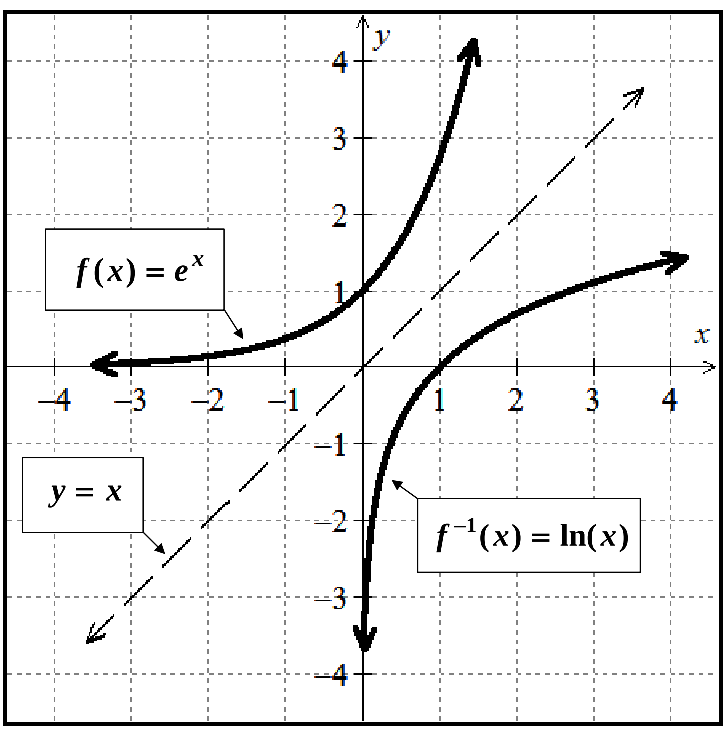
Portland Community College

REQUIRED SUPPLEMENTAL PACKET FOR MTH 111



SUPPLEMENTAL PROBLEMS FOR §1.3

DEFINITION: A function is **concave up** if its graph bends upward. The rate of change of a concave up function increases as we move from left to right along the curve.

A function is **concave down** if its graph bends downward. The rate of change of a concave down function decreases as we move from left to right along the curve.

NOTE: A function with a *constant rate of change* is a linear function and is neither concave up nor concave down.

EXAMPLE 1: In Figures 1 – 4, the graphs of four functions are given. Determine which graphs are concave up and which graphs are concave down.

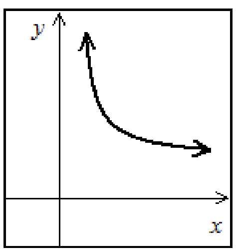


Figure 1

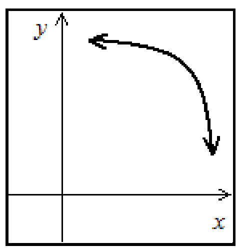


Figure 2

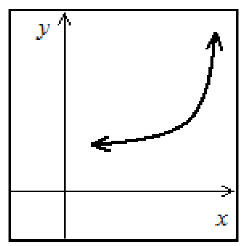


Figure 3

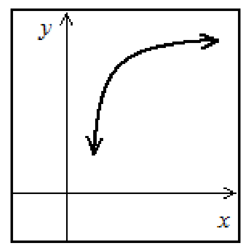


Figure 4

Solution: The graphs in Figures 1 and 3 are concave up.

The graphs in Figures 2 and 4 are concave down.

EXAMPLE 2a: A parabola that opens up is concave up. For example, the parabola graphed in Figure 5 is concave up.

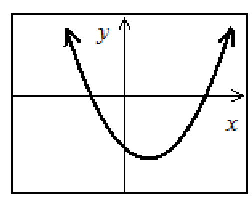


Figure 5

EXAMPLE 2b: A parabola that opens down is concave down. For example, the parabola graphed in Figure 6 is concave down.

