Math 111, Wed, 3/30

Questions on 1.1
Checkpoint 1
New material – Section 1.2

Team Mission 1 due at the beginning of class on Monday
(one neat copy per group)

Change the 3 to a 0 in the table in problem 2.

Section 1.1 Questions

79. $f(x) = \frac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{x^2}{x^2(x+h)^2} - \frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}}{h}$$

$$= \frac{\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}}{h}$$

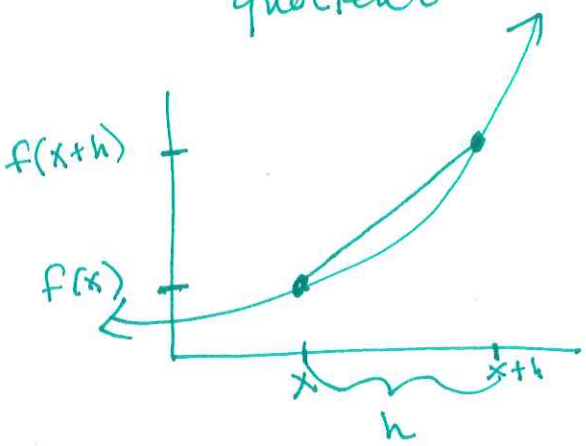
$$= \frac{x^2-(x+h)^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$\frac{f(x+h)-f(x)}{h} = \frac{y_2-y_1}{x-x_1}$

$$= \frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\cdot\frac{1}{h}$$

Difference quotient

$$= \frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$



$$= \frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$= \frac{-2x-h}{x^2(x+h)^2}, \quad h\neq 0$$

$x\neq 0$

$x+h\neq 0$

77. $f(x)=x^2-x+4$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$$

$$= \frac{(x^2+2xh+h^2)-x-h+4-x^2+x-4}{h}$$

$(x+h)(x+h)$
$x^2+xh+xh+h^2$

$$= \frac{2xh+h^2-h}{h}$$

$$= 2x+h-1, \quad h\neq 0$$

49. $f(x)=\dfrac{x}{x^2+1}$

51

$x^2+1\neq 0$

$x^2\neq -1$

$x=\pm\sqrt{-1}$

Domain: $\mathbb{R}$

51. $\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$(x+4)(x-4)\neq 0$

$x\neq \pm 4$

$x^2-16\neq 0$

$x^2\neq 16$

$x\neq \pm\sqrt{16}$

$x\neq \pm 4$

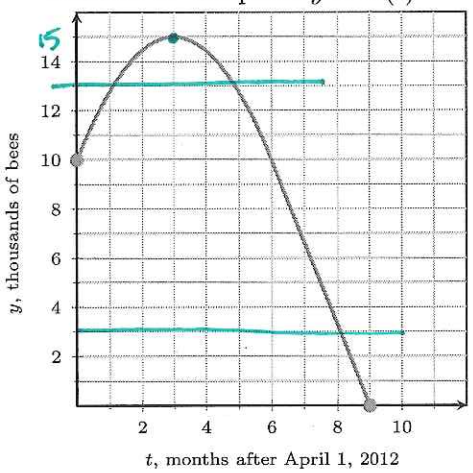
Domain: $\{x \mid x\neq \pm 4\}$

Math 111 Lecture Notes

Section 1.2: The Graph of a Function

Example 1. Bees! A population of bees was happily residing in someone's backyard last year. Let $B(t)$ the size of the bee population (in thousands) t months after April 1, 2012. This function is modeled in Figure 1.

FIGURE 1. Graph of $y = B(t)$



(a) Find $B(3)$. Explain what this function value represents in the context of the problem.

$B(3)=15$. On July 1st there were 15,000 bees.

(b) Find $B(0)$. Explain what this function value represents in the context of the problem.

$B(0)=10$. The starting population on April 1st was 10,000 bees.

(c) Solve $B(t) = 13$. Explain what this solution set represents in the context of the problem.

$B(t)=13$ when $t=1$ or 5. There were 13,000 bees on May 1st and Sept 1st.

$\{1,5\}$

(d) Solve $B(t) = 3$. Explain what this solution set represents in the context of the problem.

$B(t)=3$ when $t=8$. There were 3,000 bees on December 1st.

(e) State the domain and range of B.

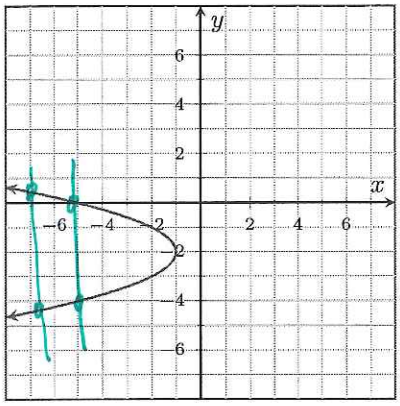
Domain: $[0, 9]$

Range: $[0,15]$

Example 2. Determine if the graphs in Figures 2-5 are functions. State the domain and range for each graph.

