Math 111 - Wed, 5/18

Return tests + go over (need to collect until monday)

New Material: 3.1

Checkpoint 7 next wed (3.1 + 3.4)

Holiday on 5/30

Class party on the last day, yes! 6/1

Final Monday, 6/6

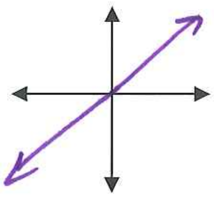
Week 8
5/21
Saturday is the last day to change grading options or withdraw

Math 111
Basic Function Library

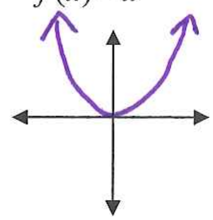
You will want to know these basic functions and their shapes / behavior. They will be important in recognizing the type of functions you might see graphically, in an equation or expression, as well as graph transformations.

Quickly sketch a graph of the basic function: Polynomials

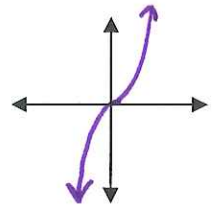
$f(x) = x$



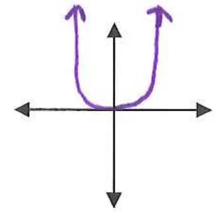
$f(x) = x^2$



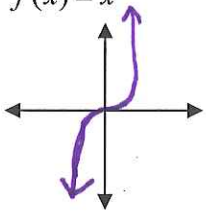
$f(x) = x^3$



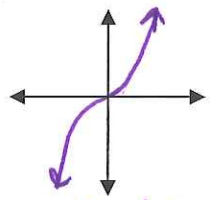
$f(x) = x^4$



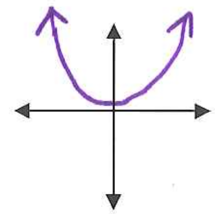
$f(x) = x^5$



$f(x) = x^n$ where n is odd.

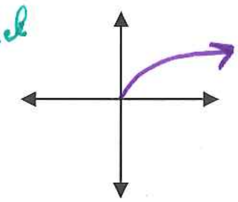


$f(x) = x^m$ where m is even.

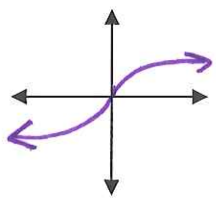


Radicals or Roots

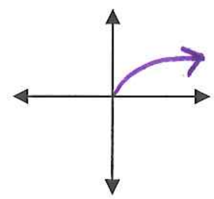
$f(x) = \sqrt{x}$



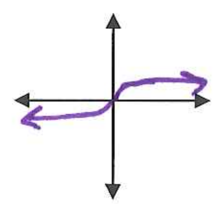
$f(x) = \sqrt[3]{x}$



$f(x) = \sqrt[4]{x}$

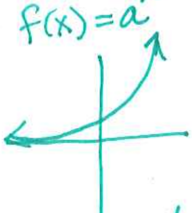


$f(x) = \sqrt[5]{x}$

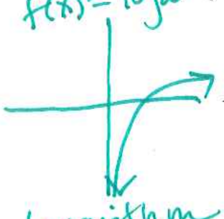


Exponential

$f(x)=a^x$



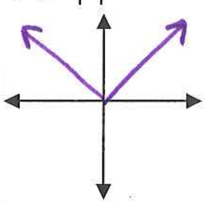
$f(x)=\log_a x$



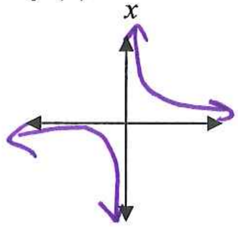
Logarithm

Rational

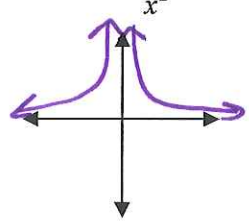
$f(x) = |x|$



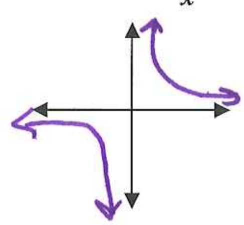
$f(x) = \frac{1}{x}$



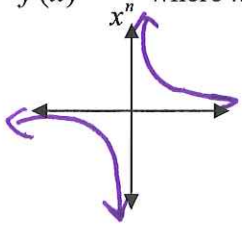
$f(x) = \frac{1}{x^2}$



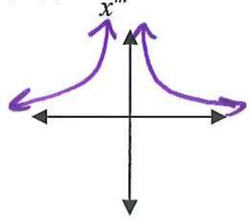
$f(x) = \frac{1}{x^3}$



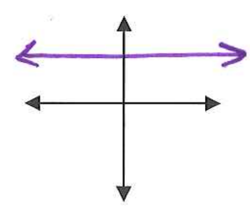
$f(x) = \frac{1}{x^n}$ where n is odd.



