1. Evaluate.

a. $\log 1000$ $= 3$ b. $\log_4 1$ $= 0$ c. $\log_3 27$ $= 3$ d. $\log_2\left(\frac{1}{4}\right)$ $= -2$

2. Solve for x.

a. $\log_4 x = 2$

$4^2 = x$

$x = 16$

$\{16\}$

b. $\log_{\frac{1}{3}} x = 4$

$\left(\frac{1}{3}\right)^4 = x$

$\left\{\frac{1}{81}\right\} = x$

c. $\log(2x+1) = 2$

$10^2 = 2x+1$

$100 = 2x+1$

$\frac{99}{2} = \frac{2x}{2}$

$\left\{\frac{99}{2}\right\}$

d. $\ln e^{3x-1} = 5$

$3x-1=5$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

$\{2\}$

3. Use log properties to solve the equation. $\log_3 x = \log_3 7 + \log_3 3$.

$\log_3 x = \log_3 21$

$x = 21$

$\{21\}$

4. Use log properties to simplify the expression. $2\log_{10} 5 + \log_{10} 8 - \log_{10} 2$

$\log\left(\frac{25 \cdot 8}{2}\right)$

$= \log(100) = 2$

5. Use log properties to expand as much as possible.

a. $\log_2(6x^{-3}y^7)$

$= \log_2 6 - 3\log_2 x + 7\log_2 y$

b. $\ln\frac{e^y}{\sqrt[3]{x}}$

$= \ln e^y - \frac{1}{3}\ln x$

$= y - \frac{1}{3}\ln x$

c. $\log_5\left(\frac{x^2-7x-18}{5}\right)$

$= \log_5(x-9)(x+2) - \log_5 5$

$= \log_5(x-9) + \log(x+2) - 1$

6. Solve and write the exact solution(s) in a solution set. Show domain restrictions where appropriate.

a. $2^{4-3x} = \frac{1}{128}$

$2^{4-3x} = 2^{-7}$

$4-3x=-7$

$\frac{-3x}{-3} = \frac{-11}{-3}$

$x = \frac{11}{3}$

$\left\{\frac{11}{3}\right\}$

b. $\log_4(x-7) + \log_4(5x+9) = 4$ —— check domain $x-7>0$, $x>7$; $5x+9>0$, $x>-\frac{9}{5}$

$\log_4(x-7)(5x+9) = 4$

$4^4 = (x-7)(5x+9)$

$256 = 5x^2+9x-35x-63$

$256 = 5x^2-26x-63$

$0 = 5x^2-26x-319$

$0 = (x-11)(5x+29)$

$x-11=0$ $\quad$ $5x+29=0$

$x=11$ $\quad$ $\frac{5x}{5} = \frac{-29}{5}$

$\{11\}$ $\quad$ $\cancel{x = -\frac{29}{5}}$

7. LaShonda invests into a corporate bond at 4.5 percent compounded quarterly. If she wants to double her money, how many years will it take? How long will it take to triple her money?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}}{\cancel{P}}\left(1+\frac{.045}{4}\right)^{4t}$

$2 = \left(1+\frac{.045}{4}\right)^{4t}$

$\ln 2 = \ln\left(1+\frac{.045}{4}\right)^{4t}$

$\ln 2 = 4t\ln\left(1+\frac{.045}{4}\right)$

$\frac{\ln 2}{4\ln\left(1+\frac{.045}{4}\right)} = t$

$15.49 \approx t$

It will take about 15.5 years to double

triple

$t = \frac{\ln 3}{4\ln\left(1+\frac{.045}{4}\right)}$

$t \approx 24.55$

It will take about 24.6 years to triple.

8. The half-life of Carbon 14 is 5, 715 years. What percent of the Carbon 14 will remain 12,000 years after its creation?

$N = N_0 e^{kt}$

Find k: $\frac{\frac{1}{2}\cancel{N_0}}{\cancel{N_0}} = \frac{\cancel{N_0}}{\cancel{N_0}} e^{k(5715)}$

$\frac{1}{2} = e^{k(5715)}$

$\ln\frac{1}{2} = \ln e^{k(5715)}$

$\frac{\ln\frac{1}{2}}{5715} = \frac{5715k}{5715}$

$-.000121 = k$

$N = N_0 e^{-.000121t}$

$N = 1e^{-.000121(12,000)}$

$N = e^{-.000121(12,000)}$

$\approx .234$

About 23% will be remaining after 12,000 years.

