Math 111 – Monday, 4/18

Please turn in MISSIONS!!
Hand back checkpoints
Questions on 1.5 + supplement
New Material: Section 4.1

1st Boss/Midterm next Monday, 4/25
Review/Study Guide handout (sections 1.1–1.5, 4.1–4.2)
Checkpoint 4 this Wednesday on 1.5

1.5 91. $p(x) = -.05x^2 + 100x - 2000$

$Y_1 = p(x) - 10{,}000$ tax of $10,000 per day

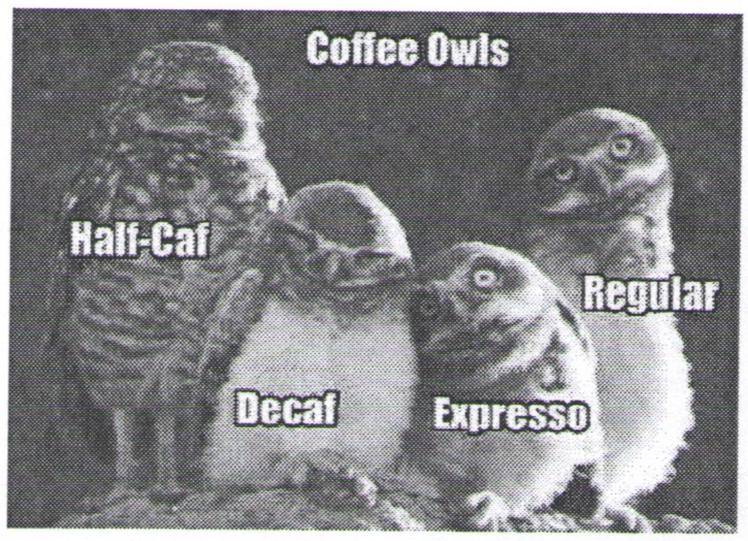
$Y_2 = (1 - .10)p(x)$

$.9p(x)$

Math 111 Lecture Notes

Section 4.1: Function Composition

When a person consumes caffeine, it is absorbed into their blood. Over time, the amount of caffeine in the bloodstream decreases (assuming they stop consuming caffeine). The result of caffeine being in the bloodstream is that the person's heart rate is elevated. This "chain reaction" is a simple example of a *composite function*. The person's heart rate depends on the amount of caffeine in their bloodstream, which depends on the amount of time since it was consumed. It makes sense then that we should be able to combine these two functions and determine person's heart rate at a given time.



Example 1. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

$$g(x) = -10x + 90, \qquad h(y) = 3y - 90$$

(a) Find and interpret $g(3)$.

$g(3) = -10(3) + 90$

$= -30 + 90$

$= 60$ mg

After 3 hours there are 60 mg of caffeine in your bloodstream

(b) Find and interpret $h(60)$.

$h(60) = 3(60) - 90$

$= 180 - 90$

$= 90$ bpm

not realistic

with 60mg of caffeine your heart rate is 90 bpm

(c) Find and interpret $h(g(3))$.

$h(g(3)) = h(60)$

$= 90$ bpm

$(h \circ g)(x)$

"h of g of x"

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read "f composed with g" or "f of g") is defined by

$$(f \circ g)(x) = f(g(x))$$

The function g is referred to as the *inside function* and the function f is referred to as the *outside function.*

In determining the domain for the composite function, the domain of the inside function and the domain for the resultant composite function must be accounted for.

Example 2. Use the functions f and g given in Table 1 to determine the following.

TABLE 1

x	-2	-1	0	1	2
$f(x)$	5	4	-3	2	0
$g(x)$	0	-2	6	9	-1

(a) $(g \circ f)(2)$

$g(f(2)) = g(0)$

$f(2)=0 \quad = 6$

(c) $(g \circ g)(-1)$

$g(g(-1)) = g(-2)$

$= 0$

(b) $(f \circ g)(2)$

$= f(g(2)) = f(-1)$

$g(2)=-1 \quad = 4$

(d) $(f \circ f)(-2)$

$f(f(-2)) = f(5)$

$=$ undefined

Example 3. Use Figure 1 to complete the following, if they exist.