$f(x) = \frac{1}{x^m}$ where m is even.



$f(x) = c$ where c is a constant.



Section 3.1: Polynomial Functions

standard form

$f(x) = 3x^2 + 2x^2 - \frac{1}{2}$

factored form

$f(x) = (x-2)(x+4)$

A **power function** is of the form $f(x) = a_n x^n$ where a_n is a real number and n is a non-negative integer.

A **polynomial function** is of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a non-negative integer.

The **leading term** is $a_n x^n$. This determines the long-run behavior of the function.

The **degree** of the polynomial is n. highest power

Basic Power Functions

Figure 1. $y = x^2$

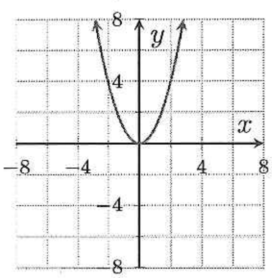


Figure 2. $y = x^3$

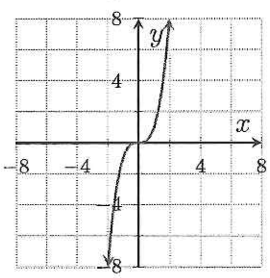


Figure 3. $y = x^4$

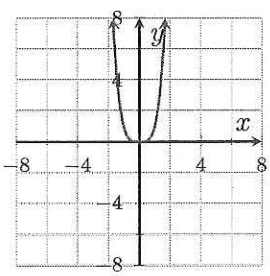
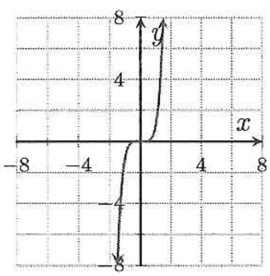


Figure 4. $y = x^5$



Basic Power Functions (close up)

Figure 5. Even Powers

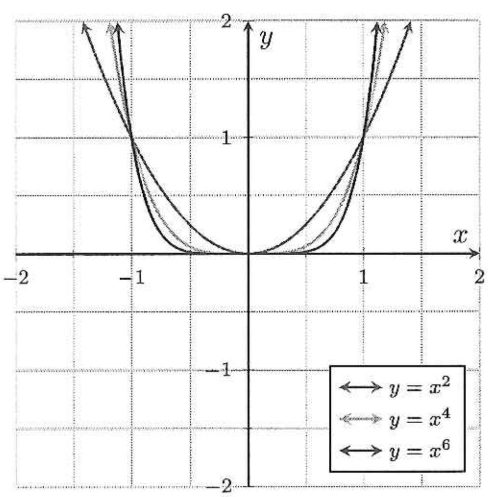
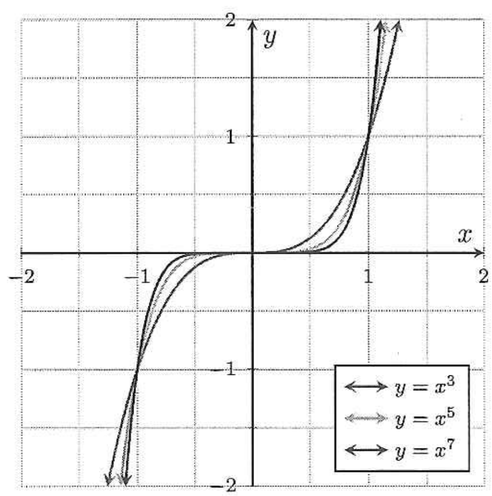


Figure 6. Odd Powers



General Polynomial Functions

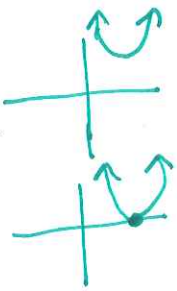
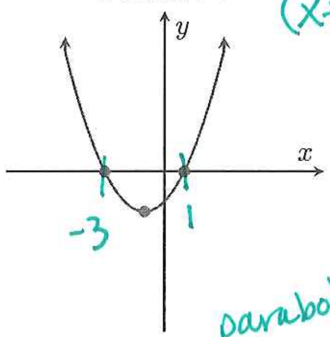


