Math 111 – Mon, 5/9

Please turn in Mission 3 + Bonus

Q's on 4.5 (or other homework)

New material: 4.6

Checkpoint 6 on wed (4.4, 4.5) memorize log properties

Project due on wed

Big Boss #2 next Monday (sections 4.3 – 4.8)

Study guide handout

4.5 #23. $\log_2 6 \cdot \log_6 8 = 3$

$$\frac{\log 6}{\log 2} \cdot \frac{\log 8}{\log 6}$$

$$= \frac{\log 8}{\log 2}$$

$$= \log_2 8$$

$$= 3$$

Math 111 Lecture Notes

Section 4.6: Logarithmic and Exponential Equations

green diamond 7 on calculator = log

Example 1. Suppose the formula $D = 5e^{-0.4h}$ can be used to find the number of milligrams D of a certain drug in a patient's bloodstream h hours after the drug was administered. When the number of milligrams reaches 1.5, the drug is to be administered again. What is the time between injections?
In this problem, we solve the equation $1.5 = 5e^{-0.4h}$. We isolate the exponential expression and then convert to logarithmic form:

$\frac{1.5}{5} = \frac{5e^{-.4h}}{5}$

$.3 = e^{-.4h}$

$\log_e .3 = -.4h$

$\frac{\ln .3}{-.4} = \frac{-.4h}{-.4}$

$3.01 \approx h$

$a^x = y \iff x = \log_a y$

$e \approx 2.7\ldots$

$\log_e$

$\ln$

The time between injections is 3 hours.

What if the base was not 10 or e? What if we had the following instead: $1.5 = 5 \cdot 2^{-0.4h}$?

$\frac{1.5}{5} = \frac{5}{5} \cdot 2^{-.4h}$

$.3 = 2^{-.4h}$

$\frac{\log_2(.3)}{-.4} = \frac{-.4h}{-.4}$

$\left(\frac{\ln(.3)}{\ln 2}\right) = h$

$\frac{}{-.4}$

$4.34 \approx h$

$\frac{1.5}{5} = \frac{5}{5} \cdot 2^{-.4h}$

use base 10 or base e

$.3 = 2^{-.4h}$

$\ln(.3) = \ln 2^{-.4h}$ log property

$\frac{\ln(.3)}{-.4\ln 2} = \frac{-.4h \ln 2}{-.4 \ln 2}$

$4.34 \approx h$

Example 2. A population of fruit flies initially has 2 fruit flies and grows exponentially. The population P of fruit flies after t days can be modeled by $P = 2(1.2)^t$. When will the population reach 100 fruit flies?

$\frac{100}{2} = \frac{2(1.2)^t}{2}$

$50 = (1.2)^t$

$\log 50 = \log(1.2)^t$

$\log 50 = t \log(1.2)$

$\frac{\log 50}{\log 1.2} = \frac{t \log(1.2)}{\log 1.2}$

$t = \frac{\log 50}{\log 1.2}$

≈ 21.46

The population will reach 100 fruit flies in 21.46 day

Example 3. Julie and Mia each made investments in 1999.[1] Let $J(t)$ be Julie's investment t years after 1999 and let $M(t)$ be Mia's investment t years after 1999. Their respective investments can be modeled by the functions

$$J(t) = 50000(1.092)^t \quad \text{and} \quad M(t) = 45000e^{0.09t}$$

(a) When will Julie's investment have doubled in value?

$\frac{100,000}{50,000} = \frac{50,000(1.092)^t}{50,000}$

$2 = (1.092)^t$

$\log 2 = \log 1.092^t$

$\frac{\log 2}{\log 1.092} = \frac{t \log 1.092}{\log 1.092}$

$7.88 \approx t$

After about 7.88 years

(b) When will the value of Mia's investment reach \$100,000? Solve this equation algebraically and then using your graphing calculator.

$\frac{100,000}{45,000} = \frac{45,000e^{.09t}}{45,000}$

$\frac{20}{9} = e^{.09t}$

$\ln(20/9) = \ln e^{.09t}$ ← log property; all in exponent together

$\ln(20/9) = .09t(\ln e)$ $\ln e = 1$

$\frac{\ln(20/9)}{.09} = \frac{.09t}{.09}$

$t \approx 8.87$ years

Solve graphically

$y_1 = 100,000$

$y_2 = 45,000e^{.09t}$

$t \approx 8.87$

use intersection

[1] This was the year when Mia Hamm, Julie Foudy, and Brandi Chastain won the Women's World Cup and netted \$50,000 bonuses, but this problem is otherwise fictional.