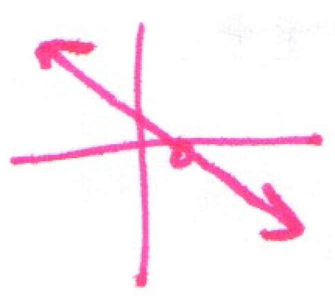
9. For each of the following functions:

- State the domain of the function
- Describe the end behavior of the function.
- If the function has any asymptotes, state the equation for each asymptote and identify the type of asymptote (vertical, horizontal, or oblique).
- If the function has any holes, state the location of each hole.

a. $f(x) = \dfrac{2x - 6x^2}{3x - 1}$

$= \dfrac{2x(1-3x)}{3x-1}$

$= \dfrac{-2x(\cancel{3x-1})}{(\cancel{3x-1})}$ hole at $\frac{1}{3}$



$= -2x, \; x \neq \frac{1}{3}$ Line with negative slope

Domain: $\{x \mid x \neq \frac{1}{3}\}$

As $x \to \infty$, $y \to -\infty$

As $x \to -\infty$, $y \to \infty$

No Asymptotes

b. $f(x) = -2x^2 + 3x - 4$

Domain: $\mathbb{R}$

End behavior:

$-2x^2$

As $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to -\infty$

No holes

No Asymptotes

c. $f(x) = \dfrac{3x - 1}{3 + x^2}$

$3 + x^2 \neq 0$

$x^2 \neq -3$

$x \neq \pm\sqrt{-3}$

imaginary number

Domain: $\mathbb{R}$

No vertical asymptote

H.A: $\dfrac{3x}{x^2} = \dfrac{3}{x} \to 0$

$y = 0$ as $x \to \pm\infty$

as $x \to \infty$, $y \to 0$

as $x \to -\infty$, $y \to 0$

No holes

d. $f(x) = 3x^2 - 1$

Domain: $\mathbb{R}$

End behavior: $3x^2$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to \infty$

No asymptotes

no holes

e. $f(x) = \dfrac{(x+2)^2(x-2)(x+4)}{(1-x^2)(x+4)}$

$= \dfrac{(x+2)^2(x-2)}{(1-x)(1+x)}, \; x \neq -4$

Domain: $\{x \mid x \neq -4, -1, 1\}$

Vertical asymptotes: $x = -1$

$x = 1$

Horizontal asymptotes:

$\dfrac{x^3}{-x^2} = -x$ no H.A.

