Math 111, Thurs, 5/5

Welcome Julia!

Questions on 4.2 and 4.3

Checkpoint 5 (on 4.2 and 4.3) ← up through page 5 of notes packet

Page 9 from 4.3 packet

New material:

4.4 + Supplement

Bonus self-reflection due Tues, 5/10

Project due Thurs, 5/12

x	f(x)
-2	8
-1	4
0	1
1	2
2	3

$f^{-1}(x)$ →

x	$f^{-1}(x)$
8	-2
4	-1
1	0
2	1
3	2

$f^{-1}(-2) =$

#33 (4.2)

$f(x) = 3x+4 \qquad g(x) = \frac{1}{3}(x-4)$

$f(g(x)) = 3\left[\frac{1}{3}(x-4)\right] + 4$

$= 1(x-4)+4$

$= x-4+4$

$3\left(\frac{1}{3}x - \frac{4}{3}\right)+4$

$x-4+4$

From midterm

x	1	2	3	4	5
f(x)	5	4	0	1	3
g(x)	4	3	2	0	-1

$f(g(1)) = f(4)$
$= 1$

$g^{-1}(4) = 1$

$f^{-1}(g(1)) = f^{-1}(4)$
$= 2$

$g(f(3)) = g(0)$
$=$ undefined

$f(4) = 1$
input x
output = f(x)

$f^{-1}(4) = 2$
input: f(x)
outputs: x

$\frac{a^m}{a^n} = a^{m-n}$

$(x-1)^3 = (x-1)(x-1)(x-1)$

Math 111 Section 4.4 Logarithms

Solve each equation. Look for similarities and differences in the processes.

1. $3x^2 + 4 = 52$

$3x^2 = 48$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

$\{\pm 4\}$

2. $4 - (x-1)^3 = 0$

$(-1)[-(x-1)^3] = (-4)(-1)$

$\sqrt[3]{(x-1)^3} = \sqrt[3]{4}$

$x - 1 = \sqrt[3]{4}$

$x = 1 + \sqrt[3]{4}$

$\{1 + \sqrt[3]{4}\}$

3. $\left(\sqrt[4]{x+4}\right)^4 = (7)^4$

$x + 4 = 2401$

$x = 2397$

$\{2397\}$

Similarities:

exponents & roots

inverse functions

Differences:

even roots → two solutions (±)

Inverse Functions:

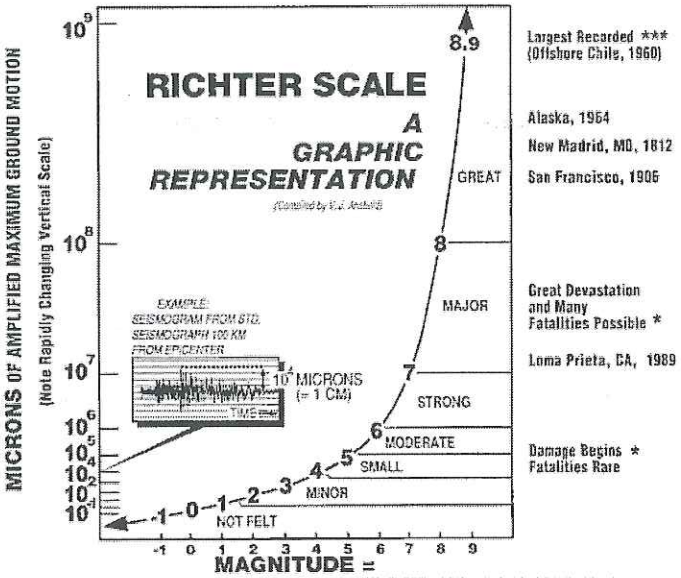
$y = x^2, x \geq 0$	$y = \sqrt{x}, x \geq 0$
Input: a number	Input: the square of a number
Output: the square of the number	Output: the number

$y = \sqrt[3]{x}$	$y = x^3$
Input: the cube of a number	Input: a number
Output: the number	Output: the cube of the number

$y = 10^x$	$y = log_{10}x$
Input: an exponent	Input: a power of the given base (ex: 100)
Output: a power of the given base	Output: the exponent that gives that power

Logarithms are Exponents! Logarithms are Fun!