Continuous

FIGURE 2

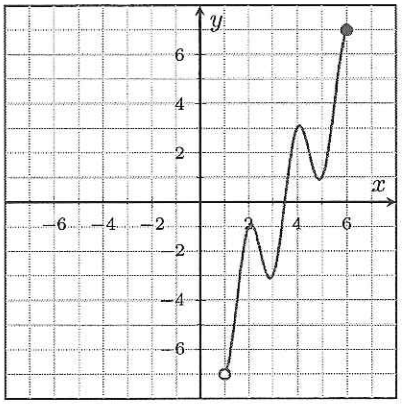


Function? NO. It fails the vertical line test

Domain: $(-\infty, -1]$

Range: $(-\infty, \infty)$ R

FIGURE 3

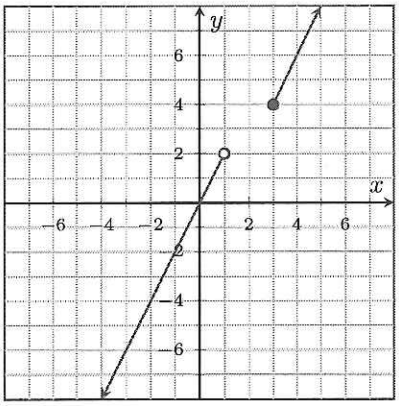


Function? Yes, it passes the vertical line test

Domain: $(1, 6]$

Range: $(-7, 7]$

FIGURE 4

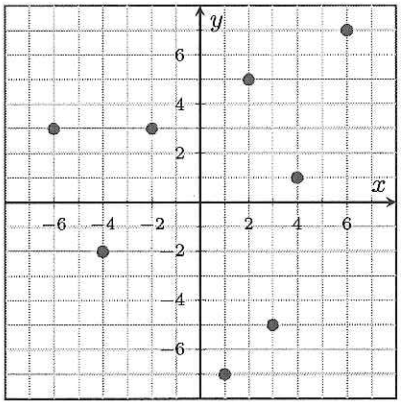


Function? yes, it passes the vertical line test.

Domain: $(-\infty, 1) \cup [3, \infty)$

Range: $(-\infty, 2) \cup [4, \infty)$

FIGURE 5



Discrete

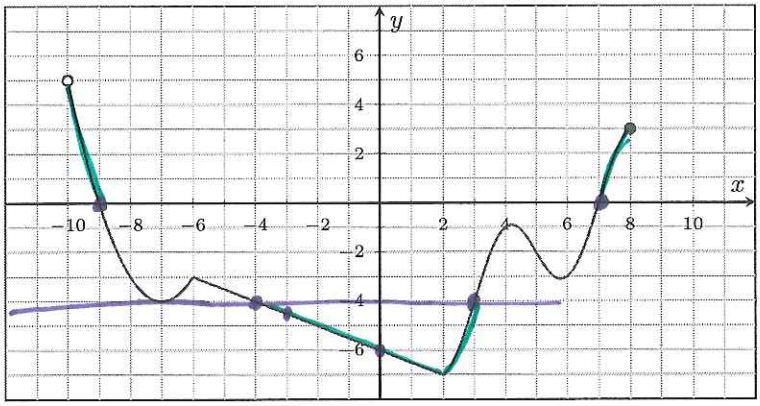
Function? yes, it passes the vertical line test

Domain: $\{-6, -4, -2, 1, 2, 3, 4, 6\}$

Range: $\{-7, -5, -2, 1, 3, 5, 7\}$

Example 3. Use the graph of $y = f(x)$ in Figure 6 to answer the following.

FIGURE 6. Graph of $y = f(x)$



(a) Find $f(0)$ and $f(-2)$.

$f(0) = -6$

$f(-2) = -5$

(b) Evaluate f at 4.

$f(4) = -1$

(c) Is $f(-3)$ positive or negative?

negative

(d) For what values of x is $f(x) = 0$?

$y = 0$

at $x = -9$ and $x = 7$

(e) State the zeros of f. (x-intercept)

-9 and 7

(f) What are the horizontal intercepts?

$(-9, 0)$ and $(7, 0)$

(g) What is the vertical intercept?

$(0, -6)$

(h) Solve $f(x) = -4$. State the solution set.

$y = -4$

$\{-7, -4, 3\}$ or $\{x \mid x = -7, -4, 3\}$

(i) For what values of x is $f(x) < -4$?

$(-4, 3)$

(j) For what values of x is $f(x) > 0$?

$y > 0$

$(-10, -9) \cup (7, 8]$

(k) State the domain of f.