Example 4. Solve the following exponential equations for x. Express irrational solutions in exact form. * — use base 10 or base e (log, ln)

(a) $3^{2x} = 15$

$\ln 3^{2x} = \ln 15$

$\frac{2x \ln 3}{2 \ln 3} = \frac{\ln 15}{2 \ln 3}$

$x = \frac{\ln 15}{2 \ln 3}$

use base e or base 10

$\left\{\frac{\ln 15}{2 \ln 3}\right\}$

(b) $4(2^{3x}) = 5$

$\frac{4}{4} \quad \frac{}{4}$

$2^{3x} = 5/4$

$\ln 2^{3x} = \ln(5/4)$

$\frac{3x \ln 2}{3 \ln 2} = \frac{\ln(5/4)}{3 \ln 2}$

$x = \frac{\ln(5/4)}{3 \ln 2}$

$\left\{\frac{\ln(5/4)}{3 \ln 2}\right\}$

(c) $2^{-x} = 14$

$\log 2^{-x} = \log 14$

$-x \log 2 = \log 14$

$\frac{x \log 2}{\log 2} = \frac{-\log 14}{\log 2}$

$= \frac{-\log 14}{\log 2}$

$\left\{-\frac{\log 14}{\log 2}\right\}$

(d) $3^x = 15^{x+1}$

$\log 3^x = \log 15^{x+1}$

$x \log 3 = (x+1) \log 15$

$x \log 3 = x \log 15 + \log 15$

$x \log 3 - x \log 15 = \log 15$

$x(\log 3 - \log 15) = \log 15$

$x \frac{\log(3/15)}{\log(1/5)} = \frac{\log 15}{\log(1/5)}$

$x = \frac{\log 15}{\log(1/5)}$

$\left\{\frac{\log 15}{\log(1/5)}\right\}$

Example 5. Solve the following exponential equations for x. Express irrational solutions in exact form and rounded to three decimal places.

$$2^{x+3} = 5^{2x+1}$$

$$\ln 2^{x+3} = \ln 5^{2x+1}$$

$$(x+3)\ln 2 = (2x+1)\ln 5$$

$$x\ln 2 + 3\ln 2 = 2x\ln 5 + \ln 5$$

$$x\ln 2 - 2x\ln 5 = \ln 5 - 3\ln 2$$

$$\frac{x(\ln 2 - 2\ln 5)}{\ln 2 - 2\ln 5} = \frac{\ln 5 - 3\ln 2}{\ln 2 - 2\ln 5}$$

$$x = \frac{\ln 5 - \ln 2^3}{\ln 2 - \ln 5^2}$$

$$= \left\{ \frac{\ln(5/8)}{\ln(2/25)} \right\}$$

$$x \approx .186$$

Example 6. Solve the following logarithmic equations for x. Clearly state the solution set.

(a) $\log_4(x+1) = \log_4(25)$ Same base

$x+1 = 25$

$x = 24$

$\{24\}$

$x+1 > 0$

Domain: $\{x \mid x > -1\}$

(b) $\frac{20}{20}\log_4(x) = \frac{10}{20}$

$\log_4 x = \frac{1}{2}$ ← exponent, ← power, ↖ base

$4^{\frac{1}{2}} = x$

$\sqrt{4} = x$

$2 = x$

$\{2\}$

Domain $\{x \mid x > 0\}$

Example 7. The loudness $L(x)$ (in decibels) of a sound of intensity x (measured in watts per square meter) is defined by $L(x) = 10\log\left(\frac{x}{I_0}\right)$, where $I_0 = 10^{-12}$ watts per square meter and represents the least intense sound that a human ear can detect.

(a) Normal conversation has an loudness level of 60 dB. What is the intensity of this sound?

$L(x)$

$\frac{60}{10} = \frac{10}{10}\log\left(\frac{x}{10^{-12}}\right)$

$6 = \log\left(\frac{x}{10^{-12}}\right)$

$(10^{-12})10^6 = \frac{x}{10^{-12}}(10^{-12})$

$x = 10^{-6}$

The intensity is 10^{-6} watts per square meter.

(b) A jet takeoff has an loudness level of 140 dB. What is the intensity of this sound?

$L(x)$

$\frac{140}{10} = \frac{10}{10}\log\left(\frac{x}{10^{-12}}\right)$

$14 = \log\left(\frac{x}{10^{-12}}\right)$

$10^{-12} \cdot 10^{14} = \frac{x}{10^{-12}} \cdot 10^{-12}$

$x = 10^2$

The intensity of the sound of a jet takeoff is 10^2 watts per square meter.