Logarithmic Scales: The Richter Scale, pH Levels and Decibels



LOGARITHM (BASE 10) OF MAXIMUM AMPLITUDE MEASURED IN MICRONS **

* EFFECTS MAY VARY GREATLY DUE TO CONSTRUCTION PRACTICES, POPULATION DENSITY, SOIL DEPTH, FOCAL DEPTH, ETC.

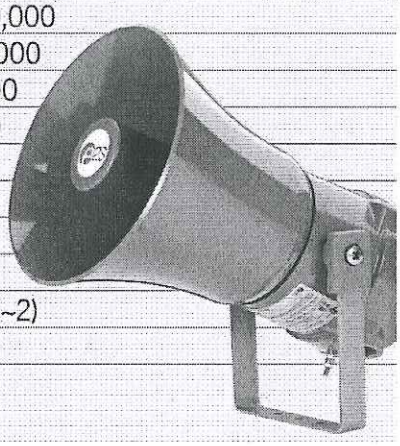
** MICRON = A MILLIONTH OF A METER

*** EQUIVALENT TO A MOMENT MAGNITUDE OF 5.1

$$pH = -\log[H^+]$$

0	10^{0}	1
1	10^{-1}	0.1
2	10^{-2}	0.01
3	10^{-3}	0.001
4	10^{-4}	0.0001
5	10^{-5}	0.00001
6	10^{-6}	0.000001
7	10^{-7}	0.0000001
8	10^{-8}	0.00000001
9	10^{-9}	0.000000001
10	10^{-10}	0.0000000001
11	10^{-11}	0.00000000001
12	10^{-12}	0.000000000001
13	10^{-13}	0.0000000000001
14	10^{-14}	0.00000000000001

dB	Power Ratio
150	1,000,000,000,000,000
140	100,000,000,000,000
130	10,000,000,000,000
120	1,000,000,000,000
110	100,000,000,000
100	10,000,000,000
90	1,000,000,000
80	100,000,000
70	10,000,000
60	1,000,000
50	100,000
40	10,000
30	1,000
20	100
10	10
6	3.981
3	1.995 (~2)
1	1 .259
0	1



Math 111 Lecture Notes

Section 4.4: Logarithmic Functions

We looked at equations such as $2^{3x-1} = 32$ in the last section by rewriting 32 with a base of 2: $2^{3x-1} = 2^5$. But what if we have an equation such as $2^x = 10$? We know intuitively that $3 < x < 4$, but can we give an *exact* answer? We will need functions that are inverse functions to exponential functions in order to solve such equations.

$2^x = 10 \rightarrow x = \log_2(10)$

The **logarithmic function to the base** a, where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a(x)$ and is defined by

$y = \log_a(x)$ if and only if $x = a^y$

(exponent, power, base)

The **common logarithmic function** is the logarithmic function with base 10 given by $f(x) = \log(x)$. We write

$$\log(x) \text{ to represent } \log_{10}(x)$$

The **natural logarithmic function** is the logarithmic function with base e given by $f(x) = \ln(x)$. We write

$$\ln(x) \text{ to represent } \log_e(x)$$

Example 1. Solve the equation $2^x = 10$ by converting from exponential form to logarithmic form.

$x = \log_2(10)$ ← exact solution

$x = \frac{\log 10}{\log 2}$ ← next section "change-of-base" formula

$x \approx 3.32$

$2^{3.32} \approx 9.99$

a little error because of rounding

Example 2. Change the exponential statement to an equivalent statement using logarithms.

(a) $3^t = 5$

$t = \log_3(5)$

(c) $10^m = 7$

$m = \log(7)$

(b) $\left(\frac{1}{4}\right)^x = 6$

$x = \log_{1/4} 6$

$(4^{-1})^x = 6$

$4^{-x} = 6$

$-x = \log_4(6)$

$x = -\log_4(6)$

(d) $e^{2t} = 5$

$2t = \ln(5)$ $\quad 2t = \log_e(5)$

$t = \frac{\ln(5)}{2}$

Example 3. Change the logarithmic statement to an equivalent statement using exponents.