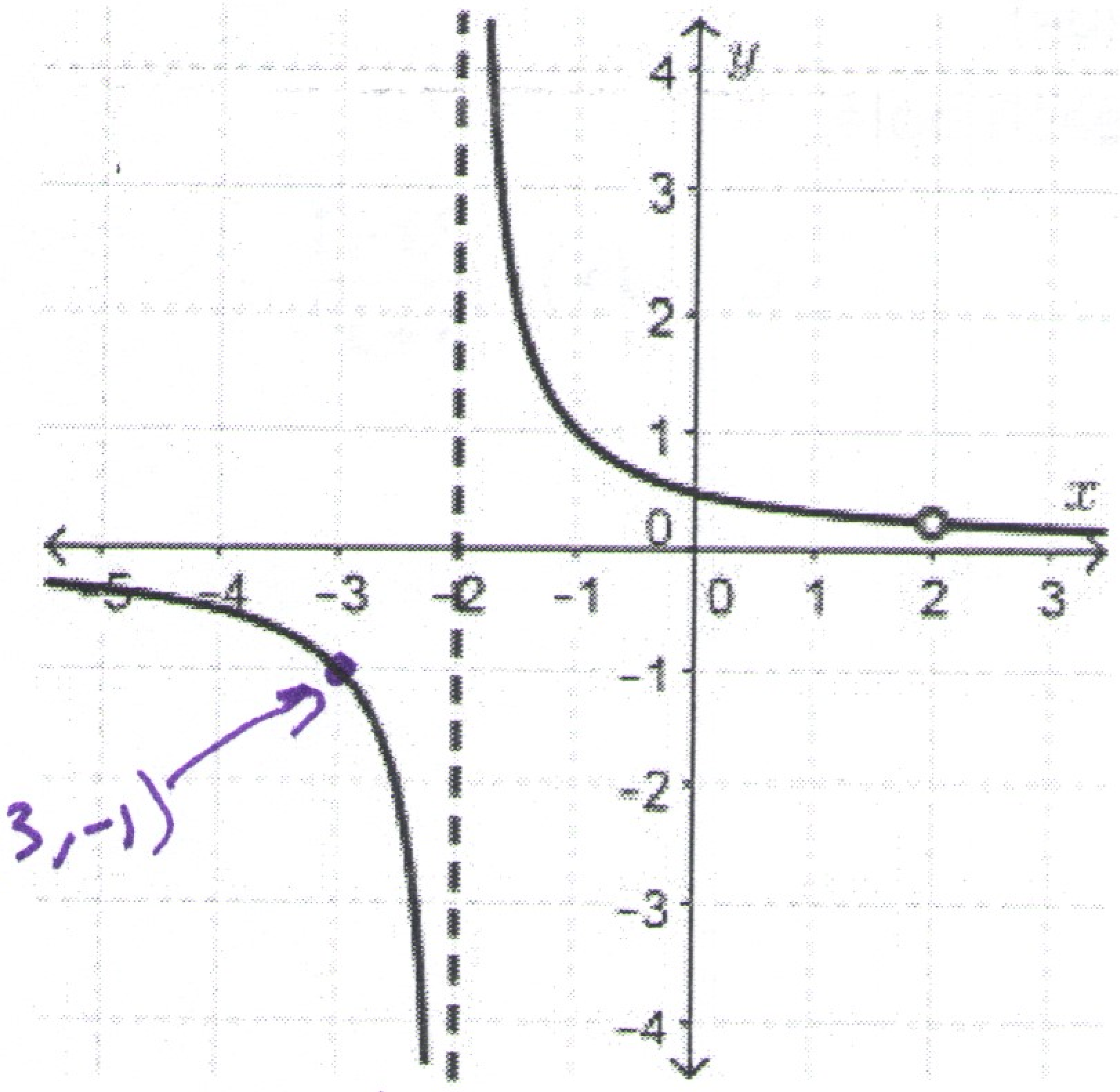
(oblique asymptote)

as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to \infty$

10. Decide whether the function shown is most likely a polynomial, rational or something else. After you have decided, find a formula for the graph, including the coefficient k.

a.



