Math 111 - Mon, 5/23

Q's on 3.1

New material: 3.4

Mission 4 Handed out → new teams

Checkpoint 7 on Wed (Section 3.1)

No class next Monday

3.1 #59 $f(x) = -2x^2(x^2-2)$

$-2x^2(x+\sqrt{2})(x-\sqrt{2})$ irrational zeros

a) $0, -\sqrt{2}, \sqrt{2}$

mult 2 1 1

b) touches (bounces) cross cross

c) 3 (degree - 1)

d) 1st term: $-2x^4$ ↙ ↓

as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to -\infty$

57. $f(x) = 3(x^2+8)(x^2+9)^2$

zeros: solve

$x^2+9=0$

$x^2=-9$

$x = \pm\sqrt{-9}$

not a real zero

$x(x^2+9)(x-3)^2$

a) no real zeros

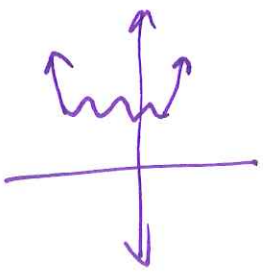
b) n/a

c) 5 (degree 6)

d) 1st term $3x^6$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to \infty$



67. $y = (x+1)(x-1)^2(x-2)$ $(0,1)$

$f(x) = k(x+1)(x-1)^2(x-2)$

$1 = k(1)(-1)^2(-2)$

$1 = k(-2)$

$\frac{1}{-2} = \frac{k(-2)}{-2}$

$-\frac{1}{2} = k$

$f(x) = -\frac{1}{2}(x+1)(x-1)^2(x-2)$

Math 111 Lecture Notes

Section 3.4: Properties of Rational Functions

A **rational function** is of the form $R(x) = \dfrac{p(x)}{q(x)}$ where p and q are polynomial functions.

Basic Rational Functions

Figure 1. Graph of $y = \frac{1}{x}$

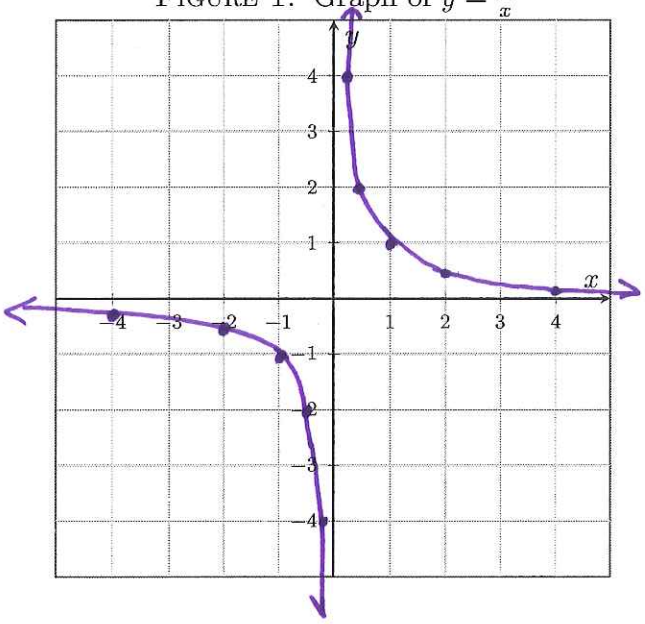


Table 1

x	$y = \frac{1}{x}$
-4	$-\frac{1}{4}$
-2	$-\frac{1}{2}$
-1	-1
$-1/2$	-2
$-1/4$	-4
0	undefined
$1/4$	4
$1/2$	2
1	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$

