

## Chapter 2 – Integers

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## Introduction to Integers

Name \_\_\_\_\_

1. Discuss each of the following situations with your group and write about what it means for a number to be positive, negative and zero.

a. Temperature

b. Checking account balance

c. Elevation

d. Profit for a business

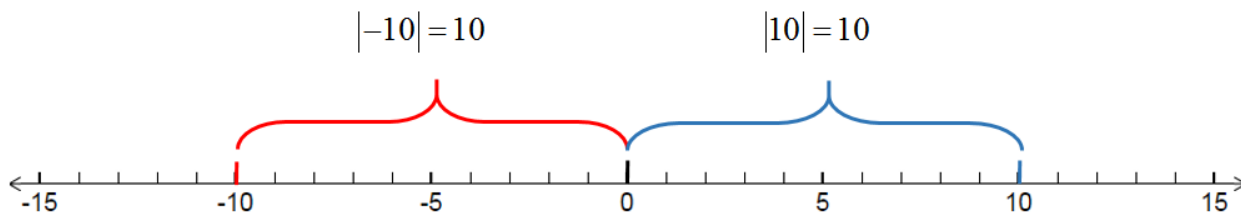
2. Which is warmer:  $-3^{\circ} F$  or  $-5^{\circ} F$  ?

3. How much warmer is  $20^{\circ} F$  than  $-10^{\circ} F$  ?

4. Write the opposite of each integer described using words and symbols.

- a. We were 1000 feet above sea level.
- b. The All-Star Brewing Company had a profit of \$23,500 in 2015.
- c. The temperature in degrees Celsius was 12 below zero.

As you noticed in problem 4, the opposite of a negative number is positive and the opposite of a positive number is negative. The **absolute value** of a number is the distance from zero to the number on a number line. In the example shown below,  $-10$  and  $10$  both have an absolute value of  $10$  because they are both ten units away from zero. The absolute value, written with the number between two vertical bars, represents the value of the number without the sign.



5. Complete the following table. The last 2 lines should not be identical.

Number	Opposite	Absolute Value
2		
3		
-5		
0		
	-7	
	11	
		13
		13

6. Arrange the following numbers in order from smallest to largest:  $11, -7, 0, 5, -3, -13, 2$

## Adding Integers with Context

Name \_\_\_\_\_

For many students it is easier to remember sign rules when they put the problems into context. The two most useful contexts for adding integers, or signed whole numbers, are money and the number line. Try both and see which one you prefer.

### MONEY

1.  $\$6 + \$7$

(you have \$6, then get \$7 more)

2.  $\$2 + (-\$7)$

(you have \$2, then spend 7)

3.  $-\$3 + (-\$9)$

(you spend \$3, then spend \$9)

4.  $-\$10 + \$2$

(you spend \$10, then get \$2 back)

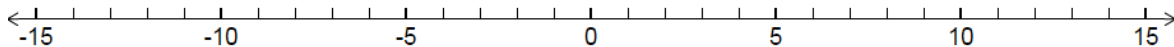
5.  $11 + (-4)$

6.  $-14 + 7$

7.  $-3 + 3$

8.  $-9 + (-9)$

### THE NUMBER LINE



9.  $4 + 5$

(start on 4, move right 5)

10.  $4 + (-5)$

(start on 4, move left 5)

11.  $-4 + (-5)$

(start on -4, move left 5)

12.  $-7 + 3$

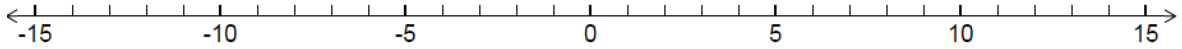
(start on -7, move right 3)

13.  $15 + (-6)$

14.  $-13 + 2$

15.  $-8 + 8$

16.  $-6 + (-6)$



Use either context to do these problems. Show your steps when there is more than one operation.

17.  $-7 + (-8)$

18.  $-12 + 5$

19.  $10 + (-6)$

20.  $3 + (-8)$

21.  $-6 + (-8) + 7 + 3$

22.  $30 + (-11) + 3 + (-16)$

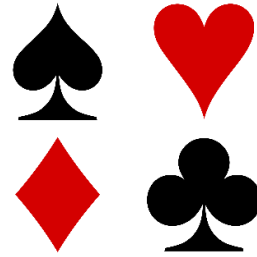
23. Which context do you prefer and why?

24. The lowest elevation on the surface of our planet is at the Dead Sea in Israel, with an elevation of 1,401 feet below sea level. The highest elevation on the surface on our planet is at the top of Mt. Everest, which is 30,430 feet above the Dead Sea. Find the elevation at the top of Mt. Everest by adding two integers. Draw a picture, show your work and write your answer in a complete sentence.