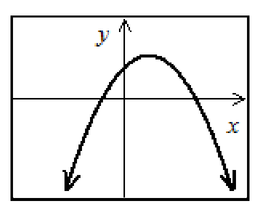


Figure 6

EXAMPLE 3: Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

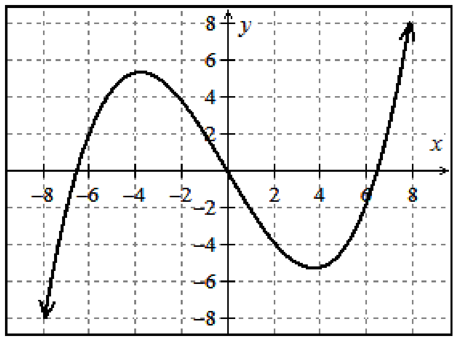


Figure 7: $y = f(x)$

b.

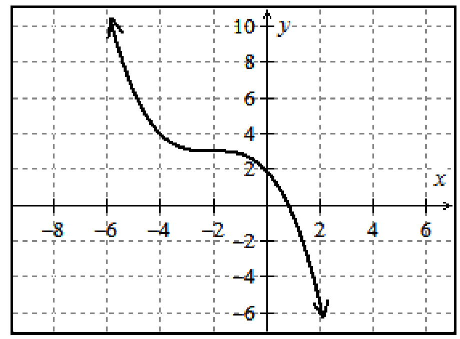


Figure 8: $y = g(x)$

Solution: **a.** f is concave up on the interval $(0, \infty)$ and concave down on the interval $(-\infty, 0)$.

b. g is concave up on the interval $(-\infty, -2)$ and concave down on the interval $(-2, \infty)$.

EXERCISES:

1. Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

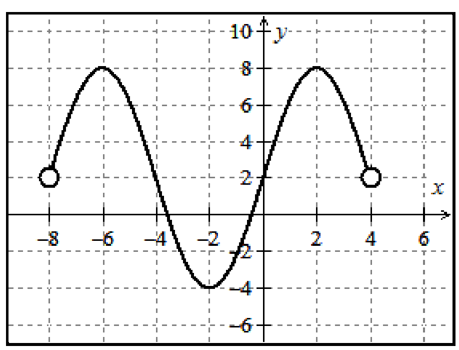


Figure 9: $y = r(x)$

b.

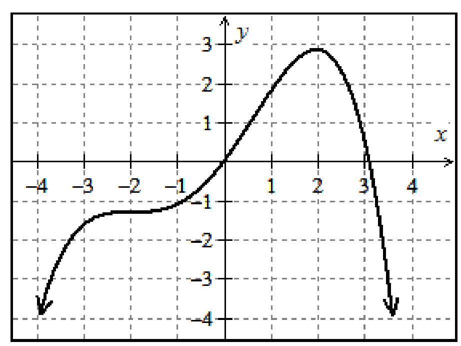


Figure 10: $y = s(x)$

c.

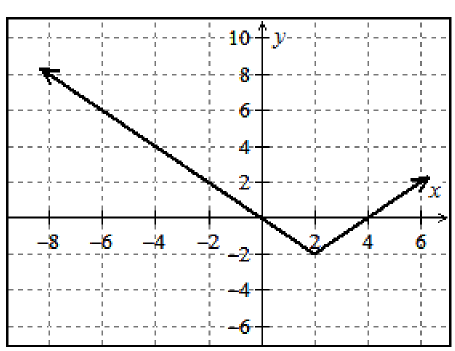


Figure 11: $y = t(x)$

d.

