Math 111 Lecture Notes

Section 3.1: Polynomial Functions

A **power function** is of the form $f(x) = a_n x^n$ where a_n is a real number and n is a non-negative integer.
A **polynomial function** is of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a non-negative integer.
The **leading term** is $a_n x^n$. This determines the long-run behavior of the function.
The **degree** of the polynomial is n.

Basic Power Functions

Figure 1. $y = x^2$

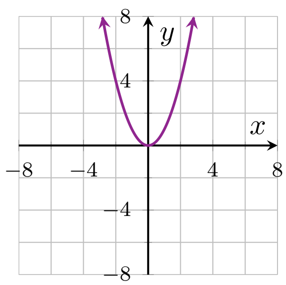


Figure 2. $y = x^3$

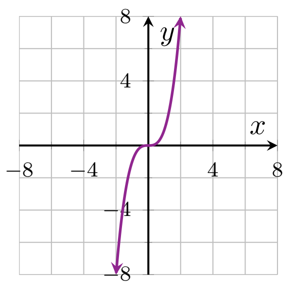


Figure 3. $y = x^4$

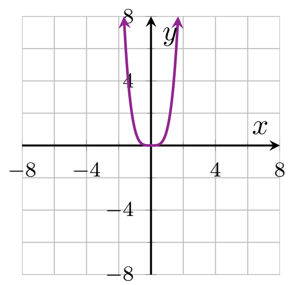
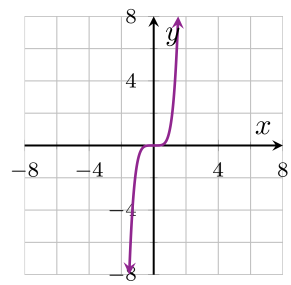


Figure 4. $y = x^5$



Basic Power Functions (close up)

Figure 5. Even Powers

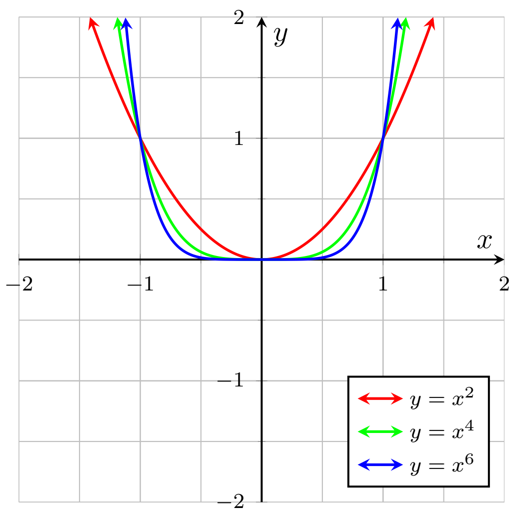
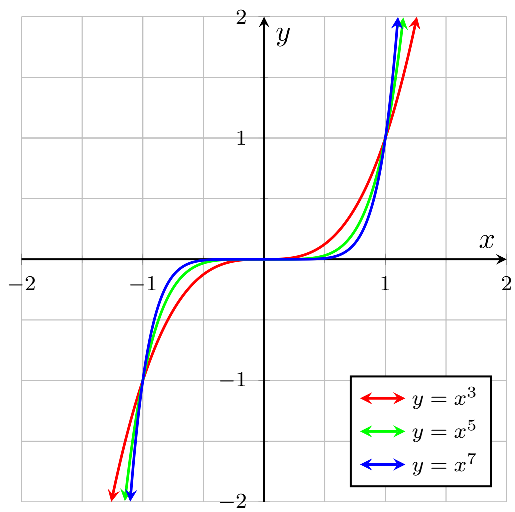
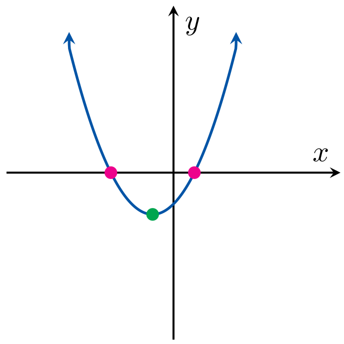


