Math 111, Tuesday, 5/3

Return Tests + go over
Calculate grades
New material: 4.3 + supplement
Project – Desmos Demo

Project due Thurs, May 12
Checkpoint 5 on Thursday (4.2 + 4.3)
inverse

Bonus self-reflection
Due next Tuesday
Substitute on Thursday (Julia Trude)

Zombie Tag!

A Zombie is loose in our classroom!

How long until we are all infected?

Example 1. Fill in the table for each scenario.

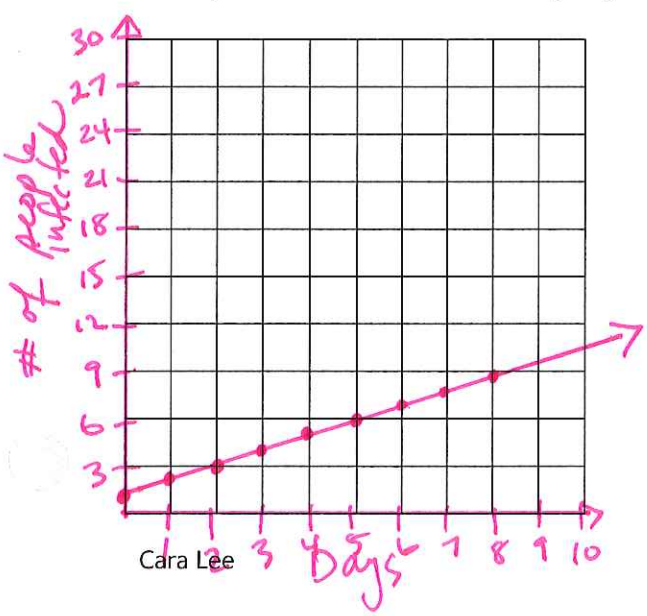
Scenario 1: The initial zombie infects one new person in our class per day. Newly infected zombies cannot infect others.

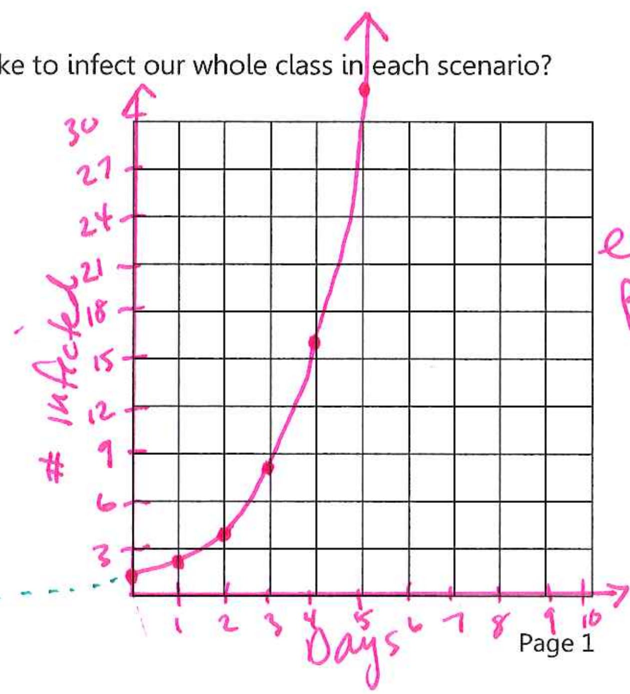
Days	# of People Infected
Day 0	1
Day 1	2
Day 2	3
Day 3	4
Day 4	5
Day 5	6
Day 6	7
Day 7	8
Day 8	9

Scenario 2: The initial zombie and each infected person infect one new person per day.

Days	# of People Infected	
Day 0	1	2^0
Day 1	2	2^1
Day 2	4	2^2
Day 3	8	2^3
Day 4	16	2^4
Day 5	32	2^5
Day 6	64	2^6
Day 7	128	2^7
Day 8	256	2^8

a. Graph each scenario. How many days will it take to infect our whole class in each scenario?





exponential pattern

b. Write an equation for each scenario:

Scenario 1:

$y = mx + b$

$y = x + 1$

day

Scenario 2:

$y = 2^x$ ← day

c. How many people would be infected on day 30?

