run behavior of the function.

The **degree** of the polynomial is n.

Basic Power Functions

Figure 1. $y = x^2$

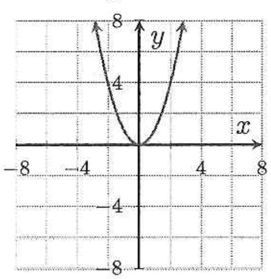


Figure 2. $y = x^3$

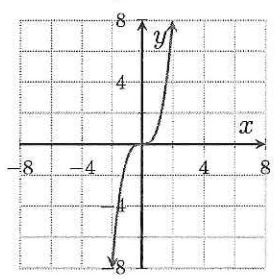


Figure 3. $y = x^4$

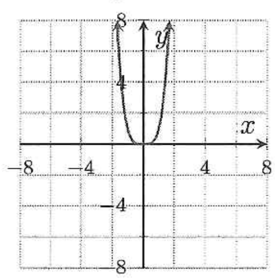
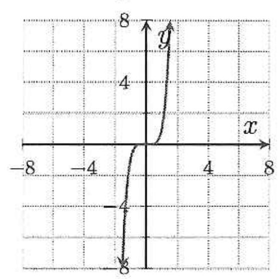


Figure 4. $y = x^5$



Basic Power Functions (close up)

Figure 5. Even Powers

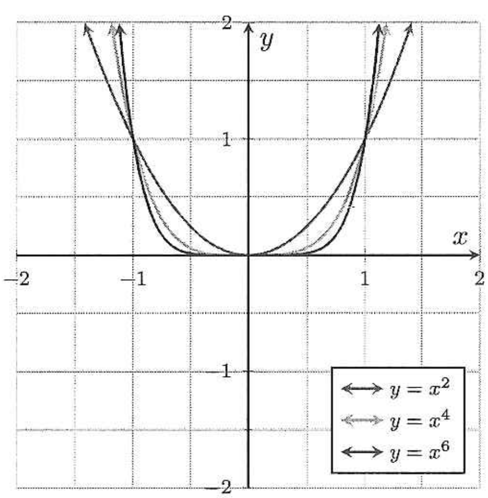
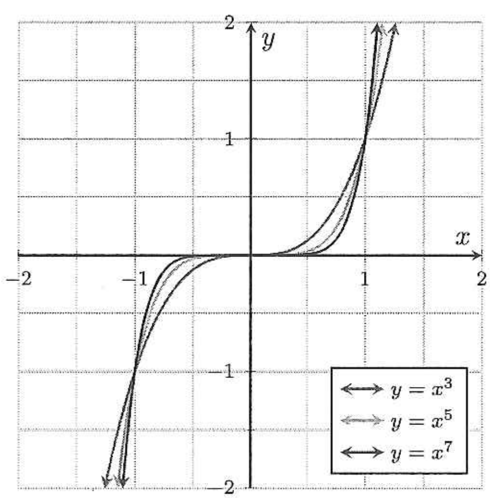