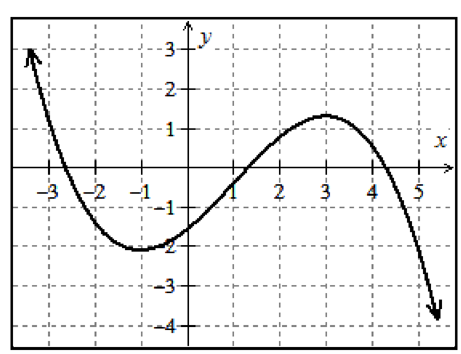


Figure 12: $y = w(x)$

SUPPLEMENTAL PROBLEMS FOR §1.5

EXAMPLE 1: The table below defines the functions f, g, and h. Express $g(x)$ and $h(x)$ in terms of f.

x	–3	–2	–1	0	1	2	3
$f(x)$	8	6	4	2	0	–1	–2
$g(x)$	–8	–6	–4	–2	0	1	2
$h(x)$	5	3	1	–1	–3	–4	–5

Solution: $g(x) = -f(x)$ and $h(x) = f(x) - 3$.

EXAMPLE 2: **a.** If $f(x) = x^2$ and $g(x) = 2x^2 + 5$, express $g(x)$ in terms of f.

b. If $f(x) = x^2$ and $h(x) = (x+5)^2 - 3$, express $h(x)$ in terms of f.

Solution: **a.** $g(x) = 2f(x) + 5$.

b. $h(x) = f(x+5) - 3$.

EXERCISES:

1. The table below defines the functions f, g, h, k, and l.

x	–2	–1	0	1	2
$f(x)$	0	1	2	3	4
$g(x)$	4	3	2	1	0
$h(x)$	0	–1	–2	–3	–4
$k(x)$	6	7	8	9	10
$l(x)$	0	3	6	9	12

a. Express $g(x)$ in terms of f and describe how the graph of $y = f(x)$ can be transformed into the graph of $y = g(x)$.

b. Express $h(x)$ in terms of f and describe how the graph of $y = f(x)$ can be transformed into the graph of $y = h(x)$.

c. Express $k(x)$ in terms of f and describe how the graph of $y = f(x)$ can be transformed into the graph of $y = k(x)$.

d. Express $l(x)$ in terms of f and describe how the graph of $y = f(x)$ can be transformed into the graph of $y = l(x)$.

2. The second row in the table below givens values for the function *f*. Complete the rest of the table. (If you don't have sufficient information to fill-in some of the cells, leave those cells blank.)

x	–4	–3	–2	–1	0	1	2	3	4
$f(x)$	–2	–1	0	1	2	3	4	5	6
$\frac{1}{2}f(x)$									
$-2f(x)$									
$f(x)+5$									
$f(x+2)$									
$f\left(\frac{1}{2}x\right)$									
$f(2x)$									
$f(x-3)$									

In **3 – 6**, first write $g(x)$ in terms of f and then compose a sequence of transformations that will transform the graph of $y = f(x)$ into the graph of $y = g(x)$.

3. $f(x) = \sqrt{x}$

$g(x) = \dfrac{\sqrt{x-7}}{4}$

4. $f(x) = \dfrac{1}{x}$

$g(x) = \dfrac{2}{x} + 3$

5. $f(x) = x^2$

$g(x) = -4\left(\frac{1}{2}x - 5\right)^2 + 3$

6. $f(x) = \sqrt[3]{x}$

$g(x) = \frac{1}{2} \cdot \sqrt[3]{10x + 30} - 6$

In **7 – 10**, the graph of $y = f(x)$ is provided; on the same coordinate plane, sketch a graph of the given function.