Scenario 1:

$y = 30 + 1$

$= 31$ zombies

Scenario 2:

$y = 2^{30}$

$= 1{,}073{,}741{,}824$ zombies!

d. On which day would the zombie outbreak infect one million people?

Scenario 1:

$1{,}000{,}000 = x + 1$

$-1 \qquad -1$

$999{,}999 = x$

Scenario 2:

$1{,}000{,}000 = 2^x$

$x \approx 20$ days

guess & check

An **exponential function** is of the form

$$f(x) = C\,a^x$$

where

- C is the initial value
- a is the growth factor and $a > 0$

a = growth factor
r = growth rate
increase of 3% $r = .03$
$a = 1 + r = 1.03$

Consequently, an exponential function is a function that increases or decreases at a constant percent rate. Let's review percent increase and decrease as we work through these examples.

Example 2. You start a new job with an initial salary of \$36,000 per year. Each year thereafter, you receive a 3% raise. Let $S(t)$ be your salary t years after you start your new job.

(a) Write the formula for $S(t)$.

$S(t) = Ca^t$
$S(t) = 36{,}000(1.03)^t$

(b) What will your salary be after 10 years?

$S(10) = 36{,}000(1.03)^{10}$
$= \$48{,}381$

After 10 years your salary would be \$48,381.

(c) When will your salary reach \$50,000? (Use your graphing calculator to solve this).

$\$50{,}000 = 36{,}000(1.03)^t$

$a > 1$, exponential growth

When $t = 12$, $S(t) = \$51{,}327.40$

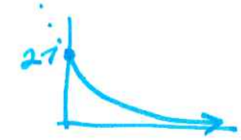
In 12 years.

Example 3. A compost pile has 27 $\boxed{\text{cubic feet}}$ (ft^3) of waste and decays at a rate of 10% per month. Let $Q(t)$ be the volume of compost (in cubic feet) t months since decay began. Write the formula for this decreasing exponential function.

$r = -.10$
$a = 1 + r = 1 + (-.10)$
$= .90$

$Q(t) = Ca^t$
$Q(t) = 27(.90)^t$

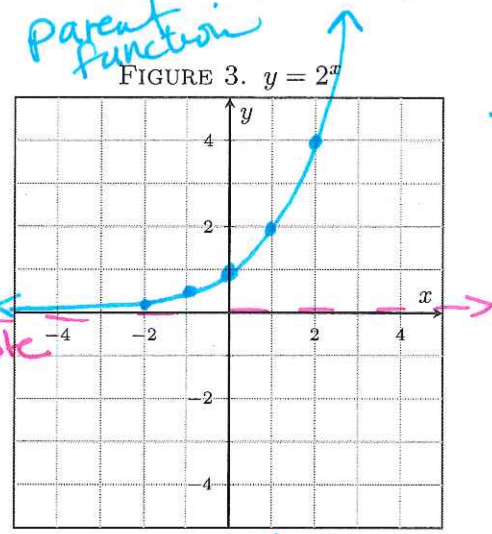
$a < 1$, exponential decay



Example 4. Graph of $y = 2^x$ in Figure 3. Use this to graph the various transformations listed.

parent function

FIGURE 3. $y = 2^x$

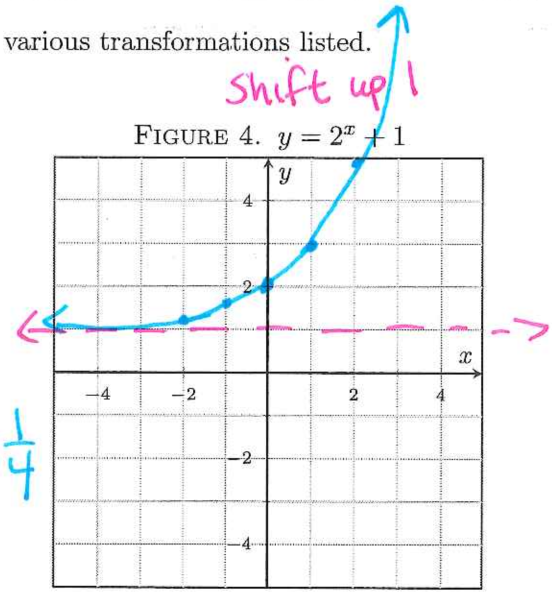


asymptote

x	y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
-1	$2^{-1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