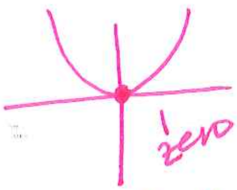
Figure 6. Odd Powers



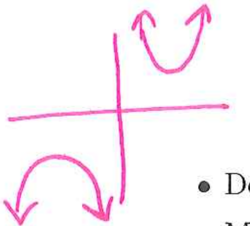
General Polynomial Functions

even multiplicity

odd multiplicity

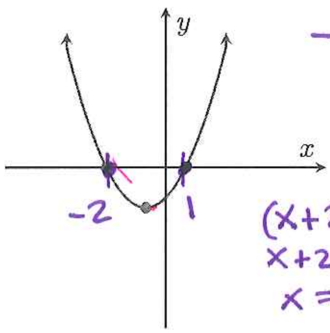


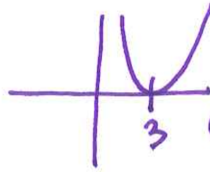
zero



no zeros

FIGURE 7





$(x-3)^2$

multiplicity of 1

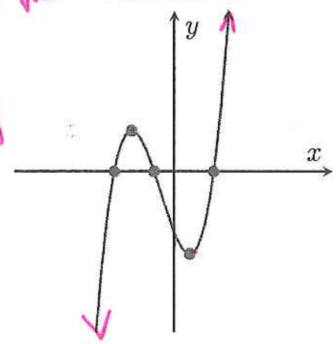
$(x+2)(x-1)=0$

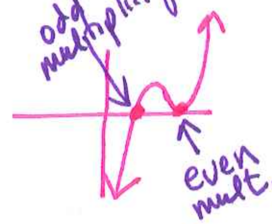
$x+2=0 \quad x-1=0$

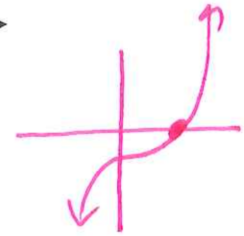
$x=-2 \quad x=1$

- Degree: 2
- Max. # of zeros: 2
- Max. # of turning points: 1

FIGURE 8

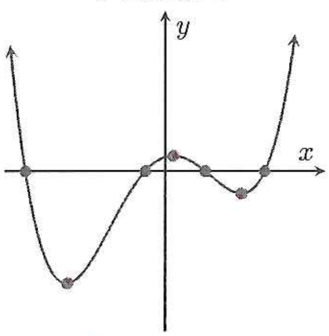






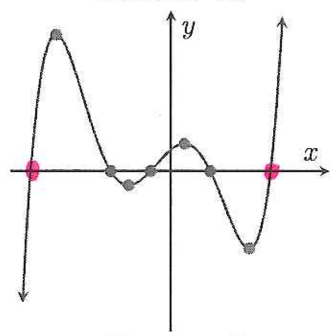
- Degree: 3
- Max. # of zeros: 3
- Max. # of turning points: 2

FIGURE 9



- Degree: 4
- Max. # of zeros: 4
- Max. # of turning points: 3

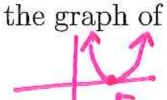
FIGURE 10

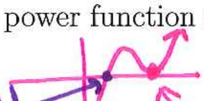


- Degree: 5 $= n$
- Max. # of zeros: 5 $= n$
- Max. # of turning points: 4 $= n-1$

A polynomial function f has a real zero r if and only if $(x-r)$ is a factor of $f(x)$.

If r is a zero of **even multiplicity**, then the factor $(x-r)$ occurs an even number of times. The graph then *looks like* the graph of an even power function at that zero. Hence the function "bounces" there.

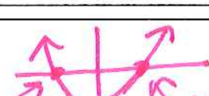




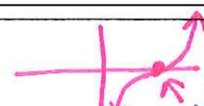
even multiplicity

odd mult.

If r is a zero of **odd multiplicity**, then the factor $(x-r)$ occurs an odd number of times. The graph then *looks like* the graph of an odd power function at that zero. Hence, if $(x-r)$ occurs once, the function passes "straight through" at that zero and if $(x-r)$ occurs any other odd number of time, the function "flattens" there.



multiplicity 1



multiplicity 3

$4(x-0)$ $4x=0$, $x=0$

Example 1. Let $f(x) = 4x(x-7)^2(x+1)^5(x+2)^3$. Determine the following:

(a) the zeros and their respective multiplicities

x-intercepts

Zeros	multiplicity
7	2
-1	5
-2	3
0	1

11^{th} degree

(b) the degree and long-run behavior

ends of the function

tails

1^{st} term : $4x^{11}$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$

$-\frac{1}{2}(x^2 + 4x + -2x - 8)$

Example 2. Graph the polynomial function defined by $f(x) = -\frac{1}{2}(x-2)(x+4)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