25. At the end of last month your checking account balance was \$307. So far this month you got paid \$515, spent \$284 on groceries, \$28 on coffee at PCC, and returned an item to the store for a credit of \$25. What is your new balance? Show all of your steps in an organized way and write your answer in a complete sentence.

**Zero Sum Card Game**

**Materials:** A deck of standard playing cards for each group of 2-4 players



**Directions**

- Leave the jokers in the deck. One player shuffles the deck and deals seven cards to each player. Place the remaining cards in the center of the table for the draw pile. Turn the top card face up and place it next to the draw pile for the discard pile.
- Red cards (hearts and diamonds) represent negative numbers and black cards (spades and clubs) represent positive numbers. Number cards (2 through 10) have a value equal to that number. The other cards have values as shown in the table.

Joker = 0	Ace = 1	Jack = 11	Queen = 12	King = 13
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- The object of the game is to get rid of your cards by playing any number of cards that add up to zero. Here are some examples:
  - 3 of hearts + 3 of spades is equivalent to  $-3+3=0$
  - 10 of hearts + 7 of clubs + 3 of clubs + Joker is  $-10+7+3+0=0$
  - King of spades + Ace of hearts + 4 of diamonds + 8 of hearts is equivalent to  $13+(-1)+(-4)+(-8)=0$
- On your turn, choose the top card from the discard pile or draw the top card from the draw pile. Then play as many cards as you can that sum to zero. Lay the cards out on the table and show all players so they can practice and verify the sum. Then set the cards aside. At the end of your turn, discard one card onto the discard pile.

- If you run out of cards in the draw pile, shuffle all of the played cards with the discard pile to make a new draw pile.

### Winning the Round

The first person to play all of the cards from their hand is the winner of that round. The last card may be played or discarded.

### Scoring

- Each person with cards left in their hand adds the values of all the cards remaining. Take the absolute value of the sum so that all scores are positive.

Here are some examples:

- 4 of diamonds + King of clubs is a score of 9.  
 $(-4)+13=9$  and  $|9|=9$
  - 5 of hearts is a score of 5.  
 $|-5|=5$
  - 10 of hearts + 2 of clubs + Ace of spaces is a score of 7.  
 $-10+2+1=-7$  and  $|-7|=7$
- Each person keeps a running total of their own score. The person with the lowest score at the end of the time allotted is the winner.



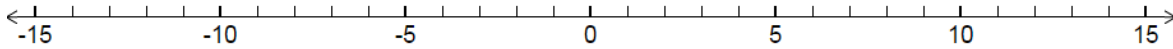
## Subtracting Integers with Context

Name \_\_\_\_\_

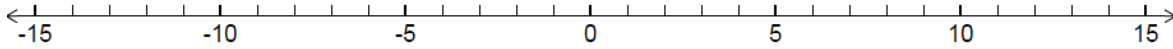
Try the number line and money contexts to subtract signed numbers and see which one you prefer.

1. Find each answer using the number lines provided.

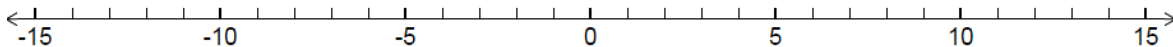
a.  $5 + (-8) =$  (Start at five, move to the \_\_\_\_\_ eight)



b.  $5 - 8 =$  (Start at five, move to the \_\_\_\_\_ eight)



c.  $5 - (-8) =$  (Start at five, move to the \_\_\_\_\_ eight)



2. Notice that the answers to 1.a and 1.b are the same. If you prefer addition you can change all subtractions to adding the opposite of the number being subtracted, as shown in Examples A and B. In 1.c you can see that subtracting a negative number is the same as adding because you move to the right.

Example A.

$$\begin{aligned} 3 - 10 &= 3 + (-10) \\ &= -7 \end{aligned}$$

Example B.

$$\begin{aligned} -6 - (-10) &= -6 + 10 \\ &= 4 \end{aligned}$$

**Use the number line context to do these problems:**

3.  $4 - 5$   
(right 4, left 5)

4.  $-5 - 11$   
(left 5, left 11)

5.  $-4 - (-3) = -4 + 3$   
(left 4, right 3)

6.  $10 - (-4)$   
(right 10, right 4)

7.  $15 - (-3)$

8.  $-9 - 1$

9.  $-5 - 5$

10.  $-4 - (-4)$

**Use the money context to do these problems:**

11.  $\$3 - \$8$   
(have 3, spend 8)

12.  $-\$9 - \$6$   
(spend 9, spend 6)

13.  $-\$1 - (-\$4)$   
(spend 1, make a return of 4, removing a debt)