7. $k_1(x) = f(2x)$

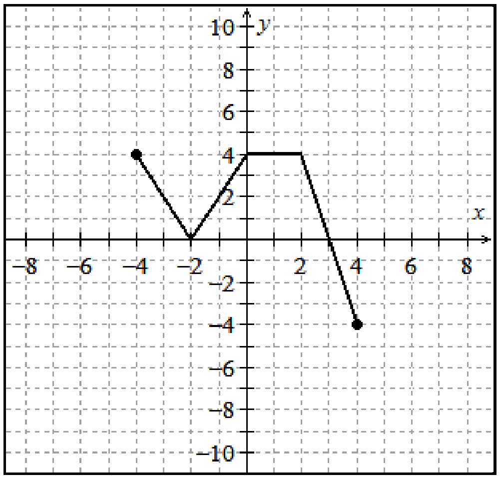


Figure 13: $y = f(x)$

8. $k_2(x) = 2f(-2x) - 1$

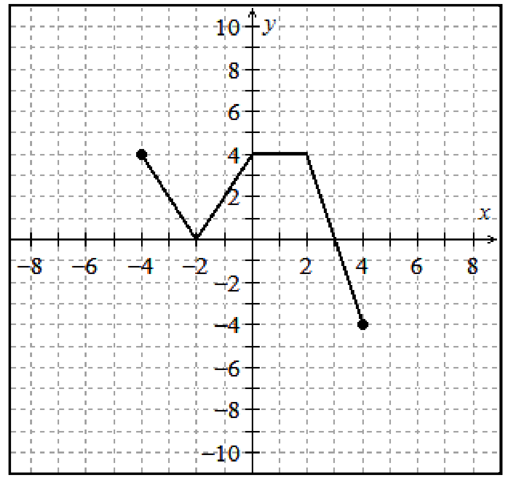


Figure 14: $y = f(x)$

9. $k_3(x) = -2f(2x + 4)$

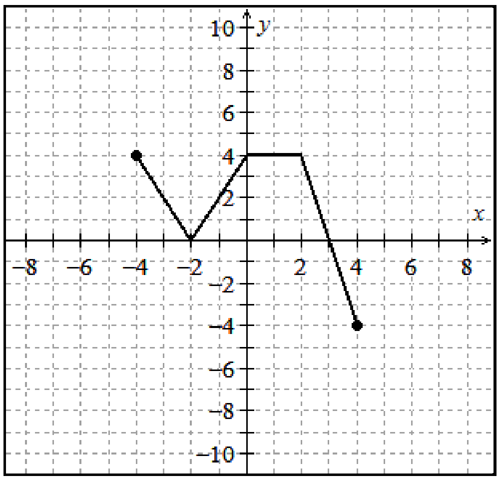


Figure 15: $y = f(x)$

10. $k_4(x) = f\left(\frac{1}{2}x\right) + 2$

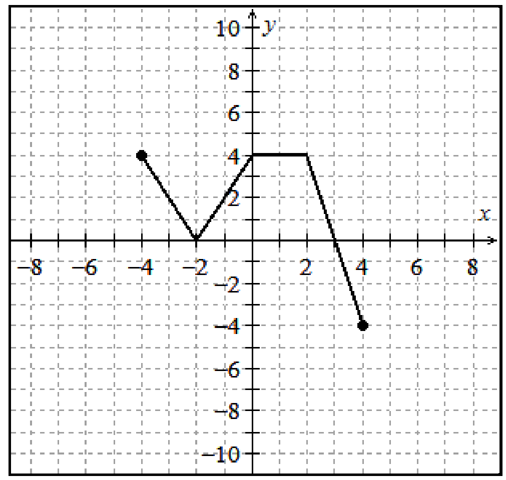


Figure 16: $y = f(x)$

SUPPLEMENTAL PROBLEMS FOR §4.2

EXERCISES:

1. The table below defines the function m. Is m an invertible function? Why or why not? If your answer is "yes", construct a table-of-values for m^{-1}.

x	1	2	3	4	5
$m(x)$	0	5	10	15	20

2. The table below defines the function p. Is p an invertible function? Why or why not? If your answer to part (a) is "yes", construct a table-of-values for p^{-1}.

x	1	2	3	4	5
$p(x)$	4	0	–2	0	2

SUPPLEMENTAL PROBLEMS FOR §4.3

EXERCISES:

1. Find an algebraic rule for an exponential function f that passes through the given two points.

a. $(0, 50)$ and $(3, 400)$

b. $(0, 4)$ and $\left(4, \frac{1}{4}\right)$

c. $\left(-1, \frac{2}{3}\right)$ and $(2, 18)$

d. $\left(-2, \frac{125}{8}\right)$ and $(1, 8)$

e. $(-2, 125)$ and $\left(3, \frac{1}{25}\right)$

f. $\left(-3, \frac{27}{16}\right)$ and $\left(3, \frac{4}{27}\right)$

2. A population increases at a constant rate of 1.3% per year. Find the approximate value for the following:

a. 1-year factor of growth and 1-year rate of growth.

b. 5-year factor of growth and 5-year rate of growth.

c. 1-month factor of growth and 1-month rate of growth.