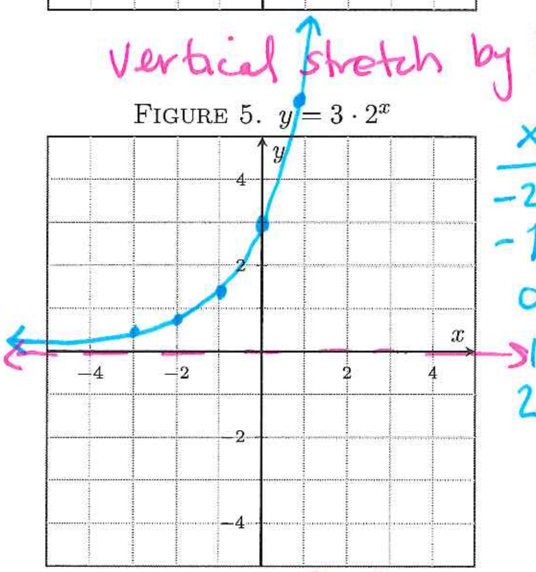
shift up 1

FIGURE 4. $y = 2^x + 1$



vertical stretch by 3

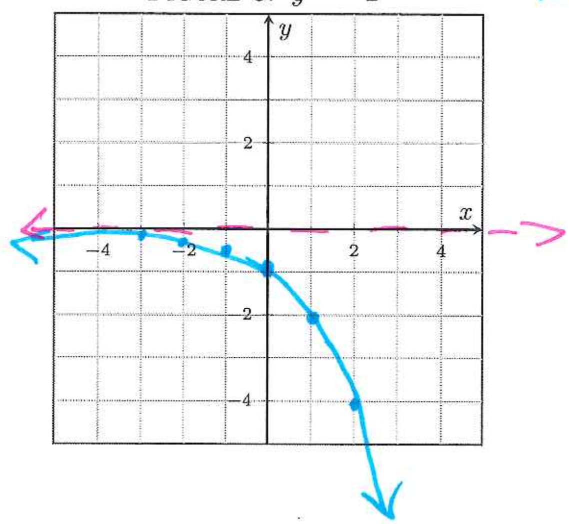
FIGURE 5. $y = 3 \cdot 2^x$



x	y	3·y
-2	$\frac{1}{4}$	$\frac{3}{4}$
-1	$\frac{1}{2}$	$\frac{3}{2}$
0	1	3
1	2	6
2	4	12

vertical flip over the x

FIGURE 6. $y = -2^x$



horizontal flip across the y

FIGURE 7. $y = 2^{-x}$

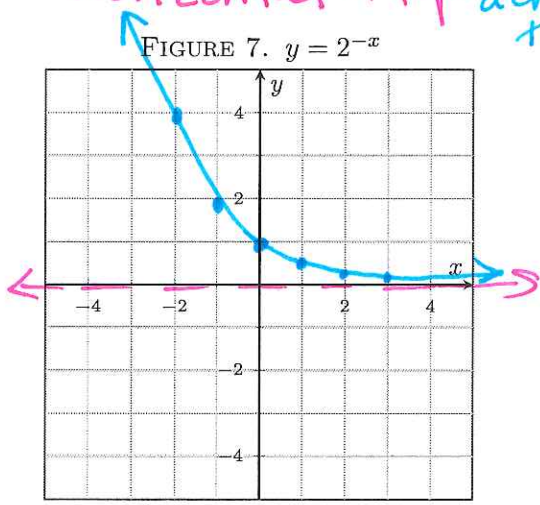
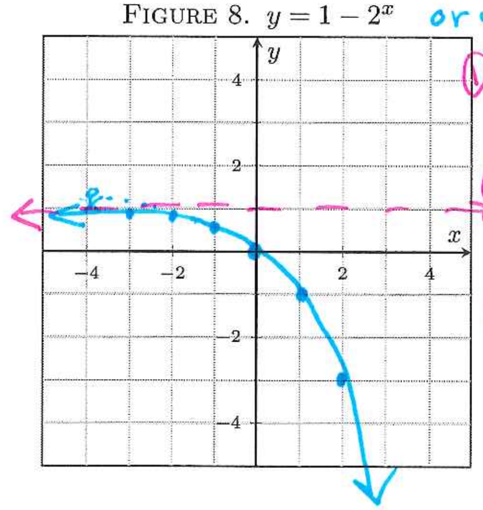