14.  $-9 - (-8)$

15.  $11 - (-9)$

16.  $-3 - 5$

17.  $-10 - 10$

18.  $-6 - (-6)$

**Use either context to do these longer problems. Show all of your steps.**

19.  $-5 - (-4) + 10 - 1$

20.  $(-11 - 2) + (-12 - 1)$

21.  $15 + (-7) + 5 - (-14)$

22.  $4 - 5 - (-11) - (-9)$

23. The balance you owed on your last credit card statement was \$201. Since then you were charged \$25 in interest, made purchases of \$184 and \$31, returned two items for credits of \$11 and \$16, and made a payment of \$150. What is your new balance? Show your work and answer in a complete sentence.

24. Show how the returned items can be written as addition statements and as the subtraction of negative numbers.

## Mixed Addition and Subtraction – Mixer Activity

Name \_\_\_\_\_

**Mixer Activity.** Walk around the room and solve each set of problems with a **different partner**. Choose people that you don't know very well. You may ask other pairs for help. Sign each other's papers when you agree on the answers.

Problems	Partner
1. $-3 + (-6) =$ 2. $-10 + 18 =$ 3. $1 - 5 - (-2) - 6 =$	
4. $5 - (-10) =$ 5. $16 + (-30) =$ 6. $20 + (-2) + (-8) - 10 =$	
7. $-5 - 7 =$ 8. $-3 + (-12) =$ 9. $12 - (-8) + 4 - 5 =$	
10. $-5 - (-6) =$ 11. $18 - 27 =$ 12. $-14 + 7 - (-1) - 6 =$	
13. $-21 - (-42) =$ 14. $-10 + 7 =$ 15. $35 + (-7) - 10 + 8 =$	



## Multiplying Integers

Name \_\_\_\_\_

Write each multiplication as an equivalent addition problem and make up a context for it. If needed you can change the order of the multiplication. Two examples are shown for you.

1.  $3(\$10) = \$10 + \$10 + \$10 = \$30$  I earned \$10 per hour for three hours for a total of \$30.

2.  $4(-\$3) = (-\$3) + (-\$3) + (-\$3) + (-\$3) = -\$12$  I spent \$3 on a latte for four days. I spent \$12 in total.

3.  $5(-\$7) =$

4.  $(-11)(4) =$

5. a. Using what you learned from problems 1-4, complete these problems and observe the pattern in the answers.

$$-5 \cdot 4 =$$

$$-5 \cdot 3 =$$

$$-5 \cdot 2 =$$

$$-5 \cdot 1 =$$

$$-5 \cdot 0 =$$

b. What can you say about the sign of a negative number multiplied by a positive number?

6. a. Use the pattern from problem 5 to continue filling in the answers.

$$-5(-1) =$$

$$-5(-2) =$$

$$-5(-3) =$$

$$-5(-4) =$$

b. What can you say about the sign of a negative number multiplied by a negative number?

7. Summarize the sign rules for multiplication.



Evaluate each expression.

8.  $5(-7)$

9.  $-3(14)$

10.  $(-2)(-7)$

11.  $-23(-100)$

12.  $-5 \cdot 12$

13.  $(-20)(40)$

14.  $(-2)(-3)(-5)$

15.  $10(-10)(10)(-10)(10)$

Expand each problem into a multiplication statement by identifying the base and use the sign rules to determine the answer. Two examples are shown for you.

16.  $(-4)^2 = (-4)(-4) = 16$

The base is  $-4$

17.  $-4^2 = -(4 \cdot 4) = -16$

The base is  $4$

18.  $(-5)^4 =$

19.  $-5^4 =$

20.  $(-2)^3 =$

21.  $-2^3 =$

22.  $(-1)^{804} =$

23.  $(-1)^{805} =$

What can you say about the sign of your answer if you have a negative base with an exponent?

Write the problem as a product of signed numbers, then find the answer and state it in a complete sentence.

24. All-Star Widget Company lost \$5,000 in 2014. The following year, they lost 7 times as much. Write All-Star's balance at the end of 2015 as a signed number.



## Dividing Integers

Name \_\_\_\_\_

Fill in the blank in the multiplication problem and use it to answer the corresponding division problem.

1. a.  $3 \cdot \square = 15$

b.  $15 \div 3 =$

2. a.  $3 \cdot \square = -15$

b.  $-15 \div 3 =$

3. a.  $-3 \cdot \square = 15$

b.  $15 \div (-3) =$

4. a.  $-3 \cdot \square = -15$

b.  $-15 \div (-3) =$

What can you say about the sign rules for division?

Fill in the blank in the multiplication problem and use it to answer the corresponding division problem involving zero.