$R(x) = -\frac{2}{x}$

$\frac{1}{2}$ 1 2 4 8 undef −8 −4 −2 −1 $-\frac{1}{2}$

Figure 2. Graph of $y = \frac{1}{x^2}$

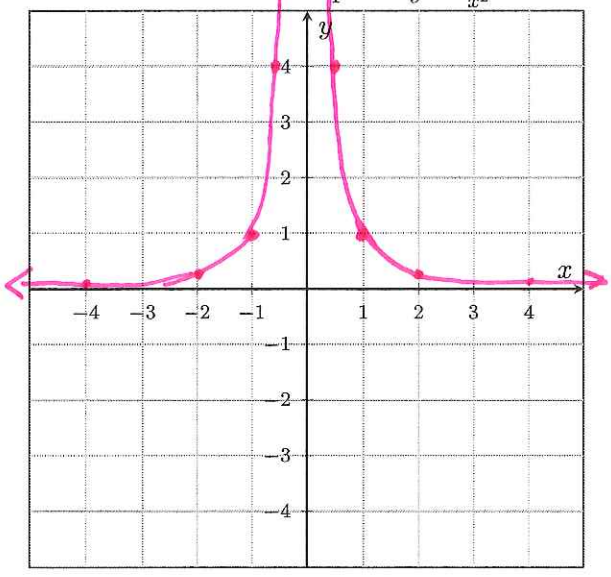


Table 2

x	$y = \frac{1}{x^2}$
-4	$\frac{1}{(-4)^2} = \frac{1}{16}$
-2	$\frac{1}{(-2)^2} = \frac{1}{4}$
-1	$\frac{1}{(-1)^2} = 1$
$-1/2$	4
$-1/4$	16
0	undefined
$1/4$	16
$1/2$	4
1	1
2	$\frac{1}{4}$
4	$\frac{1}{16}$

R(x)

Basic Rational Functions (close up)

FIGURE 3. Odd Powers

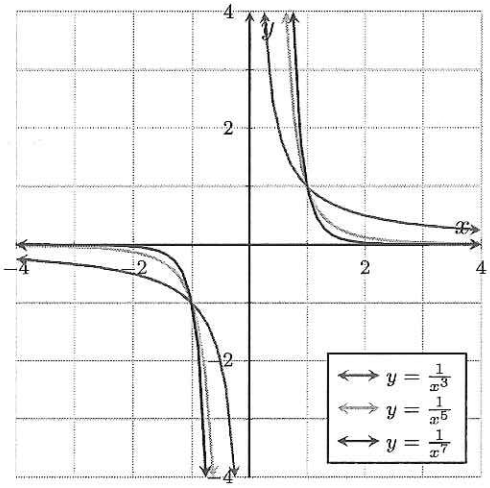
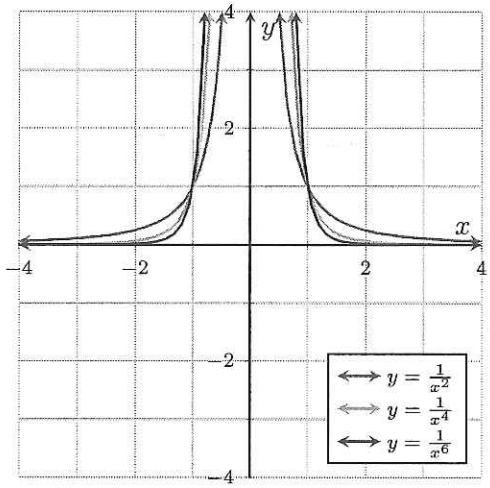


FIGURE 4. Even Powers



Example 1. Use transformations to graph the functions below. Clearly label any horizontal and vertical asymptotes.

