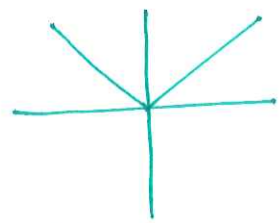
Math 111, Wed, 4/6

Q's on 1.3 + supplement (and 1.2)
Checkpoint 2
New material: 1.4

why is $|-x| = |x|$?



even function
$f(-x)=f(x)$

$\sqrt{x^2} = |x|$

$|x| = \sqrt{x^2}$

$$|-x| = \sqrt{(-x)^2} = \sqrt{x^2} = |x|$$

1.3 37. $F(x) = \sqrt[3]{x}$ $\qquad \sqrt[3]{-8} = -2$

$-\sqrt[3]{8}$

$F(-x) = \sqrt[3]{-x}$ $\qquad (-x)^{1/3}$

$= -\sqrt[3]{x}$ $\qquad -(x)^{1/3}$

$= -F(x)$

odd

61. Average rate of change is the slope

$$\frac{f(b) - f(a)}{b - a} \qquad \frac{y_2 - y_1}{x_2 - x_1}$$

$f(x) = -2x^2 + 4$

a) From $\underset{a}{0}$ to $\underset{b}{2}$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{-2(2)^2 + 4 - (-2 \cdot 0^2 + 4)}{2 - 0}$$