5. a.  $5 \cdot \square = 0$

b.  $0 \div 5 =$

6. a.  $-5 \cdot \square = 0$

b.  $0 \div (-5) =$

7. a.  $0 \cdot \square = 7$

b.  $7 \div 0 =$

8. a.  $0 \cdot \square = -7$

b.  $-7 \div 0 =$

Summarize the rules for division with zero.

Evaluate each expression.

9.  $-12 \div 3$

10.  $30 \div (-5)$

11.  $\frac{-14}{-2}$

12.  $90/10$

13.  $-800 \div (-200)$

14.  $\frac{-24000}{4}$

15.  $-77 \div (-11)$

16.  $-300 \div 6$

17.  $\frac{50}{-2}$

18.  $\frac{0}{-5}$

19.  $-3 \div 0$

20.  $0 \div (-7)$

21.  $\frac{-13}{0}$

22.  $-100000 \div 100$

Write each problem as a division of signed numbers, then find the answer and state it in a complete sentence.

23. An airplane descends 30,000 feet in 15 minutes. What is the average change in elevation for the plane during this period (in feet per minute)?

24. A company plans to split its tax deduction of \$125,000 evenly over the next 5 years. How much will they deduct per year?

# Mixed Operations with Integers Bingo

Name \_\_\_\_\_

Preparation for Order of Operations

Fill in your bingo card by randomly placing an integer from -15 to 15 in each square. Use an integer only once. You will not use every integer. As your instructor puts the problems on the board, shade in the correct answer. The middle square is a free space. Call out bingo when you have five shaded squares in a row – vertically, horizontally or diagonally. Your class may also play for four corners or black-out if time permits.

B	I	N	G	O
		FREE		

## Bingo Problems and Answers – For the Instructor

Print one copy of this sheet for you and a blank bingo card for each student. Have students randomly place integers from -15 to 15 in the squares on the bingo card. They should use a number only once and will not use all of the numbers.

Put one problem at a time up on the board and give students time to calculate and color in the correct answer on their bingo card. The answers are integers from -15 to 15 in random order.

1.  $14 - 16 = -2$

2.  $-55 \div (-11) = 5$

3.  $-2 + 1 = -1$

4.  $(-2)^2 = 4$

5.  $-4 - 1 = -5$

6.  $-2 + 14 = 12$

7.  $-10 - (-24) = 14$

8.  $-3^2 = -9$

9.  $(-56)(13)(0)(11) = 0$

10.  $(-2)^3 = -8$

11.  $-6 - (-12) = 6$

12.  $-24 \div 4 = -6$

13.  $3(-4) = -12$

14.  $(-3)^2 = 9$

15.  $-45 \div (-3) = 15$

16.  $12 + (-23) = -11$

17.  $-2^2 = -4$

18.  $(13)^1 = 13$

19.  $(-2)(-5) = 10$

20.  $-24 \div 8 = -3$

21.  $(-1)^4 = 1$

22.  $-4 - 3 = -7$

23.  $-7 + 14 = 7$

24.  $19 + (-16) = 3$

25.  $-6 + (-8) = -14$

26.  $-6 - 7 = -13$

27.  $-3(5) = -15$

28.  $-18 + 20 = 2$

29.  $-1 \cdot 10 = -10$

30.  $-22 + 33 = 11$

31.  $2^3 = 8$

## Order of Operations with Integers

Name \_\_\_\_\_

Use the order of operations to complete each problem. Perform one operation at a time and write the answer in its place. Show each step vertically with an equal sign on each line. There are two examples to show you the proper form.

$$\begin{aligned}\text{Example A: } 18 + 4[2 + 3 \cdot (-2)] &= 18 + 4[2 + (-6)] \\ &= 18 + 4[-4] \\ &= 18 + (-16) \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{Example B: } \frac{-4 - 3 \cdot 6 + 2}{9 - 2^2} &= \frac{-4 - 18 + 2}{9 - 4} \\ &= \frac{-22 + 2}{5} \\ &= \frac{-20}{5} \\ &= -4\end{aligned}$$

1.  $3 + (-4) \cdot 5$

2.  $(-40) \div (-4) \cdot 2$

3.  $15 - 2[5 - (-2)]$

4.  $\frac{9 \cdot 2 + 2(-5)}{(-2)^2}$

5.  $3[1-2(10-8)]$

6.  $\frac{22+20\div(-5)}{(-4+7)^2}$

7.  $4(-9)+8\div(-2)-6\cdot 5$

8.  $-6^2-27\div 3^2\cdot 2-(-1)$

9.  $5(-3)^2-(-2)(-2)^3$

10.  $5(2-4)^2+15\div(9-6)$