Rational

$$y = \frac{k\,(x-2)}{(x+2)(x-2)}$$

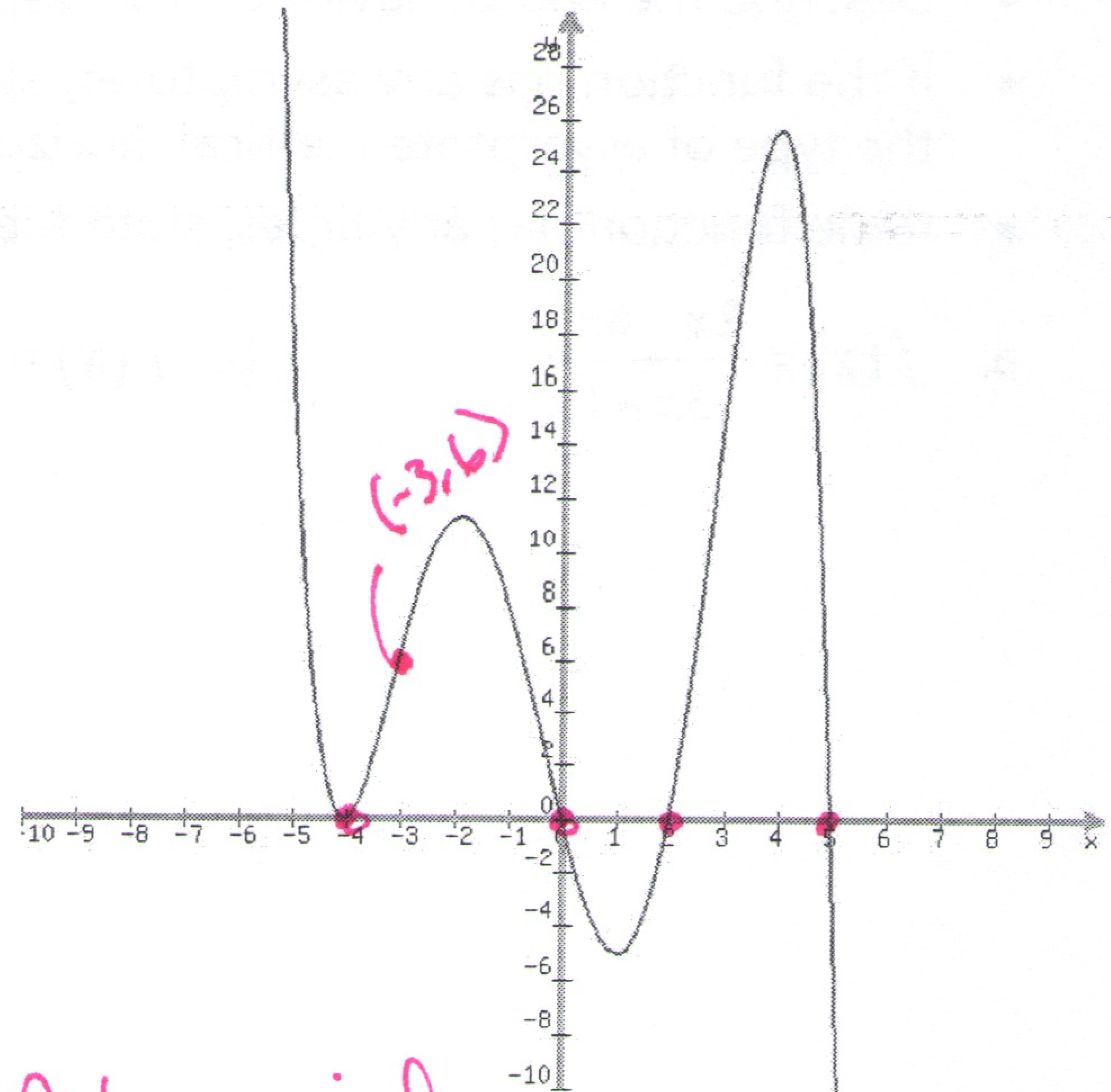
$$-1 = \frac{k(\cancel{-3-2})}{(-3+2)(\cancel{-3-2})}$$

$$-1 \cdot \quad -1 = \frac{k}{-1} \cdot -1$$

$$1 = k$$

$$y = \frac{(x-2)}{(x+2)(x-2)}$$

b.