(a) $R(x) = \dfrac{-2}{x}$ vertical flip, vertical stretch by 2

(b) $R(x) = \dfrac{1}{(x-1)^2} - 3$ right 1, down 3

FIGURE 5

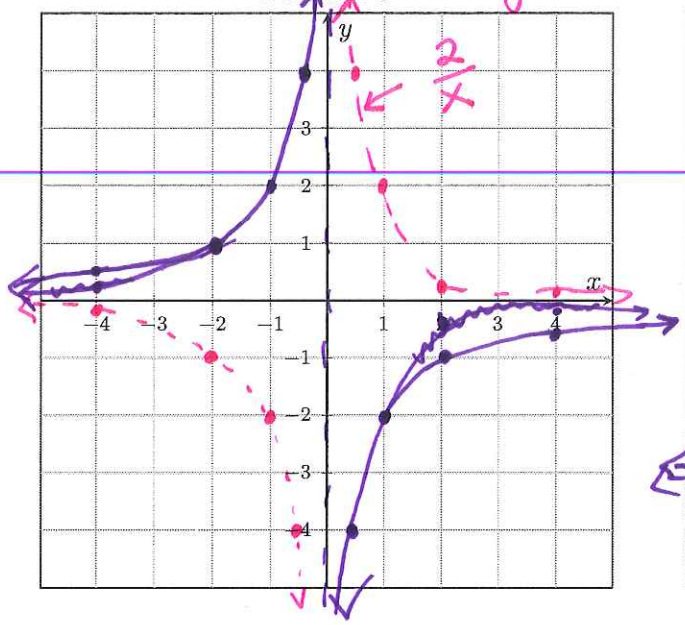
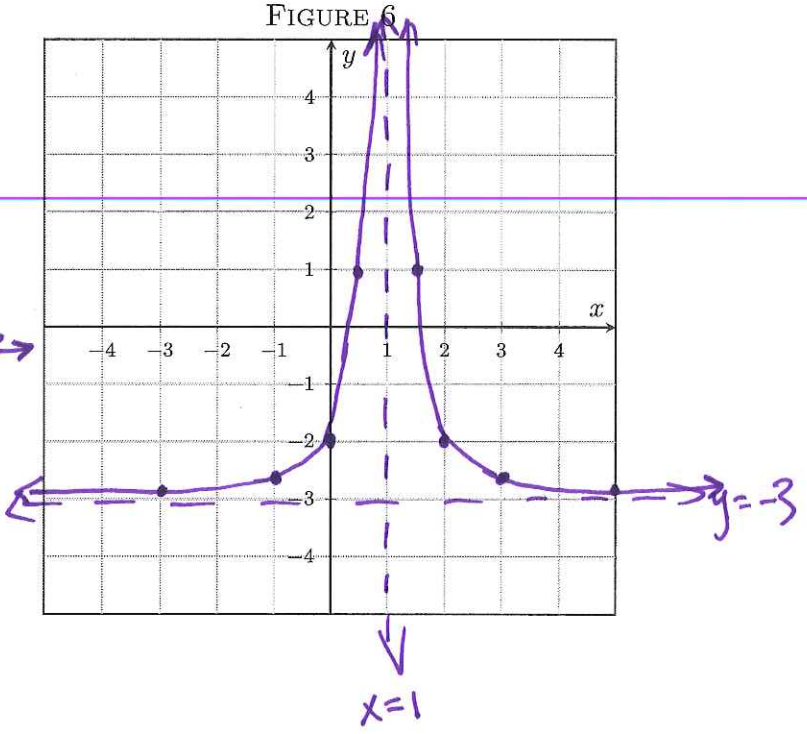


FIGURE 6



Determining the Horizontal Asymptote of a Rational Function

End Behavior

Let m be the degree of the function p in the numerator and let n be the degree of the function q in the denominator.

- If $m < n$, then the horizontal asymptote is $y = 0$.
- If $m = n$, then the horizontal asymptote is $y = c$ where c is a real number determined by the ratio of leading coefficients.
- If $m > n$, then no horizontal asymptote exists.

If $m = n + 1$, then an *oblique asymptote* exists.

1st term $\frac{ax^m}{bx^n}$

Example 2. Determine any horizontal asymptotes for the following rational functions.