FIGURE 7

$(x+3)(x-1)$
factors

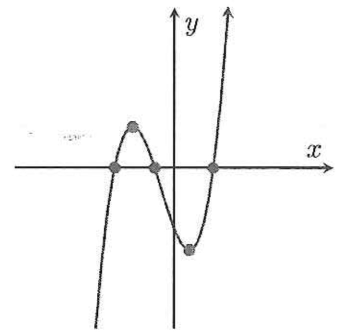


parabola

- Degree: 2
- Max. # of zeros: 2
- Max. # of turning points: 1

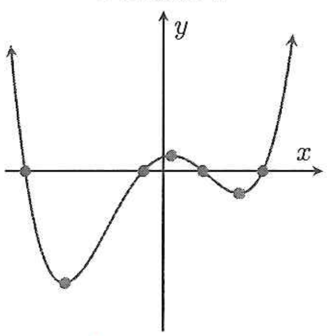
x-intercepts

FIGURE 8



- Degree: 3
- Max. # of zeros: 3
- Max. # of turning points: 2

FIGURE 9

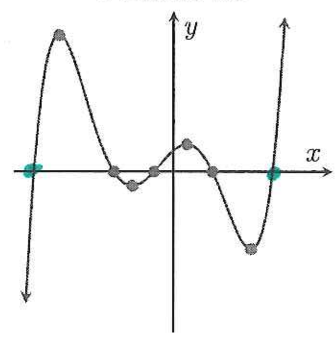


x^4

- n Degree: 4
- n Max. # of zeros: 4
- $n-1$ Max. # of turning points: 3

could have fewer

FIGURE 10



- Degree: 5
- Max. # of zeros: 5
- Max. # of turning points: 4

A polynomial function f has a real zero r if and only if $(x - r)$ is a factor of $f(x)$.

$(x-3)^2$

If r is a zero of **even multiplicity**, then the factor $(x - r)$ occurs an even number of times. The graph then *looks like* the graph of an even power function at that zero. Hence the function "bounces" there. $(x-2)^2$ bounce off the axis

$(x-5)$ or $(x-5)^3$

If r is a zero of **odd multiplicity**, then the factor $(x - r)$ occurs an odd number of times. The graph then *looks like* the graph of an odd power function at that zero. Hence, if $(x - r)$ occurs once, the function passes "straight through" at that zero and if $(x - r)$ occurs any other odd number of time, the function "flattens" there. $(x-r)$ $(x-r)^3$

cross the axis

$4x^1(x-7)^2(x+1)^5(x+2)^3=0$

$4x=0$ or $x-7=0$ or $x+1=0$ or $x+2=0$

$x=0$ $\quad x=7$ $\quad x=-1$ $\quad x=-2$

Example 1. Let $f(x) = 4x(x-7)^2(x+1)^5(x+2)^3$. Determine the following:

(a) the zeros and their respective multiplicities

zeros	multiplicity	
0	1	odd
7	2	even
−1	5	odd
−2	3	odd

(b) the degree and long-run behavior

degree 11

1st term – long-run behavior

$\underbrace{4x^{11}}$

tails

max # of zeros: 11

max # of turning points: 10

As $x \to \infty$, $y \to \infty$

As $x \to -\infty$, $y \to -\infty$

Example 2. Graph the polynomial function defined by $f(x) = -\frac{1}{2}(x-2)(x+4)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

degree: 2

long-run behavior

1st term: $-\frac{1}{2}x^2$

As $x \to \infty$, $y \to -\infty$

As $x \to -\infty$, $y \to -\infty$

max turning points: 1

horizontal intercepts

zeros: 2, multiplicity 1

−4, multiplicity 1

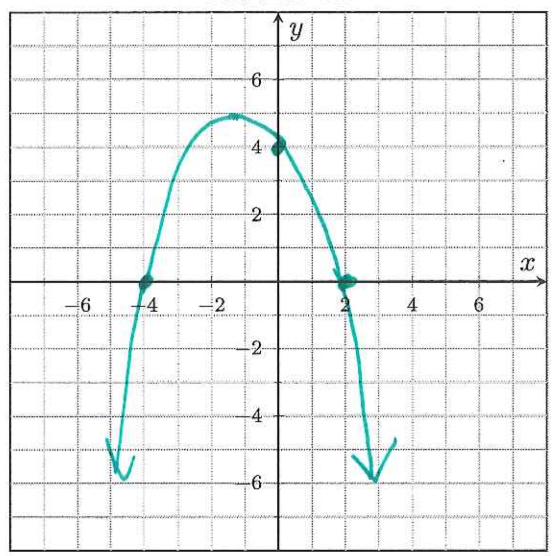
vertical intercept:

$f(0) = -\frac{1}{2}(0-2)(0+4)$

$= -\frac{1}{2}(-2)(4)$

$= 4$

FIGURE 11

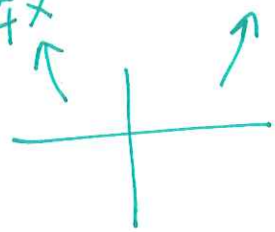


Example 3. Graph the polynomial function defined by $f(x) = \frac{1}{4}(x+1)^2(x+2)(x-5)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

degree : 4

1^{st} term : $\frac{1}{4}x^4$

long-run behavior



As $x \to \infty$, $y \to \infty$

As $x \to -\infty$, $y \to \infty$

maximum turning points : 3

zeros : -1, mult 2

-2, mult 1

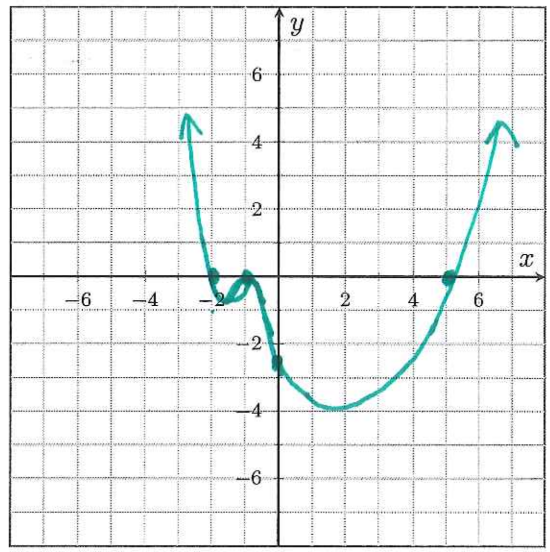
5, mult 1

y-int : $f(0) = \frac{1}{4}(0+1)^2(0+2)(0-5)$

$= \frac{1}{\cancel{4}_{2}}(1)(\cancel{2})(-5)$

$= -\frac{5}{2}$

FIGURE 12



Example 4. Graph the polynomial function defined by $f(x) = -\frac{1}{2}x(x+3)(x-2)^3$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