FIGURE 8. $y = 1 - 2^x$ or $y = -2^x + 1$

(1) vertical flip
(2) up 1
or
figure 6 +1



if $a^x = a^y$, then $x = y$

Example 5. Solve the following equations. List your solution set.

(a) $5^x = 5^{-6}$

$x = -6$

$\{-6\}$

(d) $2^{2x-1} = 4$

$2^{2x-1} = 2^2$

$2x - 1 = 2$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$ $\left\{\frac{3}{2}\right\}$

(b) $4^{2x-5} = \frac{1}{16}$ $\frac{1}{4^2} = 4^{-2}$

$4^{2x-5} = 4^{\boxed{-2}}$

$2x - 5 = -2$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$

$\left\{\frac{3}{2}\right\}$

(e) $2^{3x-1} = 32$

$2^{3x-1} = 2^5$

$3x - 1 = 5$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

$\{2\}$

(c) $5^{x^2+8} = 125^{2x}$

$5^{x^2+8} = (5^3)^{2x}$

$\boxed{a^m a^n = a^{m+n}}$
$\boxed{(a^m)^n = a^{m \cdot n}}$
$\boxed{a^{-1} = \frac{1}{a}}$

$5^{x^2+8} = 5^{6x}$

$x^2 + 8 = 6x$ quadratic set to zero

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x - 4 = 0$ or $x - 2 = 0$

$x = 4$ or $x = 2$

$\{2, 4\}$

(f) $9^{2x} \cdot 27^{x^2} = 3^{-1}$

$(3^2)^{2x} \cdot (3^3)^{x^2} = 3^{-1}$

$3^{4x} \cdot 3^{3x^2} = 3^{-1}$

$3^{4x+3x^2} = 3^{-1}$

$4x + 3x^2 = -1$

$3x^2 + 4x + 1 = 0$

$(3x+1)(x+1) = 0$

$3x + 1 = 0$ or $x + 1 = 0$

$3x = -1$

$x = -\frac{1}{3}$ or -1 $\left\{-\frac{1}{3}, -1\right\}$

What's "e"?

The number e is a number that occurs in nature, and is a frequent base for exponential and logarithmic expressions. It is defined by:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

It can also be expressed by the following:

$$e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots$$

This number is irrational and is approximated by 2.718281828. The graph of the function given by $y = e^x$ looks a lot like the graphs of the functions given by $y = 2^x$ and $y = 3^x$, as shown in Figure 9. In calculus, you will study that the special property of e is that the slope of the tangent line at zero is exactly **1**, as shown in Figure 10.