(a) $R(x) = \dfrac{5x+1}{10x^2+6x}$

$\frac{5x}{10x^2} = \frac{1}{2x} \to 0$

Horizontal asymptote: $y = 0$

as $x \to \infty$, $R(x) \to 0$

as $x \to -\infty$, $R(x) \to 0$

(b) $R(x) = \dfrac{5x^2+1}{10x^2+6x}$

$\frac{5x^2}{10x^2} = \frac{1}{2}$

Horizontal asymptote: $y = \frac{1}{2}$

as $x \to \infty$, $R(x) \to \frac{1}{2}$

as $x \to -\infty$, $R(x) \to \frac{1}{2}$

(c) $R(x) = \dfrac{5x^3+1}{10x^2+6x}$

$y = mx+b$

$\frac{5x^3}{10x^2} = \frac{1}{2}x$

no horizontal asymptote

(oblique asymptote)

as $x \to \infty$, $R(x) \to \infty$

as $x \to -\infty$, $R(x) \to -\infty$

(d) $R(x) = \dfrac{5x^4+1}{10x^2+6x}$

$\frac{5x^4}{10x^2} = \frac{1}{2}x^2$

no horizontal asymptote

as $x \to \infty$, $R(x) \to \infty$

as $x \to -\infty$, $R(x) \to \infty$

The **zeros** of a rational function are the values of x for which $p(x) = 0$, as the function's value is zero where the value of the numerator is zero. Most of the time, the zeros will occur at a when the factor $(x - a)$ is in the numerator of R.

A rational function is undefined where $q(x) = 0$, as this would cause division by zero.

A **vertical asymptote** occurs when the denominator of the *simplified* form of R is equal to zero. Most of the time, the vertical asymptote $x = b$ will occur when the factor $(x - b)$ is in the denominator of the *simplified* form of R.

A **hole** occurs when <u>both</u> the numerator and denominator equal zero for some value of x. We will identify a zero at c when the linear factor $(x - c)$ occurs in both the numerator and denominator of a rational function. Note that during simplification this factor cancels and results in a domain restriction for R.

Example 3. Find the any zeros, vertical asymptotes, and horizontal asymptotes for each rational function below.

(a) $R(x) = \dfrac{x-5}{x+6}$ ← zeros ← domain vertical asymptotes

$\frac{x}{3} = 0$

zeros: $x - 5 = 0$

$x = 5$

Domain: $x + 6 \neq 0$

$x \neq -6$

$\{x \mid x \neq -6\}$

vertical asymptote

$x = -6$

Horizontal asymptote: $y = 1$

$\frac{x}{x} = 1$

(b) $R(x) = \dfrac{-10x}{5x-5} = \dfrac{-10x}{5(x-1)} = \dfrac{-2x}{x-1}$

zeros: $-10x = 0$

$x = 0$

Domain: $x - 1 \neq 0$

$x \neq 1$

$\{x \mid x \neq 1\}$

Vertical asymptote

$x = 1$

Horizontal Asymptote: $y = -2$

$\frac{-10x}{5x} = -2$

Example 4. Find the any zeros, holes, vertical asymptotes, and horizontal asymptotes for each rational function below. Factor each expression first and reduce to lowest terms if necessary.