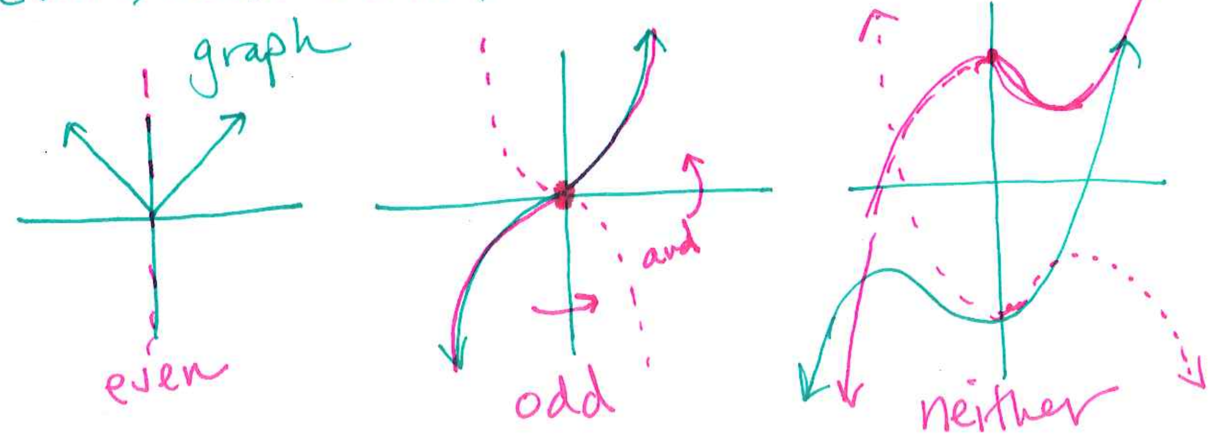
$$= \frac{-8 + 4 - 4}{2}$$

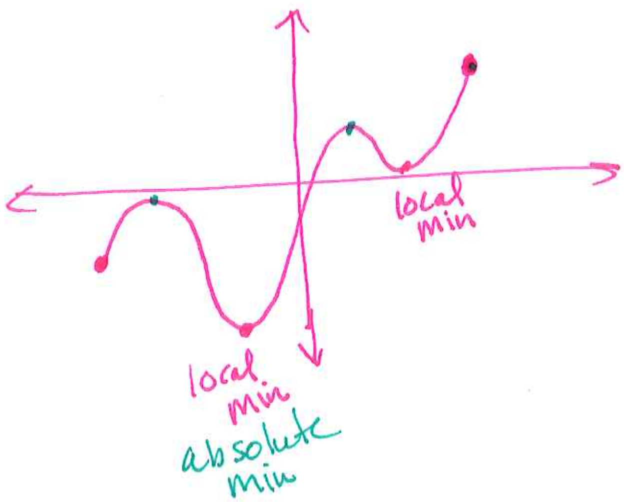
$$= \frac{-8}{2}$$

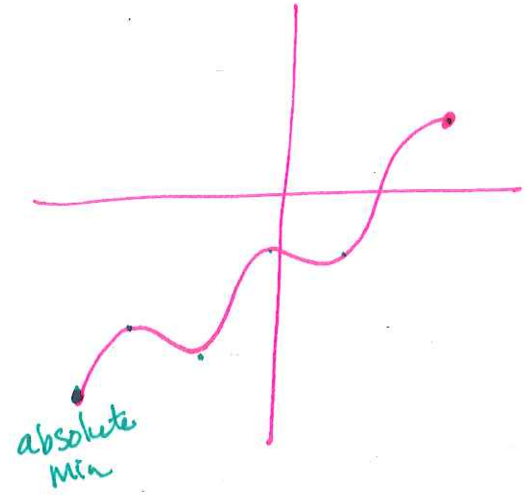
$$= -4$$

even, odd or neither

graph




local min
local min
absolute min

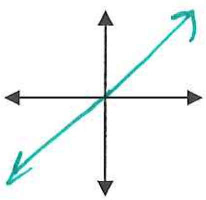

absolute min

Math 111
Basic Function Library

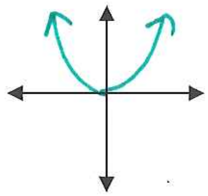
You will want to know these basic functions and their shapes / behavior. They will be important in recognizing the type of functions you might see graphically, in an equation or expression, as well as graph transformations.

Quickly sketch a graph of the basic function:

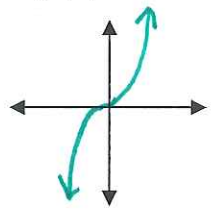
$f(x) = x$



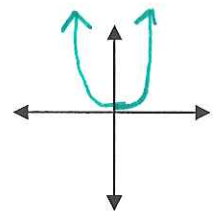
$f(x) = x^2$



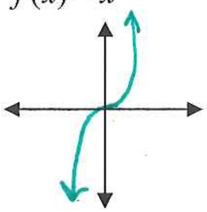
$f(x) = x^3$



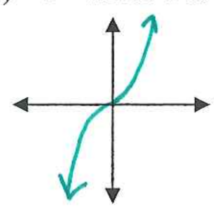
$f(x) = x^4$



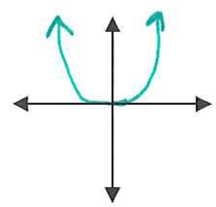
$f(x) = x^5$



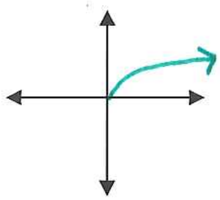
$f(x) = x^n$ where n is odd.



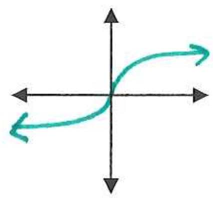
$f(x) = x^m$ where m is even.



$f(x) = \sqrt{x}$

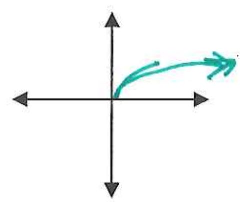


$f(x) = \sqrt[3]{x}$



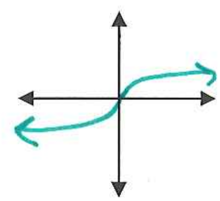
even root

$f(x) = \sqrt[4]{x}$

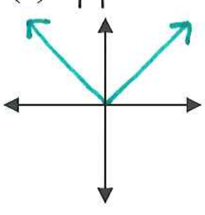


odd root

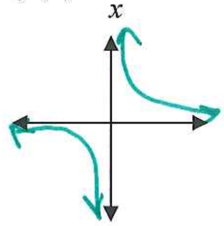
$f(x) = \sqrt[5]{x}$



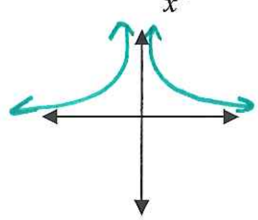
$f(x) = |x|$



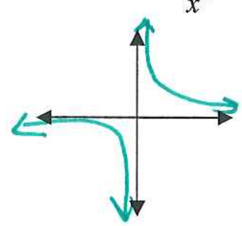
$f(x) = \frac{1}{x}$



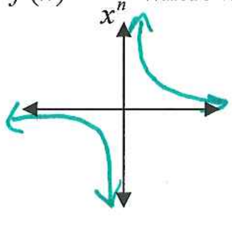
$f(x) = \frac{1}{x^2}$



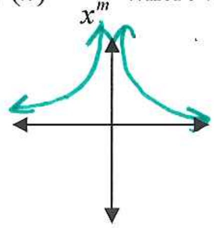
$f(x) = \frac{1}{x^3}$



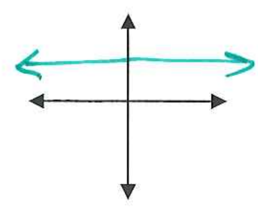
$f(x) = \frac{1}{x^n}$ where n is odd.