(a) $\log_3(5) = x$

$3^x = 5$

(c) $\log(x) = \frac{1}{2}$

$10^{1/2} = x$

(b) $\log_2(y+1) = -3$

$2^{-3} = y+1$

(d) $\ln(x) = 5$

$e^5 = x$

Example 4. If $f(x) = 2^x$, then the inverse function of f is given by $f^{-1}(x) = \log_2(x)$.

(a) Complete Table 1 and then sketch the graph of $y = f(x)$ in Figure 1.

(b) Use the properties of inverse functions to graph $y = f^{-1}(x)$.

(c) Use the properties of inverse functions to complete Table 2.

(d) Identify the domain, range and any asymptotes for each function.

FIGURE 1

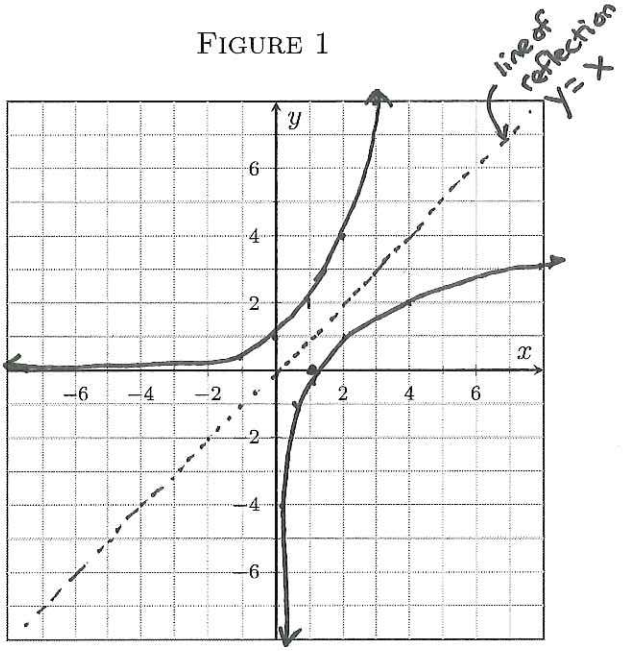


TABLE 1

x	2^x
-4	1/16
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

D: $(-\infty, \infty)$

R: $(0, \infty)$

Horizontal Asymptote $y = 0$

TABLE 2

x	$\log_2(x)$
$\frac{1}{16}$	-4
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

D: $(0, \infty)$

R: $(-\infty, \infty)$

Vertical Asymptote $x = 0$

Example 5. State the domain of the following functions using interval notation.

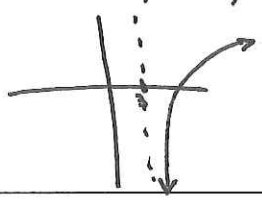
(a) $f(x) = \log_6(x - 3)$

right by 3

$x - 3 > 0$

$x > 3$

D: $(3, \infty)$



(b) $g(x) = \log(8 - x)$

$g(x) = \log(-x + 8)$

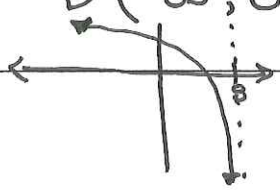
$g(x) = \log[-(x - 8)]$

right 8, reflect over the y-axis

D: $(-\infty, 8)$

$8 - x > 0$

$8 > x$



(c) $h(x) = \log_5(x^2 - 4)$

input > 0

$x^2 - 4 > 0$

$\sqrt{x^2} > \sqrt{4}$

$x > 2$ or $x < -2$

$(-\infty, -2) \cup (2, \infty)$

Example 6. Find the exact value of each logarithmic expression without using a calculator.

(a) $\log_5(25) = x$

what exponent on base 5 will give us 25?