Example 8. Solve the following logarithmic equations for x. Clearly state the solution set.

(a) $\log_6(x+4)+\log_6(x+3)=1$

$\log_6[(x+4)(x+3)]=1$

$6^1=(x+4)(x+3)$

$6=x^2+3x+4x+12$

$0=x^2+7x+6$

$0=(x+6)(x+1)$

$x+6=0$ or $x+1=0$

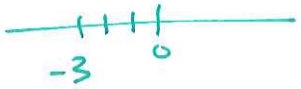
$x=\cancel{-6}$ or $x=-1$

extraneous solution

$\{-1\}$

$x+4>0$ and $x+3>0$

$x>-4$ and $x>-3$



$\log_6(-6+4)+\log(-6+3)=1$

undefined undefined

Check that solutions are in the domain when solving log equations

(b) $\log_7(x-1)-\log_7(x+5)=-1$

$\log_7\left(\frac{x-1}{x+5}\right)=-1$

$7^{-1}=\frac{x-1}{x+5}$

$\frac{1}{7}=\frac{x-1}{x+5}$ cross-multiply

$x+5=7(x-1)$

$x+5=7x-7$

$\frac{12}{6}=\frac{6x}{6}$

$2=x$

$\{2\}$

$x-1>0$ and $x+5>0$

$x>1$ $x>5$

Example 9. Solve the following logarithmic equations for x. Clearly state the solution set.

(a) $\log_2(x+1)+\log_2(x+7)=3$

$\log_2((x+1)(x+7))=3$

$2^3=(x+1)(x+7)$

$8=x^2+7x+x+7$

$0=x^2+8x-1$

$x=\dfrac{-8\pm\sqrt{64-4(1)(-1)}}{2(1)}$

$=\dfrac{-8\pm\sqrt{68}}{2}$

$=\dfrac{-8\pm\sqrt{4\cdot 17}}{2}$ $=\dfrac{-8\pm 2\sqrt{17}}{2}=-4\pm\sqrt{17}$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Domain

$x+1>0$ and $x+7>0$

$x>-1$ and $x>-7$



Domain $\{x \mid x>-1\}$

$-4-\sqrt{17}$ not in the domain $x>-8.12$

$\{-4+\sqrt{17}\}$

(b) $\ln(5x+1)-\ln(3x)=2$

$\ln\left(\dfrac{5x+1}{3x}\right)=2$

$3x\cdot e^2=\dfrac{5x+1}{3x}\cdot 3x$

$3xe^2=5x+1$

$3xe^2-5x=1$

$x(3e^2-5)=1$

$\dfrac{x(3e^2-5)}{3e^2-5}=\dfrac{1}{3e^2-5}$

$x=\dfrac{1}{3e^2-5}$

$x\approx .058$

OK

$\left\{\dfrac{1}{3e^2-5}\right\}$

Domain

$5x+1>0$ and $3x>0$

$5x>-1$ $x>0$

$x>-\frac{1}{5}$ $x>0$

$\{x \mid x>-\frac{1}{5}\}$

Example 10. Solve the following logarithmic equations for x. Clearly state the solution set.

(a) $\log_5(x+3) = 1 - \log_5(x-1)$

put the logs on the same side.

$\log_5(x+3) + \log_5(x-1) = 1$

$\log_5((x+3)(x-1)) = 1$

$5^1 = (x+3)(x-1)$

$5 = x^2 - x + 3x - 3$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x+4=0$ or $x-2=0$

$x = -4$ or $x = 2$

extraneous

$\{2\}$

Domain

$x+3>0$ and $x-1>0$

$x>-3$ and $x>1$

$\{x \mid x>-3\}$

Example 11. The following equations cannot easily be solved algebraically. Use your graphing calculator solve each equation, rounded accurately to three decimal places.

(a) $\log_2(x-1) - \log_6(x+2) = 2$

$y_1 = \frac{\log(x-1)}{\log 2} - \frac{\log(x+2)}{\log 6}$

$y_2 = 2$

$x \approx 12.149$

(b) $e^{2x} = x+2$

$y_1 = e^{2x}$

$y_2 = x+2$

2 points of intersection

$x \approx -1.981$ and $x \approx .448$

my window

$[-2.5, 1, 1]$ by $[-.5, 3, 1]$