Figure 6. Odd Powers



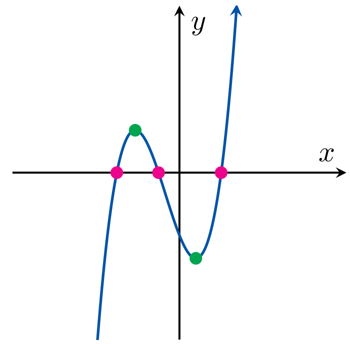
General Polynomial Functions

FIGURE 7



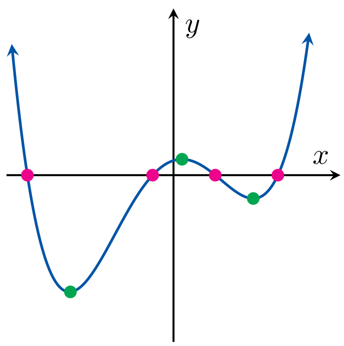
- Degree:
- Max. # of zeros:
- Max. # of turning points:

FIGURE 8



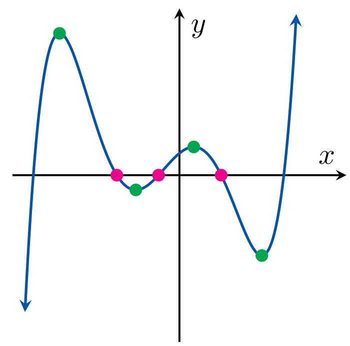
- Degree:
- Max. # of zeros:
- Max. # of turning points:

FIGURE 9



- Degree:
- Max. # of zeros:
- Max. # of turning points:

FIGURE 10



- Degree:
- Max. # of zeros:
- Max. # of turning points:

A polynomial function f has a real zero r if and only if $(x - r)$ is a factor of $f(x)$.

If r is a zero of **even multiplicity**, then the factor $(x - r)$ occurs an even number of times. The graph then *looks like* the graph of an even power function at that zero. Hence the function "bounces" there.

If r is a zero of **odd multiplicity**, then the factor $(x - r)$ occurs an odd number of times. The graph then *looks like* the graph of an odd power function at that zero. Hence, if $(x - r)$ occurs once, the function passes "straight through" at that zero and if $(x - r)$ occurs any other odd number of time, the function "flattens" there.

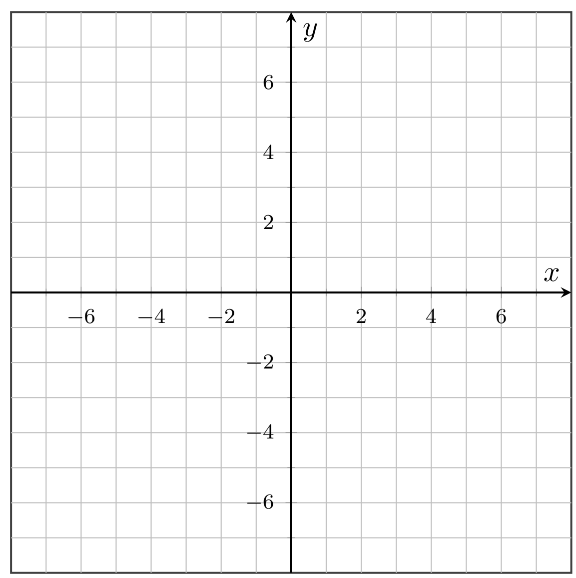
Example 1. Let $f(x) = 4x(x-7)^2(x+1)^5(x+2)^3$. Determine the following:

(a) the zeros and their respective multiplicities

(b) the degree and long-run behavior

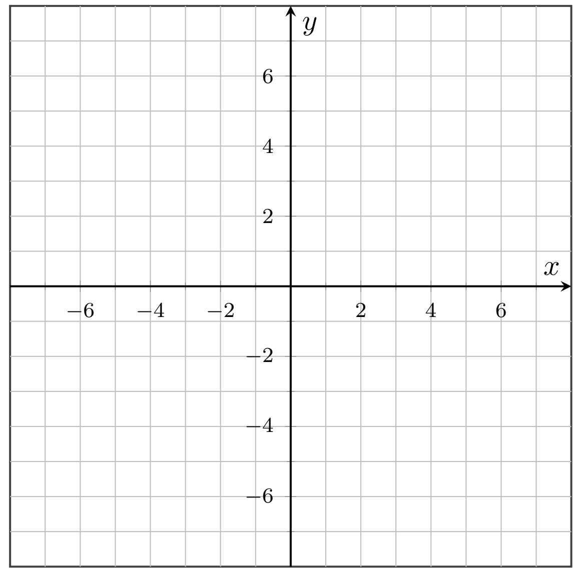
Example 2. Graph the polynomial function defined by $f(x) = -\frac{1}{2}(x-2)(x+4)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