Degree : 5

1^{st} term: $-\frac{1}{2}x^5$

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

max turning points : 4

Zeros 0, mult 1

-3, mult 1

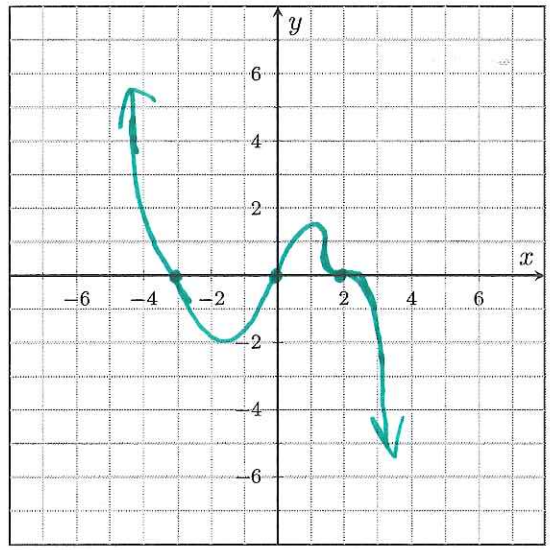
2, mult 3

$f(0) = -\frac{1}{2}(0)(0+3)(0-2)^3$

$= 0$

$(0,0)$

FIGURE 13



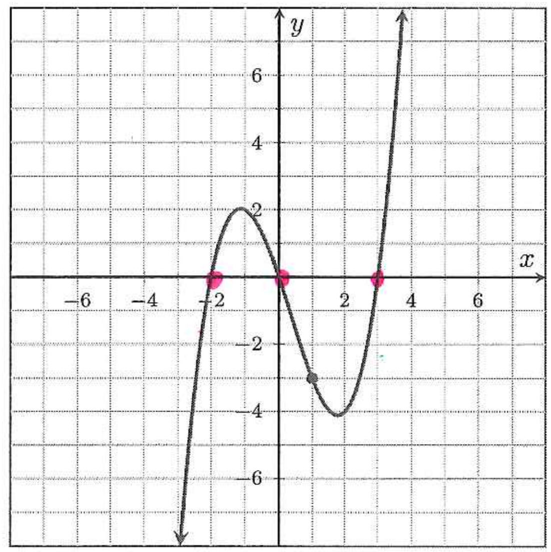
Example 5. Find a possible formula for the polynomial function graphed in Figure 14 using the zeros and their multiplicities.

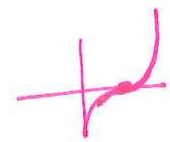
zeros: -2, mult 1

0, mult 1

3, mult 1

FIGURE 14





$(x+2)(x+0)(x-3)$

$f(x) = kx(x+2)(x-3)$

use $(1,-3)$

$-3 = k(1)(1+2)(1-3)$

$-3 = k(1\cdot 3\cdot -2)$

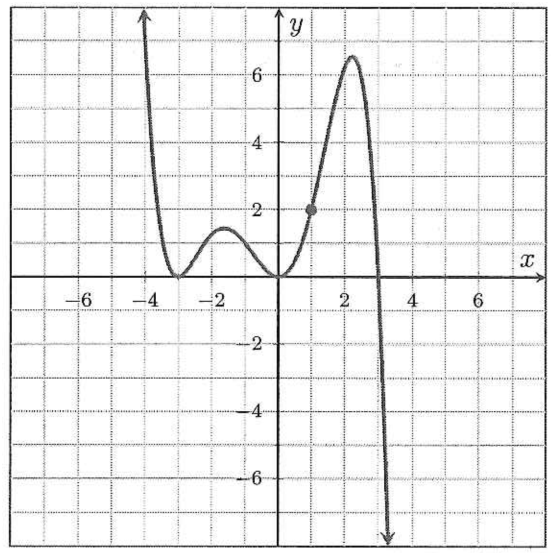
$\frac{-3}{-6} = \frac{k(-6)}{-6}$

$\frac{1}{2} = k$

$f(x) = \frac{1}{2}x(x+2)(x-3)$

Example 6. Find a possible formula for the polynomial function graphed in Figure 15 using the zeros and their multiplicities.

FIGURE 15



zeros: -3, mult 2
0, mult 2
3, mult 1

$$f(x) = k(x+3)^2(x-0)^2(x-3)$$
$$= kx^2(x+3)^2(x-3)$$

use $(1, 2)$

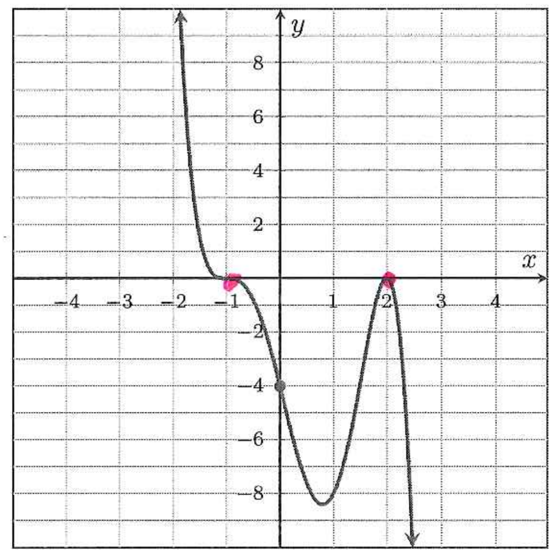
$$2 = k(1)^2(1+3)^2(1-3)$$
$$2 = k(1 \cdot 16 \cdot -2)$$
$$2 = k(-32)$$
$$-\frac{1}{16} = k$$

$$f(x) = -\frac{1}{16}x^2(x+3)^2(x-3)$$

As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to \infty$

Example 7. Find a possible formula for the polynomial function graphed in Figure 16 using the zeros and their multiplicities.