(a) $R(x) = \dfrac{x^2 - 5x - 6}{x^2 + x - 12} = \dfrac{(x-6)(x+1)}{(x+4)(x-3)}$

zeros: $6, -1$

vertical asymptotes: $x = -4, x = 3$

Domain $\{x \mid x \neq -4, 3\}$

no holes

horizontal asymptote: $y = 1$

$\dfrac{x^2}{x^2} = 1$

(c) $R(x) = \dfrac{3}{x^3 - 4x} = \dfrac{3}{x(x^2-4)} = \dfrac{3}{x(x+2)(x-2)}$

zeros: none

vertical asymptotes: $x = 0, -2, 2$

Domain: $\{x \mid x \neq 0, -2, 2\}$

no holes

horizontal asymptote: $y = 0$

$\dfrac{3}{x^3} \to 0$ as $x \to \infty$

(b) $R(x) = \dfrac{3x - 6}{x^2 + x - 6} = \dfrac{3(x-2)}{(x+3)(x-2)}$ hole

zeros: none

Vertical asymptote: $x = -3$

Domain: $\{x \mid x \neq -3, 2\}$

hole at $x = 2$

horizontal asymptote: $y = 0$

$\dfrac{3x}{x^2} = \dfrac{3}{x} \to 0$ as $x \to \infty$

(d) $R(x) = \dfrac{2(x-1)(x+7)^2}{(x-1)(x+3)(x+4)}$ hole

zeros: -7

vertical asymptotes: $x = -3, -4$

Domain: $\{x \mid x \neq 1, -3, -4\}$

hole at $x = 1$

horizontal asymptote: $y = 2$

$\dfrac{2x^3}{x^3} = 2$

Group Work 1. Find the any zeros, holes, vertical asymptotes, and horizontal asymptotes for each rational function below. Factor each expression first and reduce to lowest terms if necessary.

(a) $R(x) = \dfrac{x^2+4x+3}{2x^2-8} = \dfrac{(x+3)(x+1)}{2(x^2-4)} = \dfrac{(x+3)(x+1)}{2(x+2)(x-2)}$

zeros: $-3, -1$

vertical asymptotes: $-2, 2$

Domain $\{x \mid x \neq -2, 2\}$

no holes

horizontal asymptote: $y = \frac{1}{2}$

$\dfrac{x^2}{2x^2} = \dfrac{1}{2}$

(b) $R(x) = \dfrac{8}{x^2-25} = \dfrac{8}{(x-5)(x+5)}$

zeros: none

vertical asymptotes: $5, -5$

Domain $\{x \mid x \neq -5, 5\}$

no holes

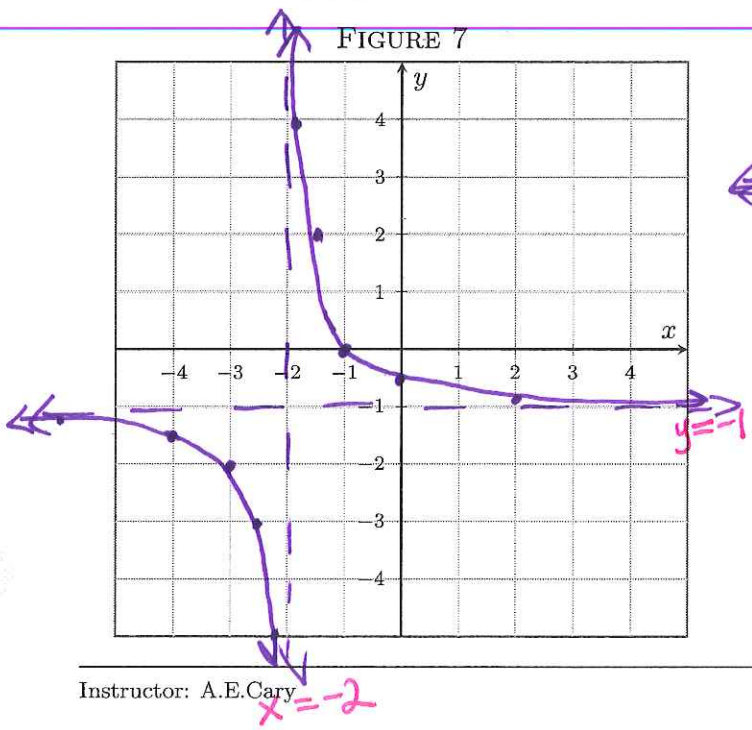
horizontal asymptote: $y = 0$

$\dfrac{8}{x^2} \to 0$

Group Work 2. Use transformations to graph the functions below. Clearly label any horizontal and vertical asymptotes.

(a) $R(x) = \dfrac{1}{x+2} - 1$ Left 2, down 1

FIGURE 7



(b) $R(x) = -\dfrac{1}{x^2} + 3$ Vertical flip, up 3

FIGURE 8

