Math 111 Lecture Notes

Section 3.5: Graphing Rational Functions

A **rational function** is of the form $R(x) = \dfrac{p(x)}{q(x)}$ where p and q are polynomial functions.

The **zeros** of a rational function are the values of x for which $p(x) = 0$, as the function's value is zero where the value of the numerator is zero. Most of the time, the zeros will occur at a when the factor $(x - a)$ is in the numerator of R.

A rational function is undefined where $q(x) = 0$, as this would cause division by zero.

A **vertical asymptote** occurs when the denominator of the *simplified* form of R is equal to zero. Most of the time, the vertical asymptote $x = b$ will occur when the factor $(x - b)$ is in the denominator of the *simplified* form of R.

A **hole** occurs when both the numerator and denominator equal zero for some value of x. We will identify a zero at c when the linear factor $(x - c)$ occurs in both the numerator and denominator of a rational function. Note that during simplification this factor cancels and results in a domain restriction for R.

The **long run behavior** and **horizontal asymptote** of R can be determined by the ratio of leading terms of p and q.

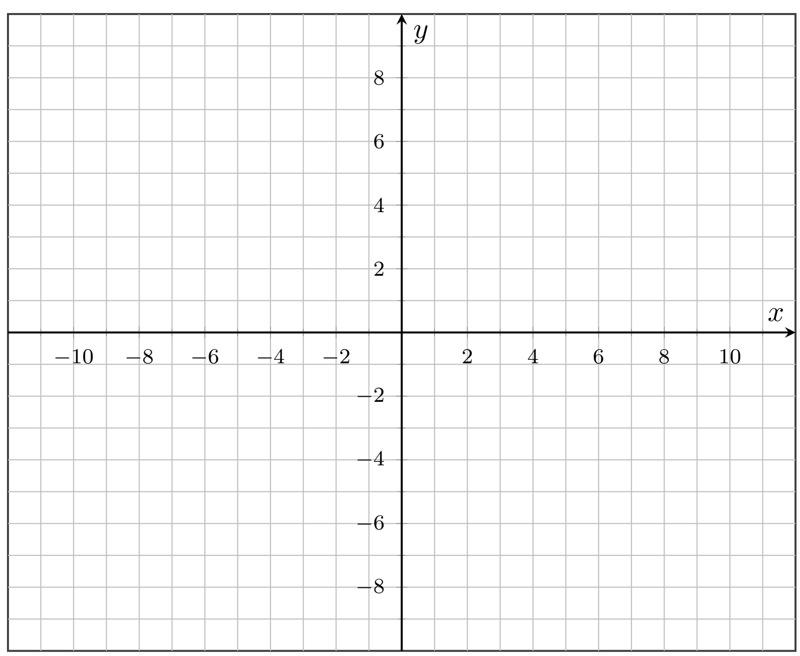
http://changestartsintheheart.wordpress.com/tag/asymptote/

http://www.theoildrum.com/node/5110

Example 1. Graph the rational function $R(x) = \dfrac{2x-6}{x+4}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

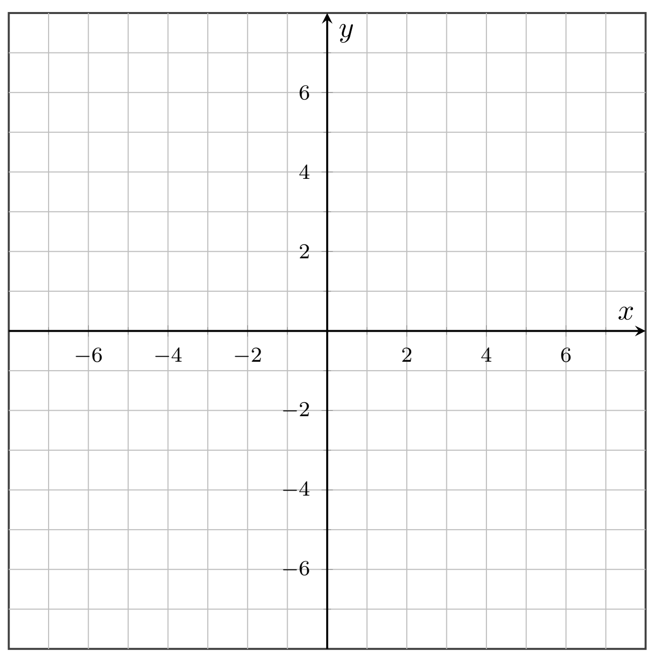
FIGURE 1



Example 2. Graph the rational function $R(x) = \dfrac{8}{x^2 - 4}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