FIGURE 16



zeros: -1, mult 3

2, mult 2

degree is 5 (or more)

$$f(x) = k(x+1)^3(x-2)^2$$

$$-4 = k(0+1)^3(0-2)^2$$

$$-4 = k(1)(4)$$

$$\frac{-4}{4} = \frac{k \cdot 4}{4}$$

$$-1 = k$$

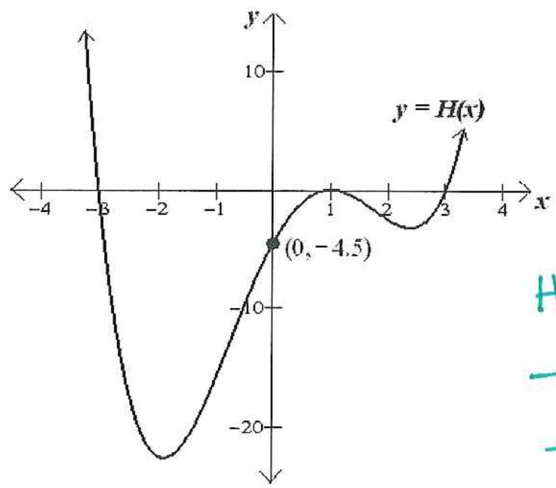
$$f(x) = -(x+1)^3(x-2)^2$$

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

1. Find a possible formula for the functions graphed below. (Check your answer by graphing on the calculator.)

a.



zeros	multiplicity
-3	1
1	2
3	1

$H(x) = k(x+3)(x-1)^2(x-3)$

$-4.5 = k(0+3)(0-1)^2(0-3)$

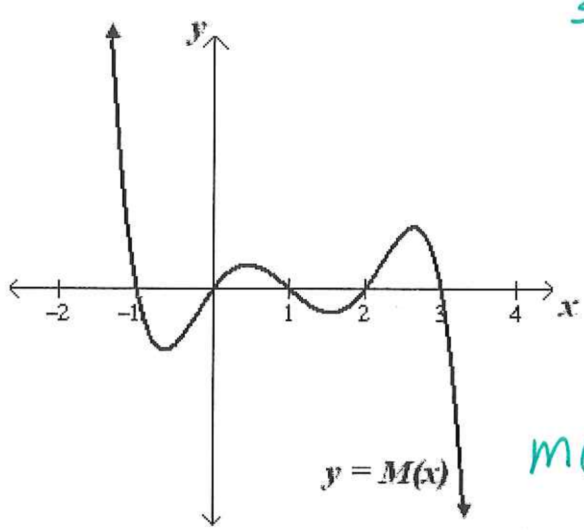
$-4.5 = k(3)(1)(-3)$

$-4.5 = k(-9)$

$\frac{1}{2} = k$

$H(x) = \frac{1}{2}(x+3)(x-1)^2(x-3)$

b. Find a possible formula given that (-2, 960) is a point on this graph.



zeros	multiplicity
-1	1
0	1
1	1
2	1
3	1

$M(x) = k(x+1)(x-0)(x-1)(x-2)(x-3)$

$= kx(x+1)(x-1)(x-2)(x-3)$

$960 = k(-2)(-2+1)(-2-1)(-2-2)(-2-3)$

$960 = k(-2 \cdot -1 \cdot -3 \cdot -4 \cdot -5)$

$960 = k(-120)$

$-8 = k$

$M(x) = -8x(x+1)(x-1)(x-2)(x-3)$

2. Given the following information about a polynomial function, find a possible formula (check your answers by graphing on the calculator).

a. f is third degree and $f(6) = -36$.
The zeros of f are 0, 5, and 8.

$$f(x) = kx(x-5)(x-8)$$
$$-36 = k(6)(6-5)(6-8)$$
$$-36 = k(6 \cdot 1 \cdot -2)$$
$$-36 = k(-12)$$
$$3 = k$$

$$f(x) = 3x(x-5)(x-8)$$

b. g is fifth degree and $g(2) = 144$.
The zeros of g are -2, 1, 4, and -1 (multiplicity 2).

$$g(x) = k(x+2)(x-1)(x-4)(x+1)^2$$
$$144 = k(2+2)(2-1)(2-4)(2+1)^2$$
$$144 = k(4 \cdot 1 \cdot -2 \cdot 3)$$
$$144 = k(-24)$$
$$-6 = k$$
$$g(x) = -6(x+2)(x-1)(x-4)(x+1)^2$$