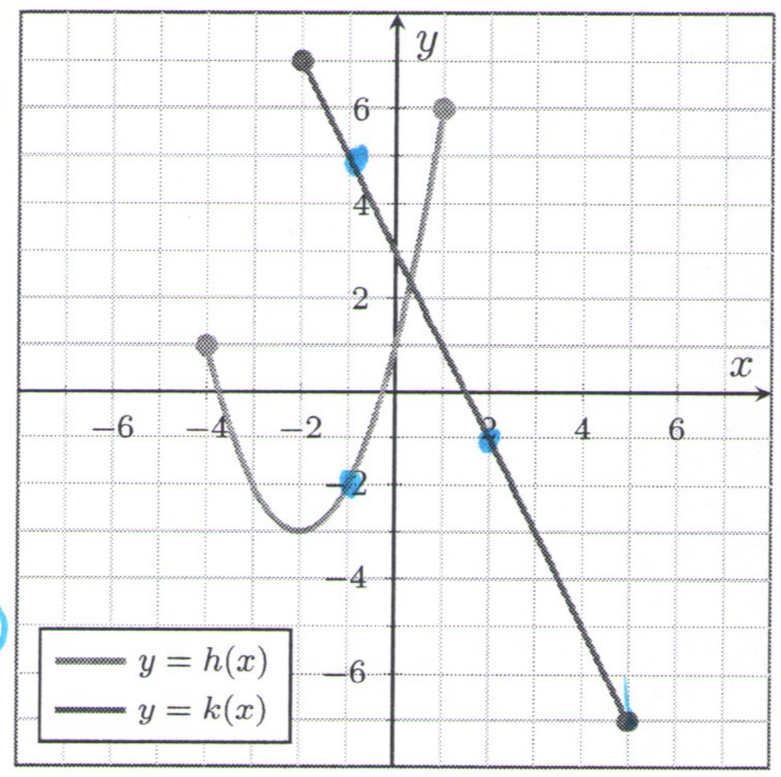
(a) $(h \circ k)(2)$

$h(k(2)) = h(-1)$
$= -2$

(c) $(h \circ h)(1)$

$h(h(1)) = h(6)$
$=$ undefined

FIGURE 1



(b) $(k \circ h)(-3)$

$k(h(-3)) = k(-2)$
$= 7$

(d) $(k \circ k)(-1)$

$k(k(-1)) = k(5)$
$= -7$

Example 4. Let $f(x) = \dfrac{\sqrt{x+4}}{3x-6}$ and $g(x) = |2x - 8|$. Compute the following:

(a) $(f \circ g)(-2)$

$f(g(-2)) = f(|2(-2)-8|)$
$= f(|-4-8|)$
$= f(|-12|)$
$= f(12)$
$= \dfrac{\sqrt{12+4}}{3(12)-6}$
$= \dfrac{\sqrt{16}}{30}$
$= \dfrac{4}{30}$
$= \dfrac{2}{15}$

(b) $(f \circ f)(-4)$

$f(f(-4)) = f\left(\dfrac{\sqrt{-4+4}}{3(-4)-6}\right)$
$= f\left(\dfrac{\sqrt{0}}{-18}\right)$
$= f(0)$
$= \dfrac{\sqrt{0+4}}{3(0)-6}$
$= \dfrac{\sqrt{4}}{0-6}$
$= \dfrac{2}{-6}$
$= -\dfrac{1}{3}$

Example 5. Let $j(x) = 5x^2 + 3x - 1$ and $k(x) = 2x + 7$. Find and fully simplify each of the following:

(a) $(k \circ j)(x)$

$k(j(x)) = k(5x^2+3x-1)$

$= 2(5x^2+3x-1)+7$

$= 10x^2+6x-2+7$

$= 10x^2+6x+5$

(b) $(k \circ k)(x)$

$k(k(x)) = k(2x+7)$

$= 2(2x+7)+7$

$= 4x+14+7$

$= 4x+21$

(c) $(j \circ k)(x)$

$j(k(x)) = j(2x+7)$

$= 5(2x+7)^2+3(2x+7)-1$

$= 5(2x+7)(2x+7)+6x+21-1$

$= 5(4x^2+28x+49)+6x+20$

$= 20x^2+140x+245+6x+20$

$= 20x^2+146x+265$

What is the domain of $k \circ j$? All real numbers

Example 6. Find $(g \circ f)(x)$ if $f(x) = \dfrac{7}{x+4}$ and $g(x) = \dfrac{3x}{2x-5}$. State the domain of $g \circ f$.