Degree : 2

1^{st} term : $-\frac{1}{2}x^2$

long-run behavior

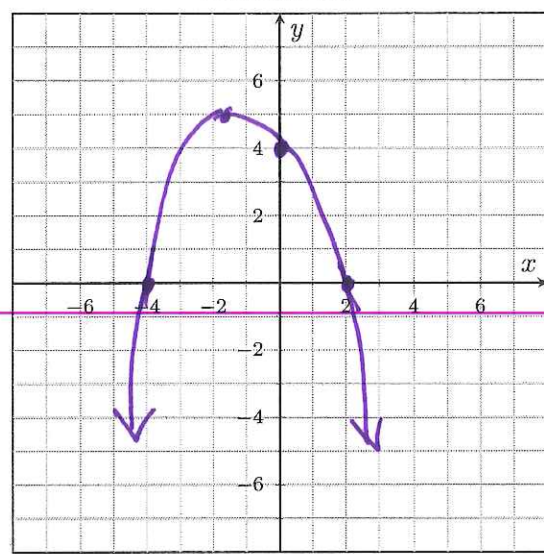
as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to -\infty$

max ~~#~~ of turning points

$n-1$ or $2-1=1$

FIGURE 11



$x = \frac{-b}{2a}$

zero	multiplicity
2	1
-4	1

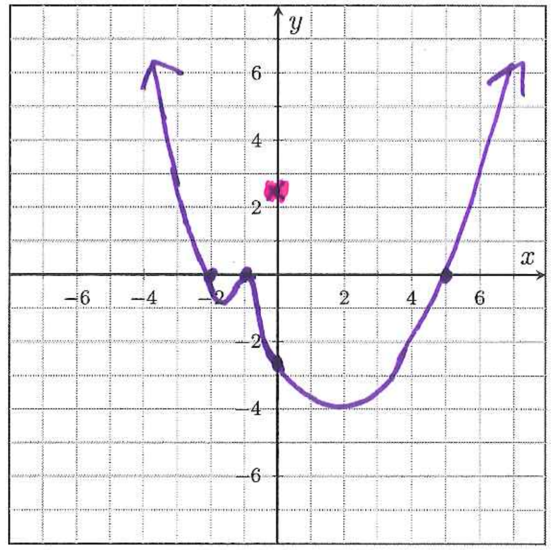
vertical or y-intercept $f(0) = -\frac{1}{2}(0-2)(0+4)$ $(0,4)$

$= -\frac{1}{\cancel{2}}(-\cancel{2})(4)$

$= 4$

Example 3. Graph the polynomial function defined by $f(x) = \frac{1}{4}(x+1)^2(x+2)(x-5)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

FIGURE 12



Degree: 4

1^{st} term: $\frac{1}{4}x^4$

long-run behavior

$\uparrow \quad \nearrow$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to \infty$

max # of turning points: 3

degree − 1

zero	mult	
−1	2	← bounce
−2	1	← cross
5	1	← cross

y-int: $f(0) = \frac{1}{4}(0+1)^2(0+2)(0-5)$

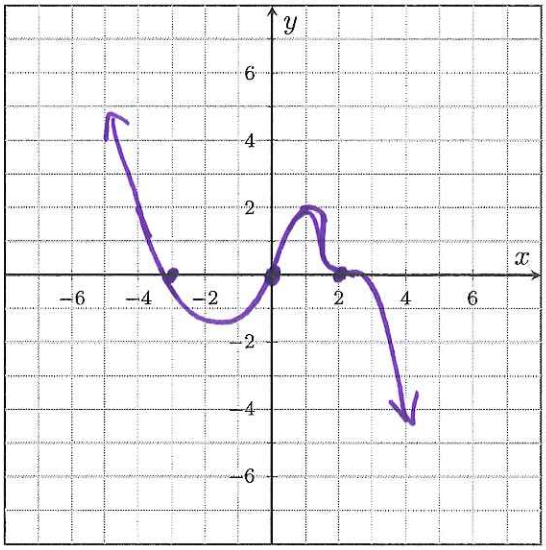
$= \frac{1}{4}(1)^2(2)(-5)$

$= \frac{1}{\cancel{4}} \cdot \frac{\cancel{2}}{1} \cdot -5$

$= -\frac{5}{2} = -2\frac{1}{2}$

Example 4. Graph the polynomial function defined by $f(x) = -\frac{1}{2}x(x+3)(x-2)^3$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