3. A population decreases at a rate of 13.2% per 5 years. Find the approximate value for the following:

a. 1-year factor of decay and 1-year rate of decay.

b. 5-year factor of decay and 5-year rate of decay.

c. 10-year factor of decay and 10-year rate of decay.

SUPPLEMENTAL PROBLEMS FOR §4.4

EXAMPLE: The graph of $f(x) = \log_a(x)$ is given in Figure 17. Find a. (Note that the points $(1, 0)$ and $(9, 2)$ are on the graph of f.)

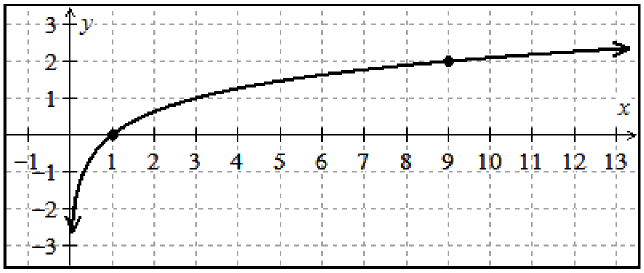


Figure 17: $f(x) = \log_a(x)$

Solution:
Since the function has form $f(x) = \log_a(x)$ and since the point $(9, 2)$ is on the graph, we know that $f(9) = 2$. Thus,

$$f(9) = 2$$

$$\Rightarrow \log_a(9) = 2 \quad \text{(since } f(9) = \log_a(9)\text{)}$$

$$\Rightarrow a^2 = 9 \quad \text{(translate the logarithmic statement into an exponential one)}$$

$$\Rightarrow a = 3 \quad \text{(take the positive square root of 9 because bases of logs are positive)}$$

Notice that we didn't attempt to use $(1, 0)$, the other obvious point on the graph of $f(x) = \log_a(x)$, to find a. Why not? (The point $(1, 0)$ is on the graph of *all* functions of the form $f(x) = \log_a(x)$ so it doesn't provide information that will help us find the particular function graphed here.)

EXERCISES:

1. The graph of $f(x) = \log_a(x)$ is given in Figure 18. Find a. (Note that the points $(1, 0)$ and $(25, 4)$ are on the graph of f.)

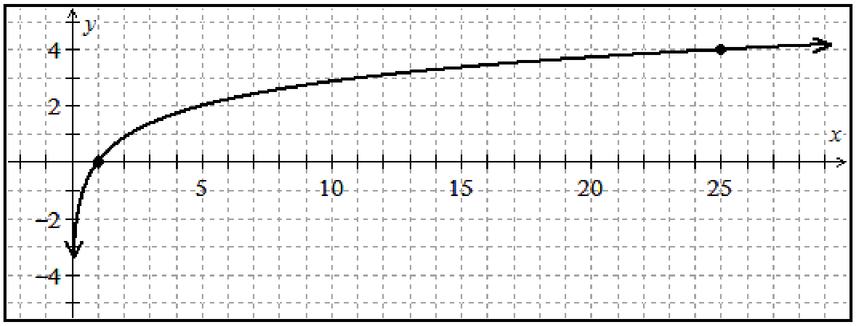


Figure 18: $f(x) = \log_a(x)$

In **2** and **3**, table-of-values for the function $f(x) = \log_a(x)$ are given. Find a.