FIGURE 11



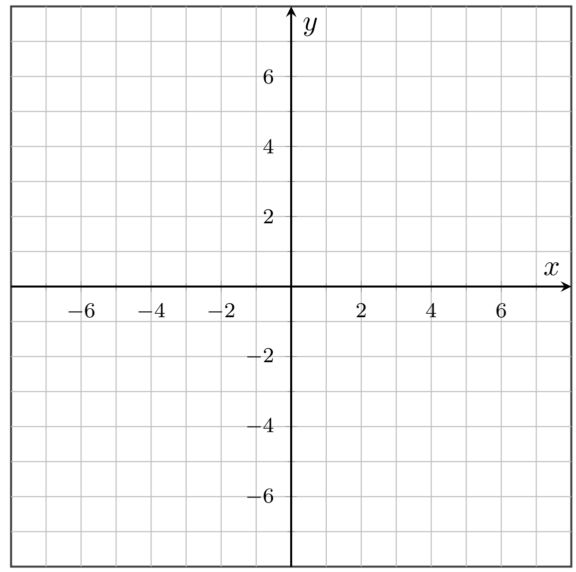
Example 3. Graph the polynomial function defined by $f(x) = \frac{1}{4}(x+1)^2(x+2)(x-5)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

FIGURE 12



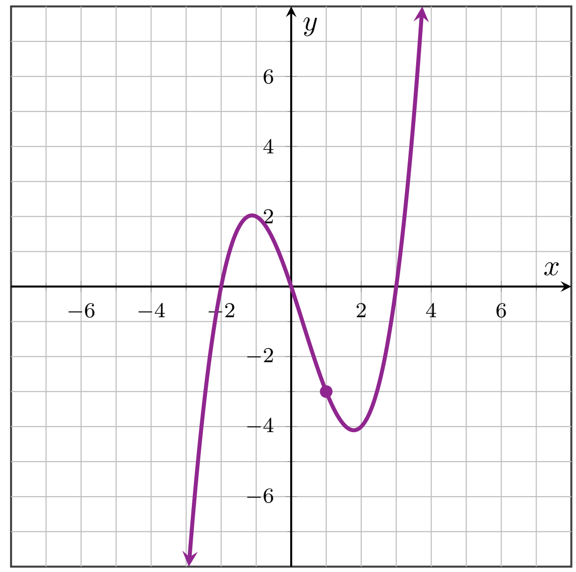
Example 4. Graph the polynomial function defined by $f(x) = -\frac{1}{2}x(x+3)(x-2)^3$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

FIGURE 13



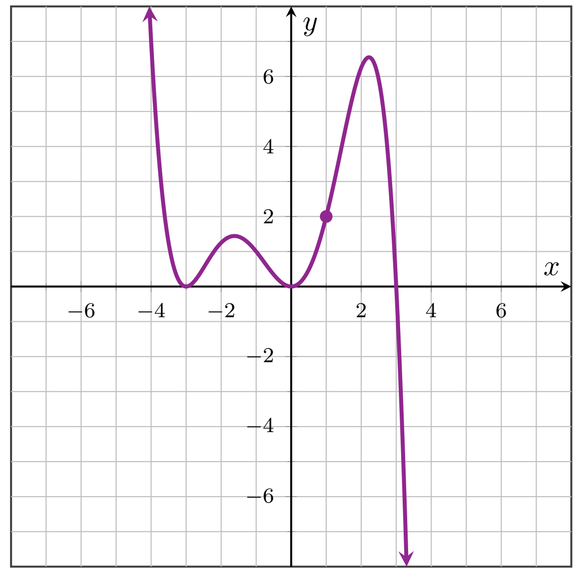
Example 5. Find a possible formula for the polynomial function graphed in Figure 14 using the zeros and their multiplicities.

FIGURE 14



Example 6. Find a possible formula for the polynomial function graphed in Figure 15 using the zeros and their multiplicities.

Figure 15



Example 7. Find a possible formula for the polynomial function graphed in Figure 16 using the zeros and their multiplicities.

FIGURE 16