$f(x) = 2.5$

$(g \circ f)(x) = g(f(x))$

$= g\left(\frac{7}{x+4}\right)$

$= \dfrac{3\left(\frac{7}{x+4}\right)}{\left(2\left(\frac{7}{x+4}\right) - \frac{5}{1}\right)} \cdot \dfrac{\frac{x+4}{1}}{\frac{x+4}{1}} = \dfrac{\frac{3 \cdot 7}{x+4} \cdot \frac{x+4}{1}}{\frac{2 \cdot 7}{x+4}(x+4) - 5(x+4)}$

$= \dfrac{21}{14 - 5(x+4)}$

$= \dfrac{21}{14 - 5x - 20}$

$= \boxed{\dfrac{21}{-6-5x}}$ or $\dfrac{21}{-5x-6}$

Domain of $g \circ f$

↑ input

Domain of $g \circ f$: $-5x - 6 \neq 0$

$-5x \neq 6$

$\frac{-5x}{-5} \neq \frac{6}{-5}$ $\quad x \neq -\frac{6}{5}$

Domain of f: $\{x \mid x \neq -4\}$

Domain of $g \circ f$ $\{x \mid x \neq -4 \text{ or } -\frac{6}{5}\}$

Example 7. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

hours ↓ caffeine mg

$g(x) = -10x + 90$, $\quad h(y) = 3y - 90$ → heart rate

caffeine ↑ caffeine

Write the composite function $(h \circ g)(x)$. What does this function represent?

$(h \circ g)(x) = h(g(x))$

$= h(-10x + 90)$

$= 3(-10x + 90) - 90$

$= -30x + 270 - 90$

$= -30x + 180$ heart rate

↑ hours

$h(g(x))$ is the heart rate after x hours.

Ex 6.

or

$$\frac{3\left(\frac{7}{x+4}\right)}{2\left(\frac{7}{x+4}\right)-5} = \frac{\frac{21}{x+4}}{\underbrace{\frac{14}{x+4}-5}_{\text{LCD}}}$$

$$= \frac{\frac{21}{x+4}}{\frac{14}{x+4}-\frac{5(x+4)}{x+4}}$$

$$= \frac{\frac{21}{x+4}}{\frac{14}{x+4}-\frac{5x+20}{x+4}}$$

$$= \frac{\frac{21}{x+4}}{\frac{14-5x-20}{x+4}}$$

$$= \frac{21}{x+4} \cdot \frac{x+4}{-6-5x}$$

$$= \frac{21}{-6-5x}$$

Group Work 1. Let $f(x) = 5x - 7$, $g(x) = \dfrac{2x}{x-3}$, and $h(x) = \sqrt{4x+8}$. Find and fully simplify each of the following. Also state the domain of $g \circ f$ and $f \circ h$.

(a) $(g \circ h)(2)$

$g(h(2)) = g(\sqrt{4\cdot 2+8})$

$= g(\sqrt{8+8})$

$= g(\sqrt{16})$

$= g(4)$

$= \dfrac{2\cdot 4}{4-3}$

$= \dfrac{8}{1} = 8$

(b) $(g \circ f)(x)$

$g(f(x)) = g(5x-7)$

$= \dfrac{2(5x-7)}{5x-7-3}$

$= \dfrac{10x-14}{5x-10}$

$5x-10 \neq 0$

$5x \neq 10$

$x \neq 2$

Domain of $g \circ f$: $\{x \mid x \neq 2\}$

(c) $(f \circ f)(-4)$

$f(f(-4)) = f(5(-4)-7)$

$= f(-20-7)$

$= f(-27)$

$= 5(-27)-7$

$= -135-7$

$= -142$

(d) $(f \circ h)(x)$

$f(h(x)) = f(\sqrt{4x+8})$

$= 5\sqrt{4x+8} - 7$

$= 5\sqrt{4(x+2)} - 7$

$= 10\sqrt{x+2} - 7$

$4x+8 \geq 0$

$\frac{4x}{4} \geq \frac{-8}{4}$

$x \geq -2$

Domain of $f \circ h$

$\{x \mid x \geq -2\}$

Group Work 2. Use Table 2 and Figure 2 to complete the following, if they exist.