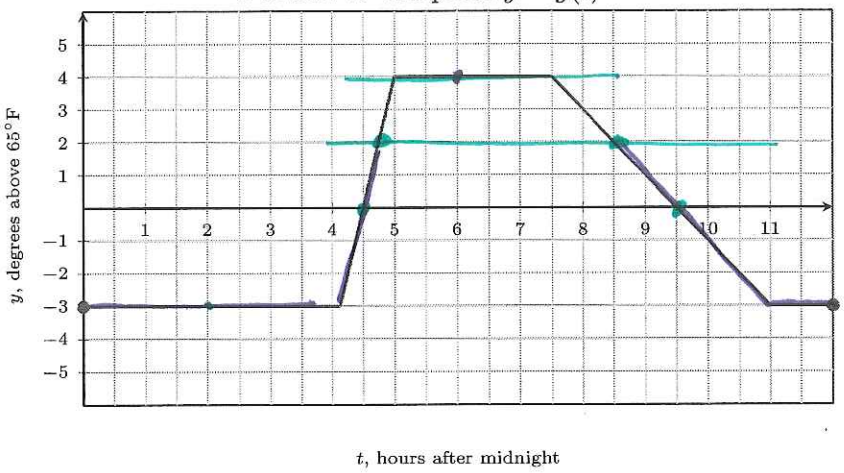
$(-10, 8]$

(l) State the range of f.

$[-7, 5)$

Group Work 1. Assume the base temperature setting for the thermostat in a house is 65°F. Let $g(t)$ be the number of degrees above 65°F t hours after midnight. Answer the following use the graph of $y = g(t)$ in Figure 7.

FIGURE 7. Graph of $y = g(t)$



(a) Find and interpret $g(6)$.

$g(6) = 4$
The temperature at 6 am is 69°F.

(b) Solve and interpret $g(t) = 2$.

$g(t) = 2$ when $t \approx 4.75$ and $t = 8.5$
The temperature is 67°F at 4:45am and 8:30am

(c) Is $g(2)$ positive or negative?

negative

(d) What are the zeros of g?

4.5 and 9.5

(e) What are the horizontal intercepts?

4.5 and 9.5
(4.5, 0) and (9.5, 0)

(f) What is the vertical intercept?

−3
(0, −3)

(g) For what values of t is $g(t) = 4$?

$[5, 7.5]$

(h) For what values of t is $g(t) < 2$?

$[0, 4.75) \cup (8.5, 12]$

(i) For what values of t is $g(t) \geq 0$?

$[4.5, 9.5]$

(j) State the domain and range of g.

Domain: $[0, 12]$
Range: $[-3, 4]$

1. Put the function $g(x) = 0.5x^2 + 5x - 35$ into your graphing calculator and find a good viewing window where the graph fills the window and you can see all of the relevant features.

a. Write down your window settings:

x-min = -20 x-max = 10 x-scale = 1

y-min= -50 y-max = 10 y-scale = 1

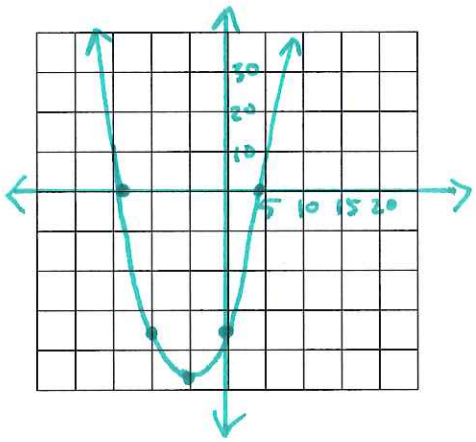
b. Use the graphing features to find the x-intercepts, y-intercepts, maximum or minimum.

x-intercepts: $(-14.7, 0)$ $(4.7, 0)$

y-intercept: $(0, -35)$

Max or min: $(-5, -47.5)$

c. Use the table feature to find some nice points to plot. You can also use your points from part b. Label your axes and scale:



2. **Effect of Elevation on Weight.** If an object weighs m pounds at sea level, then it's weight W (in pounds) at a height of h miles above sea level is given approximately by

$$W(h) = m\left(\frac{4000}{4000+h}\right)^2 = 150\left(\frac{4000}{4000+h}\right)^2$$

a. For a 150-pound person, enter the function into your graphing calculator and find a viewing window that allows you to see the relevant part of the graph. (How high above sea level is reasonable?)

x-min = 0 x-max = 5 x-scale = 1

y-min= 0 y-max = 160 y-scale = 10

(d) Another way of measuring prosperity is to calculate the per capita food supply, $R(t)$, which is defined by $\frac{N(t)}{P(t)}$. Find and simplify $R(t)$.

$$R(t) = \frac{N(t)}{P(t)}$$

$$= \frac{4+.5t}{.01t^2+2}$$

(e) Evaluate and interpret $R(45)$.

$$R(45) = \frac{4+.5(45)}{.01(45)^2+2}$$

$$= \frac{26.5}{22.25}$$

$$\approx 1.19$$

The county could feed 119% of its population.

(f) Evaluate and interpret $R(55)$.

$$R(55) = \frac{4+.5(55)}{.01(55)^2+2}$$

$$= \frac{31.5}{32.25}$$

$$\approx .98$$

The county could feed 98% of its population.

(g) The two functions are graphed in Figure 1. Use your calculator to find this point of intersection. List each coordinate accurate to three decimal places. Then interpret their point of intersection.

$P(t) = .01t^2 + 2$

$N(t) = 4 + .5t$

Use intersection on the graphing calculator

(53.723, 30.861)

The food supply equals the population in 2053, where there are 30.86 million people.

FIGURE 1