2.

x	0.000125	0.05	1	$2\sqrt{5}$	400
$f(x)$	–3	–1	0	0.5	2

3.

x	$\frac{1}{9}$	1	3	81	243
$f(x)$	–4	0	2	8	10

ANSWERS TO THE SUPPLEMENTAL PROBLEMS FOR §1.3:

1. a. r is concave up on the interval $(-4, 0)$, and it is concave down on the intervals $(-8, -4)$ and $(0, 4)$.

b. s is concave up on the interval $(-2, 0.5)$ and it is concave down on the intervals $(-\infty, -2)$ and $(0.5, \infty)$.

c. t is never concave up and it is never concave down.

d. w is concave up on the interval $(-\infty, 1)$ and it is concave down on the interval $(1, \infty)$.

ANSWERS TO THE SUPPLEMENTAL PROBLEMS FOR §1.5:

1. a. $g(x) = f(-x)$.
So we can reflect the graph of $y = f(x)$ about the y-axis to obtain $y = g(x)$.

b. $h(x) = -f(x)$.
So we can reflect the graph of $y = f(x)$ about the x-axis to obtain $y = h(x)$.

c. $k(x) = f(x) + 6$.
So we can shift the graph of $y = f(x)$ up 6 units to obtain $y = k(x)$.

d. $l(x) = 3f(x)$.
So we can stretch the graph of $y = f(x)$ vertically by a factor of 3 to obtain $y = l(x)$.

2.

x	–4	–3	–2	–1	0	1	2	3	4
$f(x)$	–2	–1	0	1	2	3	4	5	6
$\frac{1}{2}f(x)$	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$-2f(x)$	4	2	0	–2	–4	–6	–8	–10	–12
$f(x)+5$	3	4	5	6	7	8	9	10	11
$f(x+2)$	0	1	2	3	4	5	6		
$f\left(\frac{1}{2}x\right)$	0		1		2		3		4
$f(2x)$			–2	0	2	4	6		
$f(x-3)$				–2	–1	0	1	2	3

3. $g(x) = \frac{\sqrt{x-7}}{4}$

$= \frac{1}{4}\sqrt{x-7}$

$= \frac{1}{4} f(x-7)$

So we can transform $y = f(x)$ into $y = g(x)$ by...

1st: shifting right 7 units
2nd: compressing vertically by a factor of $\frac{1}{4}$
(*there are other correct answers*)

4. $g(x) = \frac{2}{x} + 3$

$= 2 \cdot \frac{1}{x} + 3$

$= 2 f(x) + 3$

So we can transform $y = f(x)$ into $y = g(x)$ by...

1st: stretching vertically by a factor of 2
2nd: shifting up 3 units
(*there are other correct answers*)

5. $g(x) = -4\left(\frac{1}{2}x - 5\right)^2 + 3$

$= -4 f\left(\frac{1}{2}x - 5\right) + 3$

$= -4 f\left(\frac{1}{2}(x-10)\right) + 3$

So we can transform $y = f(x)$ into $y = g(x)$ by...

1st: stretching horizontally by a factor of 2
2nd: shifting right 10 units
3rd: stretching vertically by a factor of 4 and reflecting about the x-axis
4th: shifting up 3 units
(*there are other correct answers*)

6. $g(x) = \frac{1}{2}\sqrt[3]{10x + 30} - 6$

$= \frac{1}{2} f(10x + 30) - 6$

$= \frac{1}{2} f(10(x+3)) - 6$

So we can transform $y = f(x)$ into $y = g(x)$ by...

1st: compressing horizontally by a factor of $\frac{1}{10}$
2nd: shifting left 3 units
3rd: compressing vertically by a factor of $\frac{1}{2}$
4th: shifting down 6 units
(*there are other correct answers*)

7.

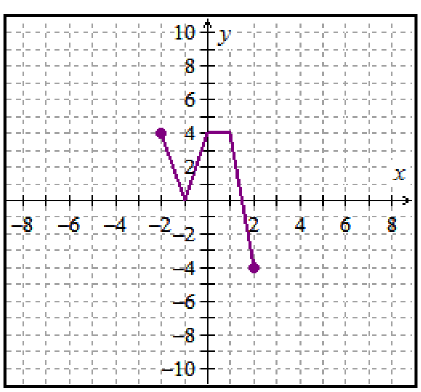


Figure 19: $k_1(x) = f(2x)$

8.