$f(x) = \frac{1}{x^m}$ where m is even.



$f(x) = c$ where c is a constant.



Math 111 Lecture Notes

Section 1.4: Piecewise-defined Functions

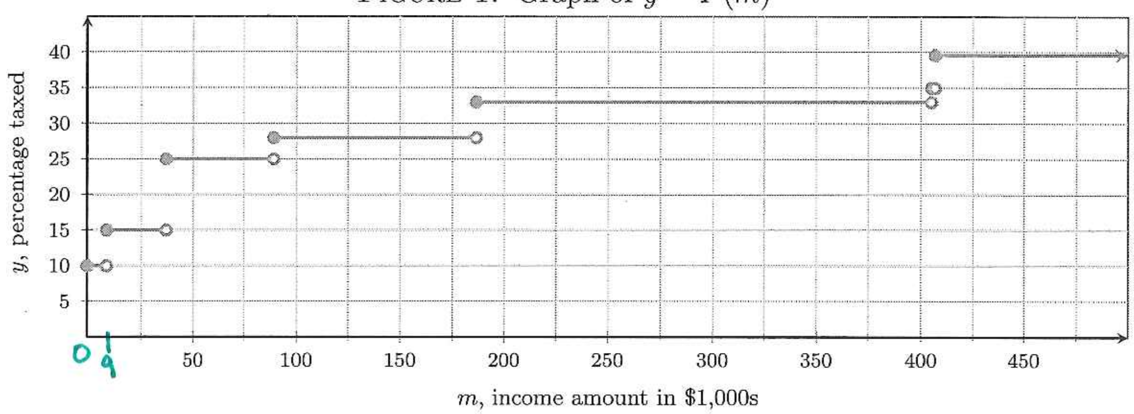
In Table 1, the 2014 federal income tax rates[1] for 2014 are shown.

Table 1. Federal Income Tax Percentage Rates for 2014 (Single Filing Status)

Income Amount (m)	Percentage of Income Taxed ($P(m)$, in %)
$0 \le m < 9075$	10
$9075 \le m < 36900$	15
$36900 \le m < 89350$	25
$89350 \le m < 186350$	28
$186350 \le m < 405100$	33
$405100 \le m < 406750$	35
$m \ge 406750$	39.6

Notice that for each interval, the percentage of income taxed as a function of income is *constant.* If we graph each *piece* over its respective interval, we obtain the following:

Figure 1. Graph of $y = P(m)$



[1]http://taxfoundation.org/article/2014-tax-brackets

> A function that is defined by different formulas on different parts of its domain is a **piecewise-defined function**.

Example 1. Use the piecewise-defined function f defined below to answer the following.

$$f(x) = \begin{cases} \dfrac{3}{x-4} & \text{if } x \leq -2 \\ \boxed{7x-8} & \text{if } -2 < x \leq 5 \\ -11 & \text{if } x > 5 \end{cases}$$

-6 is here

0 is in here

(a) $f(0)$

$f(0) = 7(0) - 8$
$= -8$

(b) $f(2) = 7(2) - 8$
$= 14 - 8$
$= 6$

(c) $f(-6)$

$f(-6) = \dfrac{3}{-6-4}$
$= \dfrac{3}{-10} = -\dfrac{3}{10}$

(d) $f(8) = -11$

(e) $f(-2) = \dfrac{3}{-2-4}$
$= \dfrac{3}{-6}$
$= -\dfrac{1}{2}$

(f) $f(5) = 7(5) - 8$
$= 35 - 8$
$= 27$

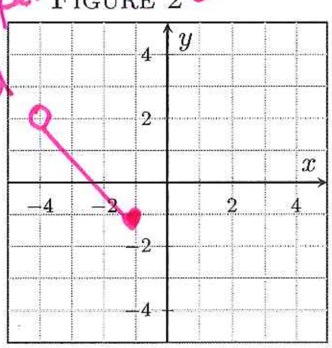
Example 2. As a prelude to graphing piecewise functions, let's graph just a few of the "pieces."