FIGURE 13



Degree: 5

1^{st} term: $-\frac{1}{2}x^5$

long-~~run~~ run behavior

↑ ↓

as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to \infty$

zeros	mult
0	1
-3	1
2	3

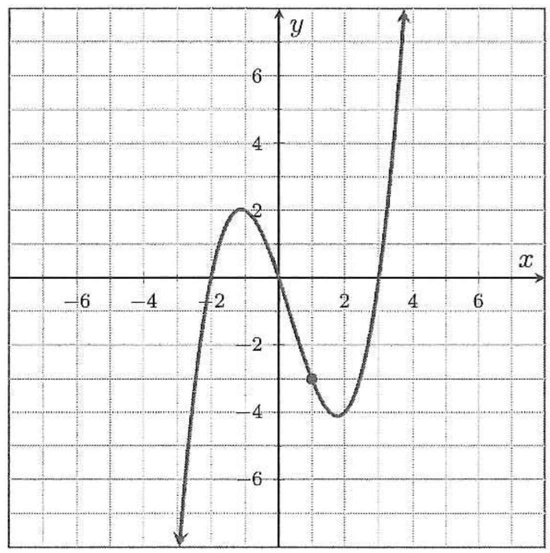
$f(0) = -\frac{1}{2}(0)(0+3)(0-2)^3$

$= 0$

Example 5. Find a possible formula for the polynomial function graphed in Figure 14 using the zeros and their multiplicities.

zeros	mult
-2	odd (1)
0	odd (1)
3	odd (1)

FIGURE 14



$h(x) = k(x+2)(x-0)(x-3)$

$= kx(x+2)(x-3)$

$(1,-3)$

$-3 = k(1)(1+2)(1-3)$

$-3 = k(1)(3)(-2)$

$\frac{-3}{-6} = \frac{k(-6)}{-6}$

$\frac{1}{2} = k$

$h(x) = \frac{1}{2}x(x+2)(x-3)$

Example 6. Find a possible formula for the polynomial function graphed in Figure 15 using the zeros and their multiplicities.

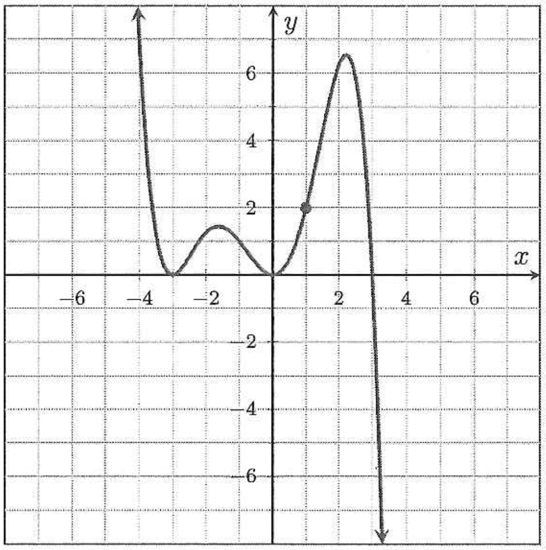
y-int $= 0$

odd degree

zeros	mult
-3	2 (even)
0	2 (even)
3	1

$(1, 2)$

FIGURE 15



$$f(x) = (x+3)^2(x-0)^2(x-3)$$
$$= x^2(x+3)^2(x-3)$$

find k

$$f(x) = kx^2(x+3)^2(x-3) \quad \text{use}(1,2)$$

$$2 = k(1)^2(1+3)^2(1-3)$$
$$2 = k(1)(16)(-2)$$
$$\frac{2}{-32} = \frac{-32k}{-32}$$
$$-\frac{1}{16} = k$$

$$f(x) = -\frac{1}{16}x^2(x+3)^2(x-3)$$

Example 7. Find a possible formula for the polynomial function graphed in Figure 16 using the zeros and their multiplicities.

zero	mult
-1	odd : 3
2	even : 2

$g(x) = k(x+1)^3(x-2)^2$

$(0,-4)$

$-4 = k(0+1)^3(0-2)^2$

$-4 = k(1)(4)$

$\frac{-4}{4} = \frac{4k}{4}$

$-1 = k$

$g(x) = -(x+1)^3(x-2)^2$

FIGURE 16