$5^x = 25$

$x = 2$

(b) $\log_3(3^{-2}) = -2$

$\log_3(3^{-2}) = x$

$3^x = 3^{-2}$

$x = -2$

(c) $\ln(e^5) = 5$

(d) $\log_2(32) = 5$

$\log_2(32) = x$

$2^x = 32$

$x = 5$

(e) $\log_2\left(\frac{1}{16}\right) = -4$

$\log_2(2^{-4})$

(f) $\ln\left(\frac{1}{e}\right) = x = -1$

$e^x = 1/e$

$x = -1$

$e^{-1} = \frac{1}{e}$

(g) $\log(1000) = 3$

$10^x = 1000$

$x = 3$

(h) $\log_4(1) = 0$

$4^x = 1$

(i) $\log_6(\sqrt{6}) = 1/2$

$6^x = \sqrt{6}$

$x = 1/2$

$\sqrt{6} = 6^{1/2}$

Group Work 1. Find the exact value of each logarithmic expression.

(a) $\log_4(64) = 3$

$4^3 = 64$

(b) $\log_6\left(\frac{1}{36}\right) = -2$

$6^{-2} = \frac{1}{36}$

(c) $\log(100) = 2$

$10^2 = 100$

(d) $\ln(e) = 1$

$e^1 = e$

$y = \log_a x$ iff $x = a^y$

Example 7. Solve the following equations. Check all proposed solutions in your calculator and state the solution set.

(a) $e^{-2x+1} = 13$

$\ln(13) = -2x + 1$

$\ln(13) - 1 = -2x$

$x = \frac{\ln(13)-1}{-2}$

$\left\{\frac{\ln(13)-1}{-2}\right\}$

Check

$e^{-2\left(\frac{\ln(13)-1}{-2}\right)+1} \stackrel{?}{=} 13$

$13 = 13$ ✓

(b) $8 \cdot 10^{5x} = 3$

$10^{5x} = 3/8$

$\log\left(\frac{3}{8}\right) = 5x$

$x = \frac{\log(3/8)}{5}$

(c) $\log_3(5x - 4) = 2$

$5x - 4 = 3^2$

$5x - 4 = 9$

$5x = 13$

$x = 13/5$

$\{13/5\}$

(d) $\log_2(x^2 + 1) = 3$

$x^2 + 1 = 2^3$

$x^2 + 1 = 8$

$\sqrt{x^2} = \sqrt{7}$

$x = \pm\sqrt{7}$

$\{\pm\sqrt{7}\}$

$a^y = x \iff \log_a x = y$ (exponent, base, power/answer)

Example 8. Let $p(h)$ be the atmospheric pressure on an object (measured in millimeters of mercury) that is h kilometers above sea level. The function p can be modeled by

$$p(h) = 760e^{-0.145h}$$

Find the height of a mountain where the atmospheric pressure is 620 millimeters of mercury.

$p(h)$

$\frac{620}{760} = \frac{760e^{-.145h}}{760}$

$\frac{31}{38} = e^{-.145h}$

$\frac{\ln(\frac{31}{38})}{-.145} = \frac{-.145h}{-.145}$

$h = \frac{\ln(31/38)}{-.145}$

≈ 1.4 km

The mountain is 1.4 km tall.

Example 9. The pH of a chemical solution is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+]$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per liter. Values of pH range from 0 (acidic) to 14 (alkaline).

(a) What is the pH of a solution for which the concentration of hydrogen ions ($[\text{H}^+]$) is 0.01?

$\text{pH} = -\log_{10}(.01)$

$= 2$

$10^{-pH} = .01$

$10^{-2} = .01$ $(\frac{1}{100})$

$ph = +2$

(b) What is the concentration of hydrogen ions ($[\text{H}^+]$) in a banana with a pH of 4.5?

$\frac{4.5}{-1} = \frac{-\log_{10}([H^+])}{-1}$

$-4.5 = \log_{10}([H^+])$

$10^{-4.5} = [H^+]$

$.000032 = [H^+]$