- Graph the linear function defined by $f(x) = -x - 2$ for values of x where $-4 < x \leq -1$.

$f(-4) = 4 - 2$
$= 2$
$f(-1) = 1 - 2$
$= -1$

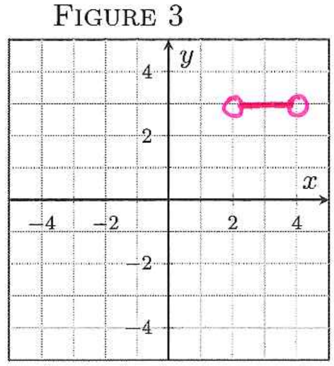
x	y
-4	2 open
-1	-1 closed

FIGURE 2



- Graph the constant function defined by $f(x) = 3$ for values of x where $2 < x < 4$.

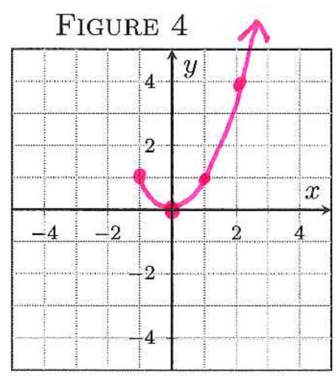
FIGURE 3



- Graph the linear function defined by $f(x) = x^2$ for values of x where $x \geq -1$.

x	y
0	0
1	1
2	4
3	9
-1	1

FIGURE 4



Example 3. Graph $y = g(x)$ in Figure 5 for the piecewise-defined function g given below.

slope $= -3 = \frac{-3}{1}$

$$g(x) = \begin{cases} -3x - 2 & \text{if} \quad x < -1 \\ 4 & \text{if} \quad -1 \le x < 2 \\ \frac{3}{2}x - 4 & \text{if} \quad 2 < x \le 4 \end{cases}$$

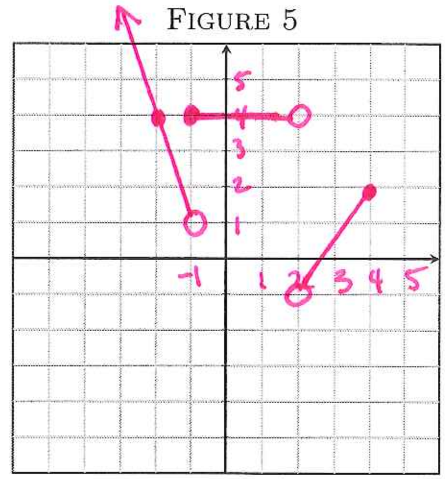
$g(-1) =$
$-3(-1) - 2$
$-3(-2) - 2$

x	y
−1	1
−2	4

x	y
2	−1
4	2

$\frac{3}{2}(2) - 4$
$3 - 4$
-1
$\frac{3}{2}(4) - 4$
$6 - 4$
$= 2$

FIGURE 5



Example 4. Graph $y = h(x)$ in Figure 6 for the piecewise-defined function h given below.

parabola

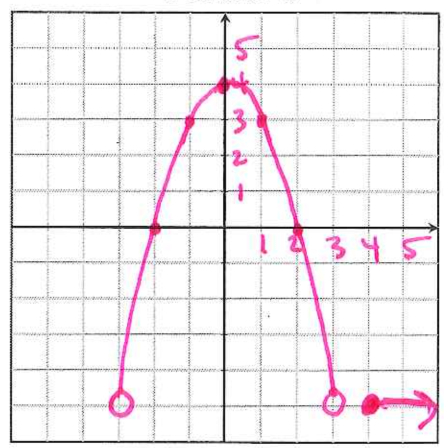
$$h(x) = \begin{cases} -x^2 + 4 & \text{if} \quad -3 < x < 3 \\ -5 & \text{if} \quad x \ge 4 \end{cases}$$

constant

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

vertex → 0

FIGURE 6

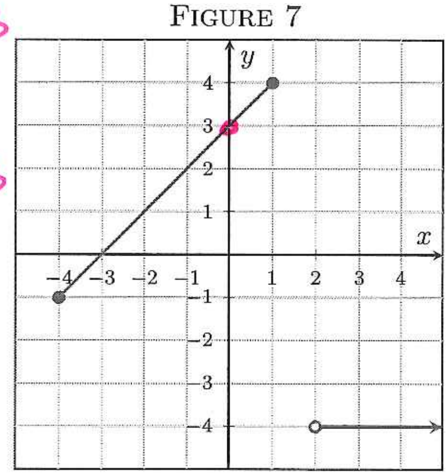


Example 5. Find the formula for the piecewise-defined function f graphed in Figure 7 below.