Figure 9

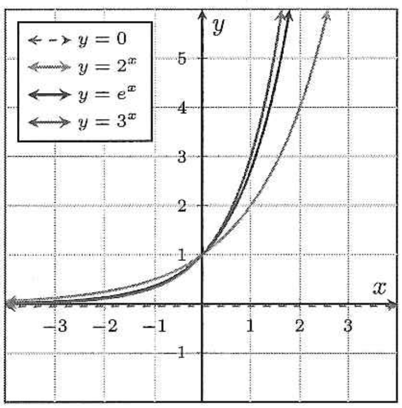
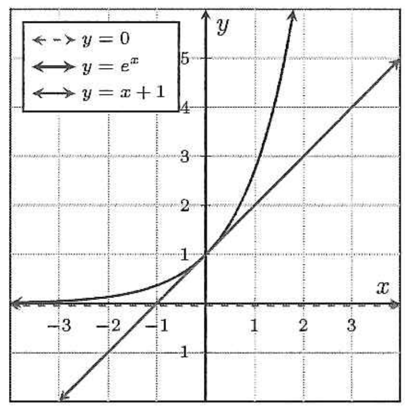


Figure 10



Example 6. Solve the following equation.

$$e^{3x} = e^{2-x}$$

$3x = 2 - x$

$\frac{4x}{4} = \frac{2}{4}$

$x = \frac{1}{2}$

$\left\{\frac{1}{2}\right\}$

Example 7. In 1990, the population of Oregon was 2.84 million people. In 2010, the population of Oregon was 3.83 million people. Let $P(t)$ be the population of Oregon in millions, where t is the number of years after 2000. This can be modeled by $P(t) = 3.298e^{0.015t}$.

(a) According to this model, what will the population be in 2020?

$P(t) = 3.298e^{.015t}$

$2020 - 2000 = 20$

$P(20) = 3.298e^{.015(20)}$

$= 4.45$ million people.

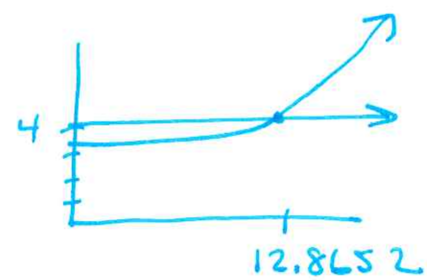
In 2020 the population according the model is 4.45 million people

(b) According to this model, when will the population reach 4 million people? Use your graphing calculator to solve this.

$4 = 3.298e^{.015t}$

graph $y1 = 4$

$y2 = 3.298e\wedge(.015x)$



use F5: Intersection

$(12.8652, 4)$

$2000 + 12.8652$

≈ 2012.9

In 2012 the population will reach (would have reached) 4 million people according to the model.

Example 8. Find an algebraic rule (or formula) for an exponential function f that passes through the points $(-1, 8)$ and $(2, 1)$. Also find the algebraic rule (or formula) for a linear function g that passes through the points $(-1, 8)$ and $(2, 1)$.