(a) $(a \circ m)(3)$ $= a(m(3))$
$= a(-2)$
$= 5$

(b) $(m \circ m)(4)$
$= m(m(4))$
$= m(1)$
$= -2$

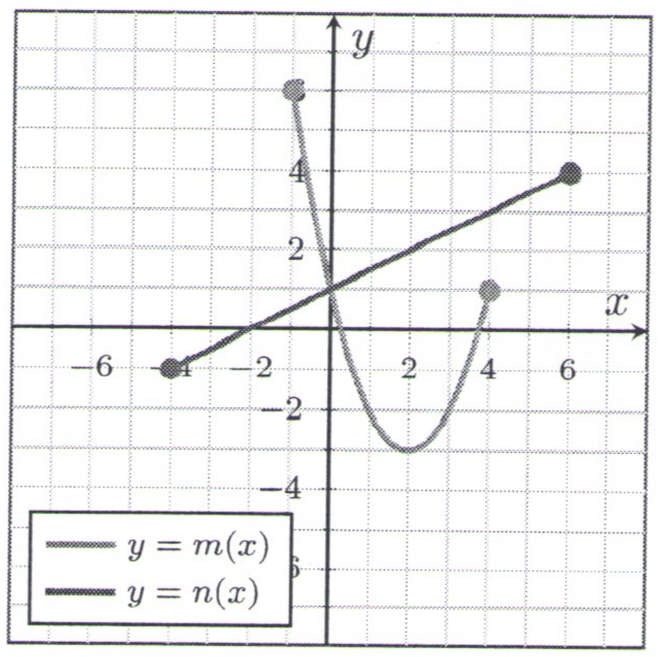
(c) $(b \circ a)(0)$
$= b(a(0))$
$= b(2)$
$= -4$

(d) $(n \circ b)(1)$
$= n(b(1))$
$= n(-9)$
$=$ undefined

TABLE 2

x	-2	-1	0	1	2
$a(x)$	5	4	2	-1	1
$b(x)$	7	2	0	-9	-4

FIGURE 2



Example 8. Let $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}(x - 5)$. Show that both $(f \circ g)(x) = x$ and that $(g \circ f)(x) = x$ for every x in the respective domains of $f \circ g$ and $g \circ f$.

$f(g(x)) = f(\frac{1}{3}(x-5))$
$= 3(\frac{1}{3}(x-5)) + 5$
$= x - 5 + 5$
$= x$

$g(f(x)) = g(3x+5)$
$= \frac{1}{3}(3x + 5 - 5)$
$= \frac{1}{3}(3x)$
$= x$

Example 9. For the following examples, find the functions f and g such that $H = f \circ g$. Do not choose $f(x) = x$ or $g(x) = x$.

(a) $H(x) = \sqrt{3x+1}$

$H(x) = f(g(x))$

$f(x) = \sqrt{x}$

$g(x) = 3x+1$

check $H(x) = f(g(x))$

$= f(3x+1)$

$= \sqrt{3x+1}$

(b) $H(x) = (5x-3)^2$

$H(x) = f(g(x))$

$f(x) = x^2$

$g(x) = 5x-3$

check $H(x) = f(g(x))$

$= f(5x-3)$

$= (5x-3)^2$

(c) $H(x) = \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}+1}$

$f(x) = \dfrac{x}{x+1}$

$g(x) = \sqrt[3]{x}$

if reversed f∘g

$\sqrt[3]{\dfrac{x}{x+1}}$

(d) $H(x) = (x^2-1)^3$

$f(x) = x^3$

$g(x) = x^2-1$

(e) $H(x) = \dfrac{2}{x-3}$

$f(x) = \dfrac{2}{x}$

$g(x) = x-3$

Difference Quotient

$$\frac{f(x+h)-f(x)}{h}, \quad h \neq 0$$

formula will be on the test

$$Af(B(x+h))+k$$