$$f(x) = \begin{cases} x + 3 & \text{if } -4 \le x \le 1 \\ -4 & \text{if } x > 2 \end{cases}$$

$y = x + 3$
$m = 1$
$b = 3$

FIGURE 7



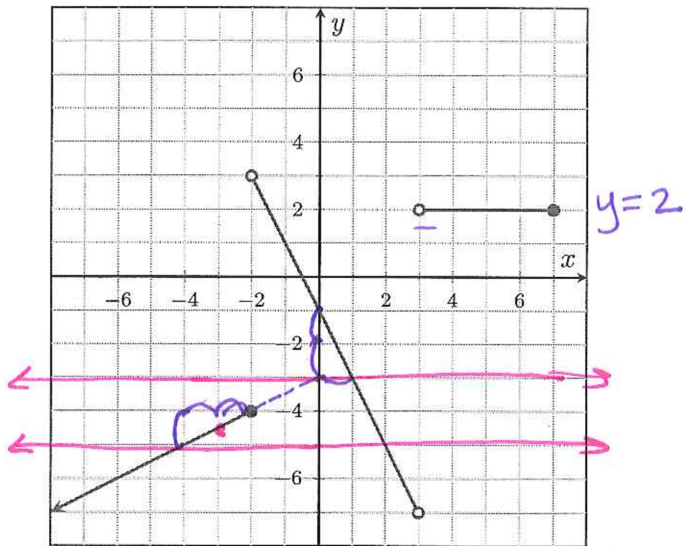
Example 6. The graph of a piecewise function g is graphed in Figure 8.

(a) State the domain and range of g.

D: $(-\infty, 3) \cup (3, 7]$

R: $(-\infty, 3)$

FIGURE 8



$y = \frac{1}{2}x - 3$ $\qquad$ $y = -2x - 1$

$\boxed{y = mx + b}$

$m = \frac{\text{rise}}{\text{run}}$

$\frac{1}{2} \qquad -\frac{2}{1} = -2$

(b) Evaluate $g(6)$.

$g(6) = 2$

(c) Evaluate $g(-2)$.

$g(-2) = -4$

(d) Solve $g(x) = -3$.

$\{1\}$

(e) Solve $g(x) = -5$.

$\{-4, 2\}$

(f) Write the formula for the function g.

$$g(x) = \begin{cases} \frac{1}{2}x - 3 & \text{if } x \le -2 \\ -2x - 1 & \text{if } -2 < x < 3 \\ 2 & \text{if } 3 < x \le 7 \end{cases}$$

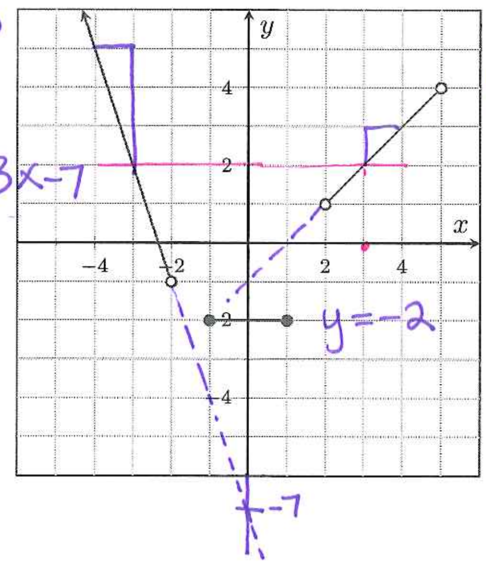
Group Work 1. The graph of the piecewise-defined function f is shown in Figure 9.

(a) Find the formula for this function.

$$f(x) = \begin{cases} -3x-7 & \text{if } x < -2 \\ -2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } 2 < x < 5 \end{cases}$$

$y - y_1 = m(x - x_1)$

FIGURE 9



(b) Find $f(1)$.

$f(1) = -2$

(c) Solve $f(x) = 2$.