Linear Equation

$y = mx + b$

$(\overset{x_1}{-1}, \overset{y_1}{8})\ (\overset{x_2}{2}, \overset{y_2}{1})$

constant slope

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1-8}{2+(+1)}$

$= \dfrac{-7}{3}$

slope & a point

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{7}{3}(x-2)$

$y - 1 = -\frac{7}{3}x + \frac{14}{3}$

$+1 \qquad\qquad + 1\left(\frac{3}{3}\right)$

$\boxed{y = -\frac{7}{3}x + \frac{17}{3}}$

Exponential Equation

$y = Ca^x$

constant rate

$(\overset{x_1}{-1}, \overset{y_1}{8})\ (\overset{x_2}{2}, \overset{y_2}{1})$

$\dfrac{y_2}{y_1} = \dfrac{\cancel{C}a^{x_2}}{\cancel{C}a^{x_1}}$

$\dfrac{1}{8} = \dfrac{a^2}{a^{-1}}$

exponent rules

$a^{2-(-1)} = a^3$

$\frac{1}{8} = a^3$ or $\sqrt[3]{\frac{1}{8}} = \sqrt[3]{a^3}$

$\left(\frac{1}{2}\right)^3 = a^3$ $\qquad \frac{1}{2} = a$

$\frac{1}{2} = a$ growth factor

$y = C\left(\frac{1}{2}\right)^x$

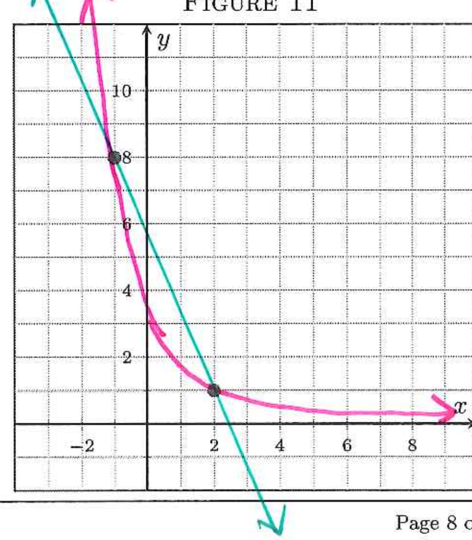
$1 = C\left(\frac{1}{2}\right)^2$

$4 \cdot 1 = C \cdot \frac{1}{4} \cdot 4$

$4 = C$

$\boxed{y = 4\left(\frac{1}{2}\right)^x}$

FIGURE 11



Example 9. Find an algebraic rule (or formula) for an exponential function f that passes through the points $(-2, \frac{3}{4})$ and $(2, 12)$.

x_1, y_1 and x_2, y_2

$(-2, \frac{3}{4})$ and $(2, 12)$

$\frac{y_2}{y_1} = \frac{\cancel{c}a^{x_2}}{\cancel{c}a^{x_1}}$

$y = c(2)^x$

$12 \div 3/4$

$12 \cdot 4/3$

$\frac{12}{3/4} = \frac{a^2}{a^{-2}}$

$12 = c(2)^2$

$\frac{12}{4} = \frac{c \cdot 4}{4}$

$\frac{4}{\cancel{3}} \cdot \frac{\cancel{12}^4}{1} = a^4$

$3 = c$

$16 = a^4$

$2 = a$

$\boxed{y = 3(2)^x}$

Example 10. Find an algebraic rule (or formula) for an exponential function f that passes through the points $(1, 8)$ and $(3, 128)$.

$x_1 y_1$ $x_2 y_2$

$\frac{y_2}{y_1} = \frac{\cancel{c}a^{x_2}}{\cancel{c}a^{x_1}}$

$y = c(4)^x$

$\frac{128}{8} = \frac{a^3}{a^1}$

$8 = c(4)^1$

$8 = c \cdot 4$

$16 = a^2$

$2 = c$

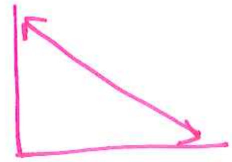
$4 = a$

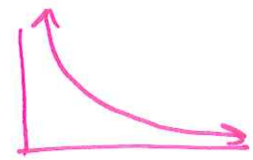
$\boxed{y = 2(4)^x}$

Example 11. After caffeine is consumed, it leaves the body at a fairly fixed rate. A person consumes 200 mg of caffeine at 8:00am. Four hours later, about 100 milligrams of caffeine are remaining in their bloodstream. Let $Q(t)$ be the number of milligrams of caffeine in the body t hours after consumption.

(a) Write the formula for the function modeling this exponential decay.

$(\underset{x_1}{0}, \underset{y_1}{200}), (\underset{x_2}{4}, \underset{y_2}{100})$





$\frac{y_2}{y_1} = \frac{Ca^{x_2}}{Ca^{x_1}}$

$\frac{100}{200} = \frac{\cancel{C}a^4}{\cancel{C}\underbrace{a^0}_{=1}}$

$\frac{1}{2} = a^4$

$\sqrt[4]{\frac{1}{2}} = a$

$.841 = a$

$y = C(.841)^x$

$200 = C\underbrace{(.841)^0}_{=1}$

$200 = C$

(b) How much caffeine will still be in the body at 8:00pm?

$Q(t) = 200(.841)^t$

$Q(12) = 200(.841)^{12}$

$= 200(.841)^{12}$

$= 25.037$ mg

$a = 1 + r$ $\quad .841 = 1 + r$
$\quad\quad -1 \quad -1$

$1 - .841 = -.159$

Losing 15.9% of the caffeine per hour.