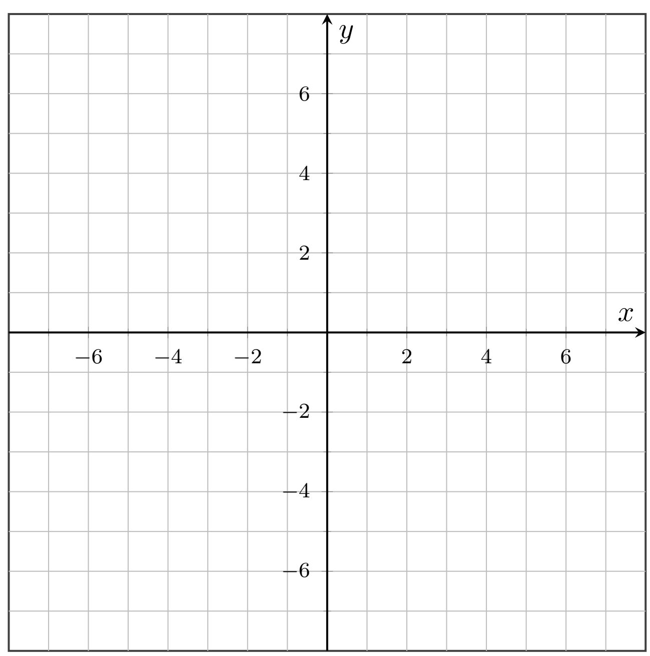
FIGURE 2



Example 3. Graph the rational function $R(x) = \dfrac{x^2 - 4x + 3}{x - 3}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

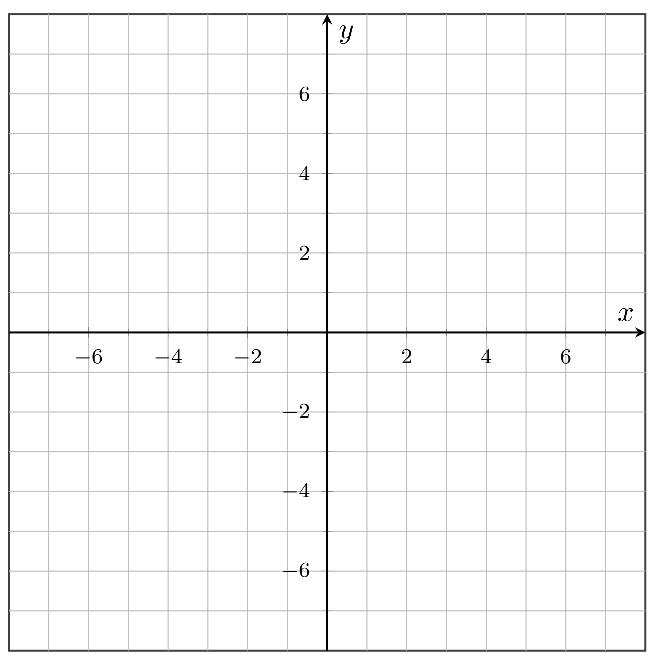
FIGURE 3



Example 4. Graph the rational function $R(x) = \dfrac{3x-6}{x^2+x-6}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

FIGURE 4

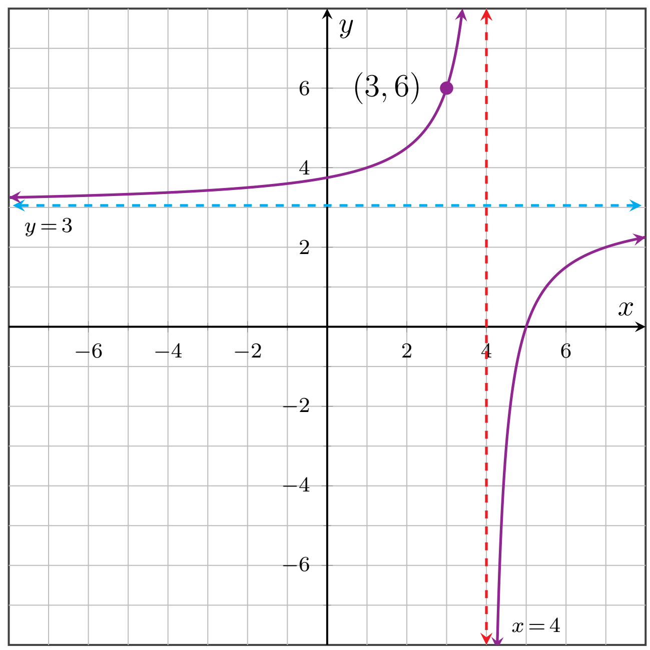


How to find a possible formula for a rational function:

- State any zeros. Use these to determine factors and the multiplicity of each factor that appears in the numerator.
- State any vertical asymptotes. Use these to determine factors and the multiplicity of each factor that appears in the denominator.
- If a "hole" appears at $x = a$, then put the factor $(x - a)$ in both the numerator and denominator.
- Use one other point to determine if there is a constant factor other than 1.

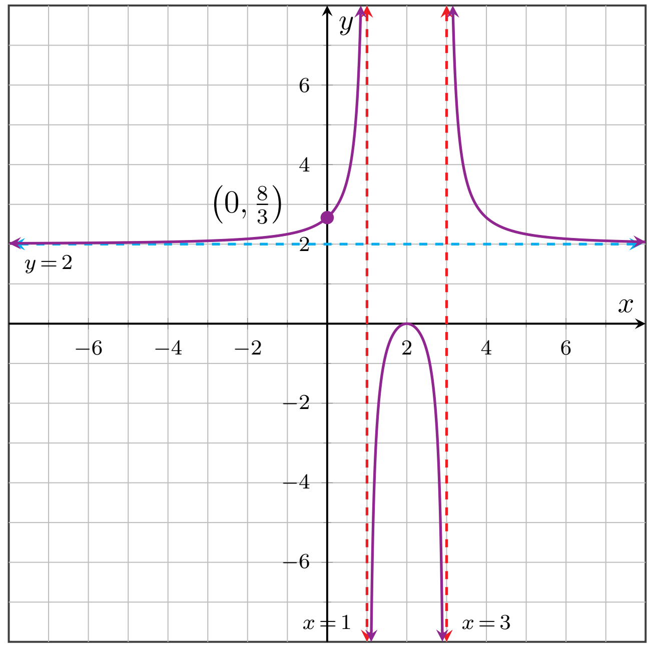
Example 5. Find a possible formula for the rational function graphed in Figure 5.

FIGURE 5



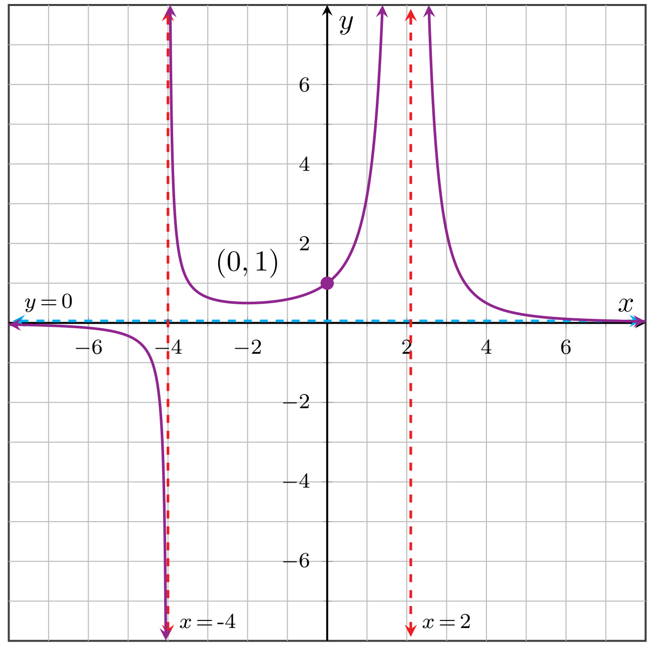
Example 6. Find a possible formula for the rational function graphed in Figure 6.