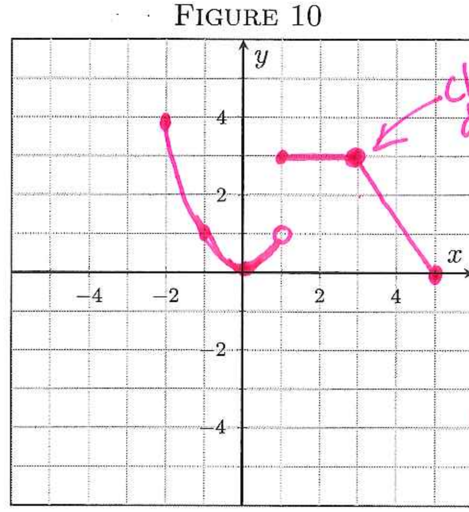
$\{-3, 3\}$

Group Work 2. Graph the function h defined below and then complete the following.

$$h(x) = \begin{cases} x^2 & \text{if } -2 \le x < 1 \\ 3 & \text{if } 1 \le x < 3 \\ -\frac{3}{2}(x-5) & \text{if } 3 \le x \le 5 \end{cases}$$

x	y
3	3
5	0

FIGURE 10



closed dot fills in the open dot and makes a continuous section

(a) State the domain and range of h.

D: $[-2, 5]$

R: $[0, 4]$

(b) State any horizontal and vertical intercepts.

Horizontal intercepts: $(0,0)$, $(5,0)$
Vertical intercepts: $(0,0)$

(c) State the absolute maximum of h and where it occurs. The absolute max is 4 at $x = -2$

Group Work 3. When calculating your electricity bill, PGE uses the follows rates: It costs 5.124 cents per kWh for the first 250 kWh used in a month. After the first 250 kWh, it costs 6.899 cents for each additional kWh used. Let $C(x)$ represent the monthly amount due (in dollars) for a PGE residential electricity bill where x kWh of energy were used that month.

change to dollars and cents by moving the decimal over.

(a) Write the formula for the piecewise-defined function C.

$$C(x) = \begin{cases} .05124x & \text{if } 0 \le x \le 250 \\ .05124(250) + .06899(x-250) & \text{if } x > 250 \end{cases}$$

1st 250 kwH

anything over 250

(b) Use that formula to determine the amount due (before taxes and other fees) when you use 325 kWh of electricity in a month.

$$\begin{aligned} C(325) &= .05124(250) + .06899(325-250) \\ &= 12.81 + .06899(75) \\ &= 12.81 + 5.17425 \\ &= \$17.98 \end{aligned}$$

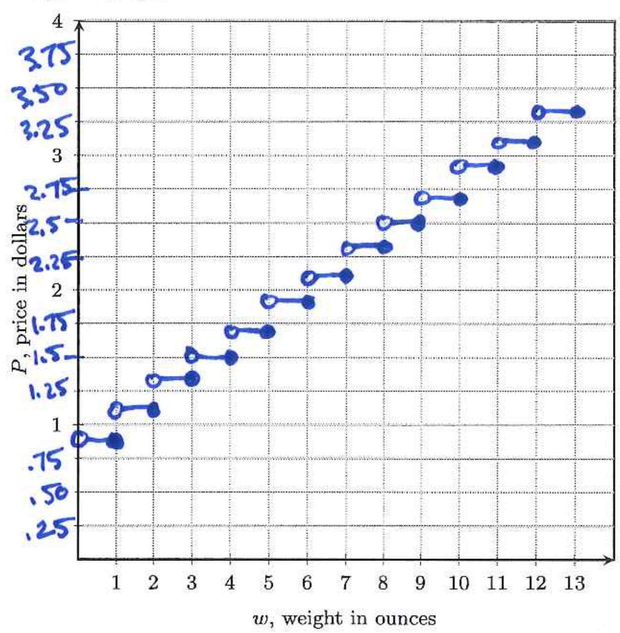
The cost for 325 kWH is \$17.98

Group Work 4. The US Postal Service rates for large envelopes are given in Table 2, according to their weight.[2] Graph the cost P (in dollars) of mailing a large envelope as a function of the weight w (in ounces) in Figure 11.

TABLE 2. US Postal Service First-Class Mail Prices, Large Envelopes

Weight Not Over (in oz.)	Price (in \$)
1	0.90
2	1.10
3	1.30
4	1.50
5	1.70
6	1.90
7	2.10
8	2.30
9	2.50
10	2.70
11	2.90
12	3.10
13	3.30

FIGURE 11. US Postal Service First-Class Mail Prices for Large Envelopes



[2] http://pe.usps.com/cpim/ftp/manuals/dmm300/Notice123.pdf