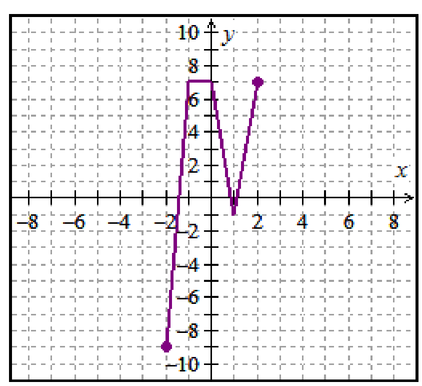


Figure 20: $k_2(x) = 2f(-2x) - 1$

9.

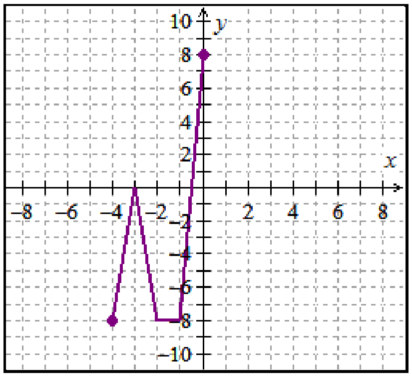


Figure 21: $k_3(x) = -2f(2x+4)$

10.

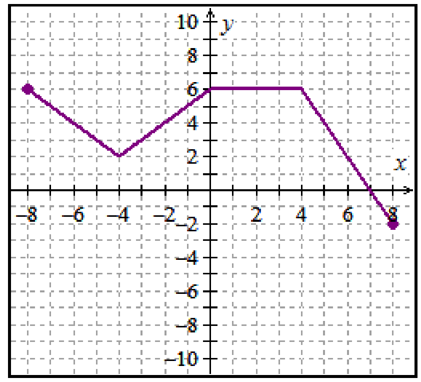


Figure 22: $k_4(x) = f\left(\frac{1}{2}x\right) + 2$

ANSWERS TO THE SUPPLEMENTAL PROBLEMS FOR §4.2

1. m is an invertible function since it is one-to-one, i.e., each output corresponds to exactly one input. Below is a table-of-values for m^{-1}.

x	0	5	10	15	20
$m^{-1}(x)$	1	2	3	4	5

2. p isn't an invertible function since it isn't one-to-one. Notice how the output 0 corresponds to TWO distinct input values.

ANSWERS TO THE SUPPLEMENTAL PROBLEMS FOR §4.3:

1. **a.** $f(x) = 50 \cdot 2^x$

b. $f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$

c. $f(x) = 2 \cdot 3^x$

d. $f(x) = 10 \cdot \left(\frac{4}{5}\right)^x$

e. $f(x) = 5 \cdot \left(\frac{1}{5}\right)^x$

f. $f(x) = \frac{1}{2} \cdot \left(\frac{2}{3}\right)^x$

2. **a.** The 1-year factor of growth is 1.013 and the 1-year rate of growth is 1.3% per year.

b. The 5-year factor of growth is $(1.013)^5 \approx 1.0667$ and the 5-year rate of growth is about 6.67% per 5 years.

c. The 1-month factor of growth is $(1.013)^{1/12} \approx 1.00108$ and the 1-month rate of growth is about 0.108% per month.

3. **a.** The 1-year factor of decay is $(0.868)^{1/5} \approx 0.972$ and the 1-year rate of decay is about 2.8% per year (since $0.972 = 1 + (-0.028)$).

b. The 5-year factor of decay is 0.868 and the 5-year rate of decay is 13.2% per 5 years.

c. The 10-year factor of decay is $(0.868)^2 \approx 0.7534$ and the 10-year rate of decay is about 24.66% per 10 years.

ANSWERS TO THE SUPPLEMENTAL PROBLEMS FOR §4.4:

1. $a = \sqrt{5}$

2. $a = 20$

3. $a = \sqrt{3}$