FIGURE 6



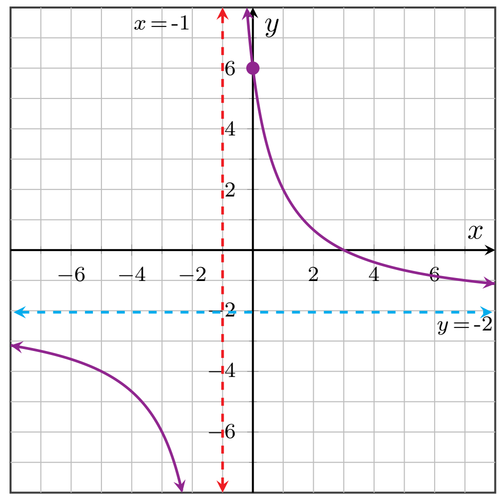
Example 7. Find a possible formula for the rational function graphed in Figure 7.

Figure 7



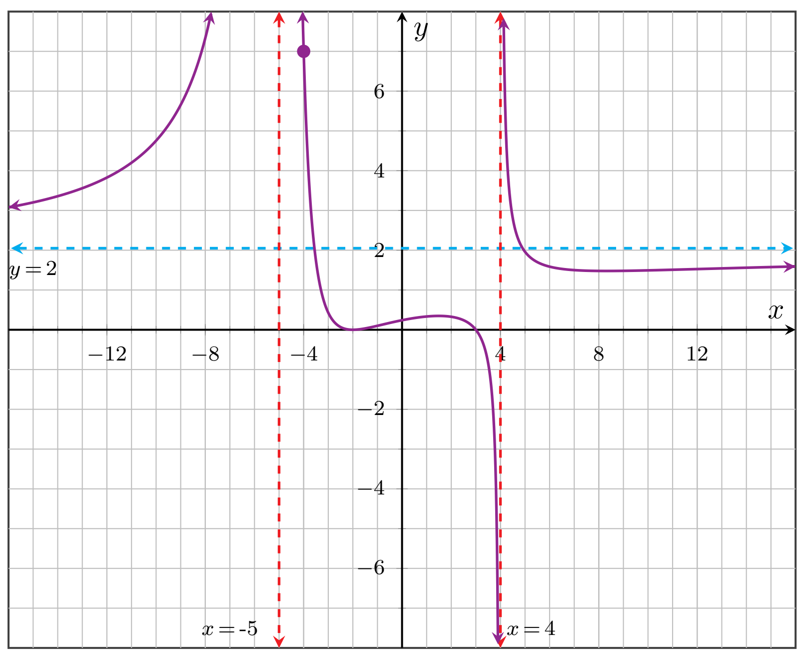
Group Work 1. Find a possible formula for the rational function graphed in Figure 8.

Figure 8



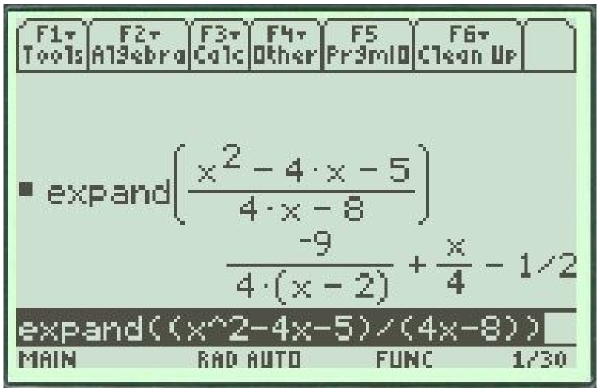
Group Work 2. Find a possible formula for the rational function graphed in Figure 9.

Figure 9



Example 8. Oblique Asymptotes

The graph of $R(x) = \frac{x^2-4x-5}{4x-8}$ "looks like" the function defined by $f(x) = \frac{1}{4}x$ in the long run. We know that this function has an oblique asymptote as the degree of the numerator is 1 greater than the degree of the denominator. To determine the equation of the oblique asymptote, either polynomial long division or a graphing calculator are needed. Using the "expand" key on a graphing calculator to perform polynomial long division, we find:



We use this to write:

$$R(x) = \frac{x^2-4x-5}{4x-8}$$
$$R(x) = \frac{-9}{4x-8} + \frac{1}{4}x - \frac{1}{2}$$

The first term in the expanded $R(x)$, which is $\frac{-9}{4x-8}$, is the *remainder*. The expression $\frac{1}{4}x - \frac{1}{2}$ is used to determine the equation of the oblique asymptote, which is $y = \frac{1}{4}x - \frac{1}{2}$. The function and its oblique asymptote are graphed in Figure 10 below.

FIGURE 10