Polynomial

$$y = k(x+4)^2(x-0)(x-2)(x-5)$$

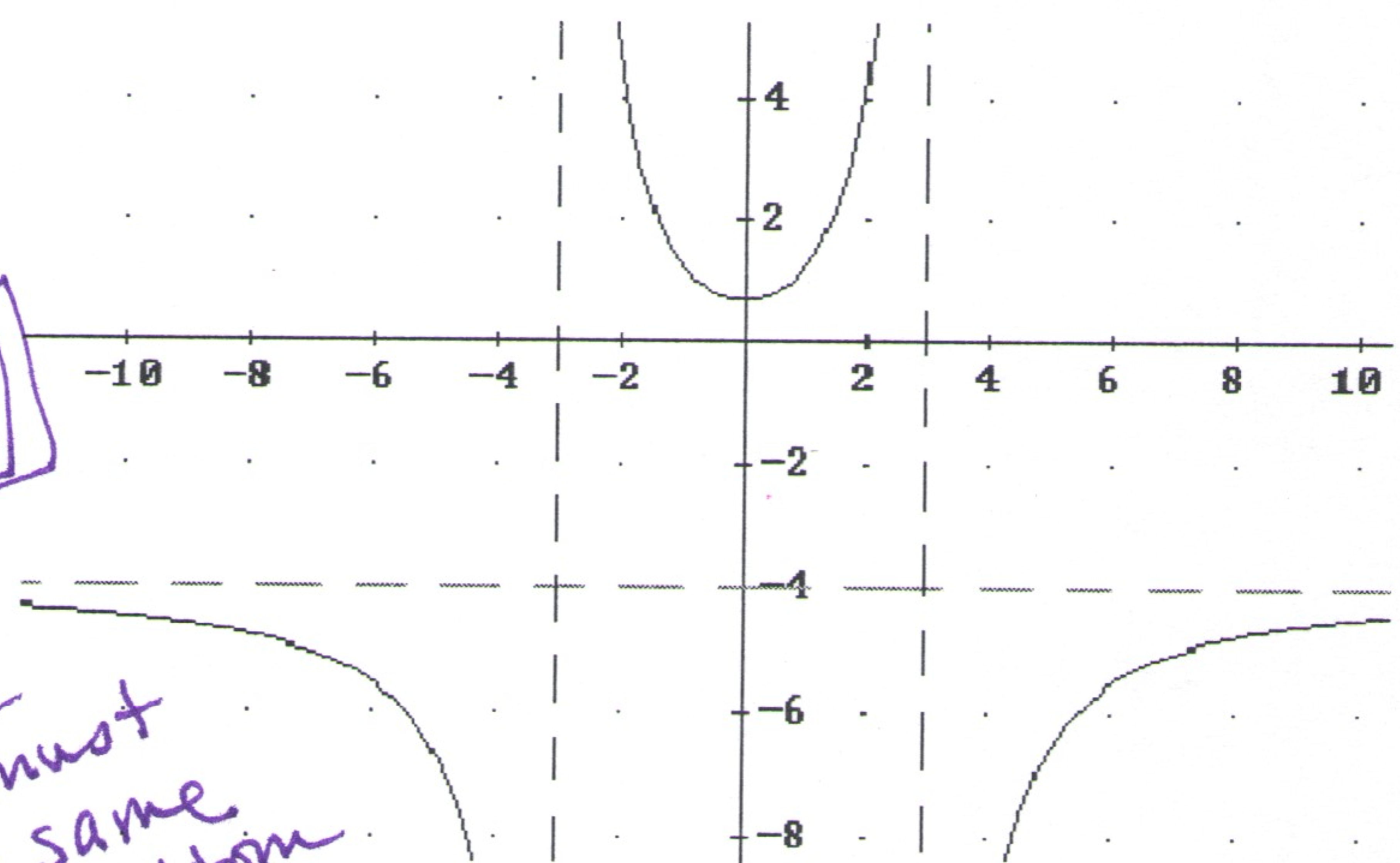
$$6 = k(-3+4)^2(-3)(-3-2)(-3-5)$$

$$6 = k(1)^2(-3)(-5)(-8)$$

$$\frac{6}{-120} = k \cdot \frac{-120}{-120} \qquad k = -\frac{1}{20}$$

$$y = -\frac{1}{20}x(x+4)^2(x-2)(x-5)$$

c.



c. Rational

$$y = \frac{k(x^2+1)}{(x-3)(x+3)}$$

$$k = -4$$

$$y = \frac{-4(x^2+1)}{(x-3)(x+3)}$$

$\boxed{\text{challenge problem}}$

no real zeros but the degree must be the same on top + bottom to get a H.A. at $y=-4$

11. Draw complete graphs of each function, following the procedures outline in the rational function and polynomial function lecture notes.

a. Graph $y = \dfrac{(x+2)^2(x-2)(x+4)}{(1-x^2)(x+4)}$

same as 10e on the previous page:

H.A. = $-x$ oblique

$\boxed{\text{Challenge problem}}$

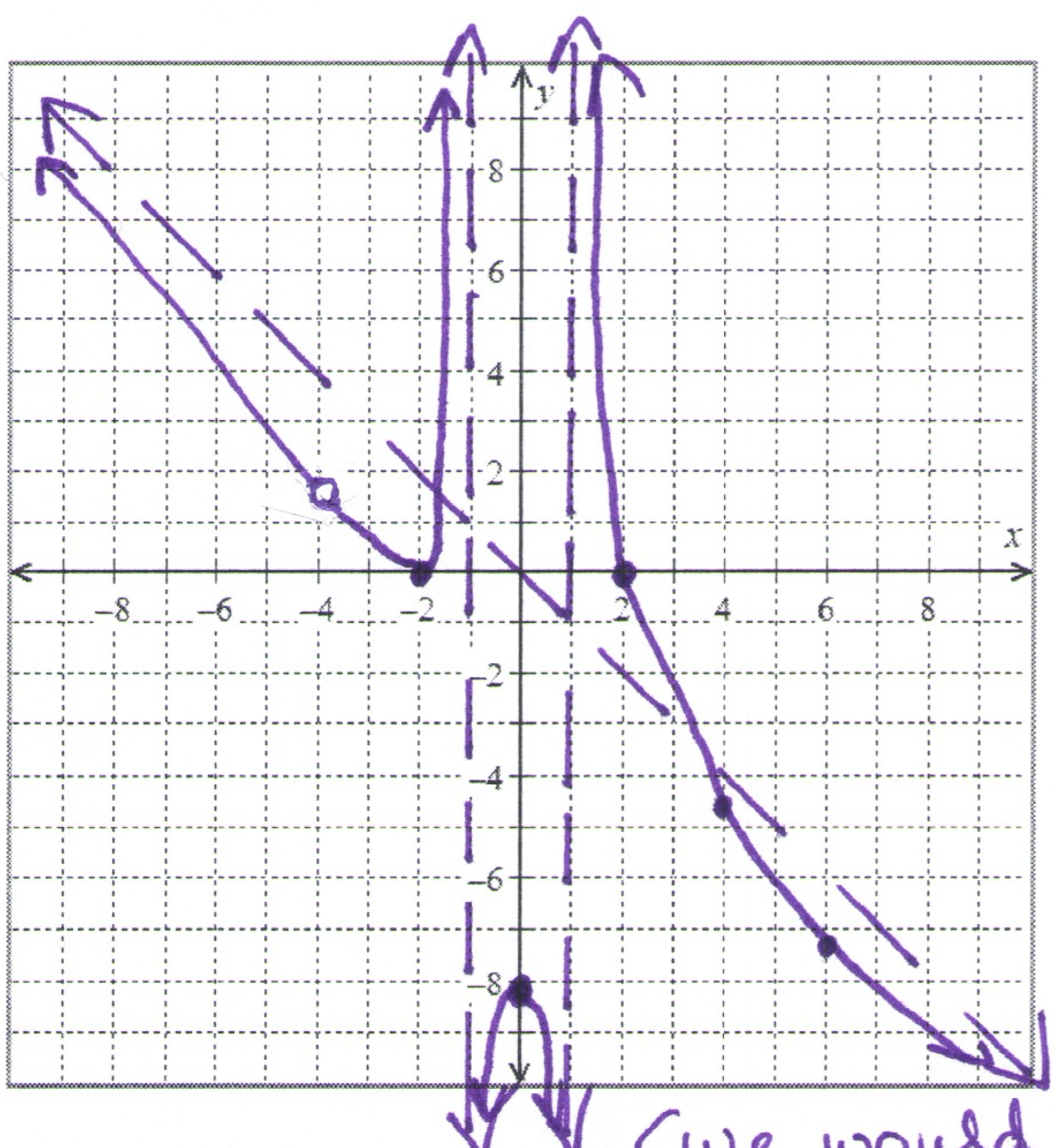
zeros: -2, mult 2

2, mult 1

Hole at $(-4, 1.6)$

$$y = \frac{(-4+2)^2(-4-2)}{(1-(-4)^2)} = \frac{4(-6)}{1-16} = \frac{-24}{-15} = 1.6$$

$$\text{y-int} = \frac{(2)^2(-2)(4)}{(1)(4)} = -8$$



(we would need to plot more points for this graph - I used table in the calculator)

b. Graph $y = -\dfrac{(x-3)^2(x+2)}{3}$

$y = -\frac{1}{3}(x-2)^2(x+2)$

zeros	mult
2	2
-2	1

end behavior: $-\frac{1}{3}x^3$

y-int: $-\frac{1}{3}(0-2)^2(0+2)$

$= -\frac{1}{3}(2)^2(2)$

$= -\frac{8}{3}$ or $-2\frac{2}{3}$