Math 111 - Wed, 4/20

Q's on 1.5
Checkpoint 4 (on 1.5) ← Q's on 4.1
New material : 4.2 + Supplement
Inverse matching activity

1st Boss/Midterm on Monday
1 - 3:20 (You may leave when finished)
Part 1 - No Calculator
Part 2 - Graphing Calculator Needed
Review Problems in the Book
Review Packet - Solutions will be posted later in the week.

Math 111 Lecture Notes

Section 4.2: Inverse Functions

Example 1. Temperature in degrees Fahrenheit, F, can be written as a function of temperature in degrees Celsius, C. This relationship is given by $F = g(C) = \frac{9}{5}C + 32$.

(a) Find and interpret $g(100)$.

$$g(\underset{C}{100}) = \frac{9}{\cancel{5}}\left(\frac{\cancel{100}^{20}}{1}\right) + 32$$
$$= 180 + 32$$
$$= 212°F$$

100°C is equivalent to 212°F

(b) Solve and interpret the solution to $g(C) = 32$.

$$g(C) = 32°F$$
$$\frac{9}{5}C + 32 = 32$$
$$\frac{9}{5}C = 0$$
$$C = 0°C$$

0°C is equivalent to 32°F

(c) Solve the equation $F = \frac{9}{5}C + 32$ for C.

$$F = \frac{9}{5}C + 32$$
$$\frac{5}{9}(F - 32) = \frac{\cancel{9}}{\cancel{5}}C \cdot \frac{\cancel{5}}{\cancel{9}}$$
$$\frac{5}{9}(F - 32) = C$$

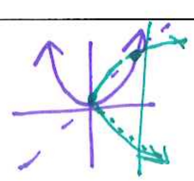
$$\left.\begin{aligned} \overset{\text{output}}{C} &= \frac{5}{9}(\overset{\text{input}}{F} - 32) \\ \underset{\text{output}}{F} &= \frac{9}{5}\underset{\text{input}}{C} + 32 \end{aligned}\right\} \text{inverse functions.}$$

A function f is said to be **one-to-one** if for every y-value in the range of f there is exactly one x-value in the domain of f.

A function must be one-to-one in order to have an inverse. The **inverse function of** f reverses the process of the original function. In other words, the input and output switch roles. The original function is given by $y = f(x)$. The inverse function is given by $x = f^{-1}(y)$. If we want to graph both of these functions in the (x, y)-plane, then we use $y = f^{-1}(x)$.

$f^{-1}(x)$ = "f inverse of x"

The inverse function of f is denoted by f^{-1}. It is important to note that this notation *is not* denoting a reciprocal. That is, $f^{-1}(x) \neq \dfrac{1}{f(x)}$.



$3^{-1} = \frac{1}{3}$ reciprocal

Example 2. Write the definition for $g^{-1}(F)$ for Example 1.

$\cancel{g(F)=}$

$f(x) \quad g(C) = \frac{9}{5}C + 32$

$f^{-1}(x) \quad g^{-1}(F) = \frac{5}{9}(F-32)$

Example 3. The function f defined by $f(x) = 3x + 2$ is one-to-one. Find its inverse. Then graph $y = f(x)$ and $y = f^{-1}(x)$ in Figure 1. Include the graph of $y = x$ also.

$f(x) = 3x + 2$

$y = 3x + 2$ switch x and y

$x = 3y + 2$ Solve for y

$\frac{x-2}{3} = \frac{3y}{3}$ or $\frac{1}{3}(x-2) = 3y\left(\frac{1}{3}\right)$

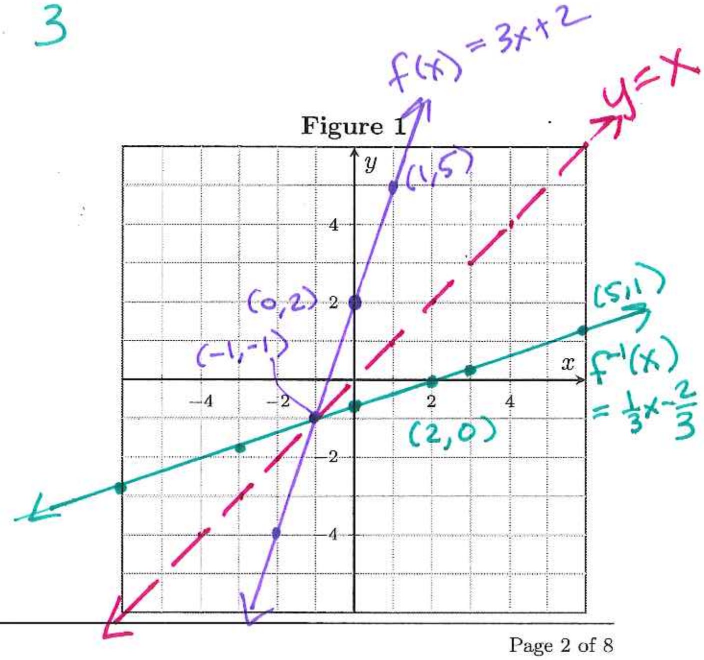
$\frac{x-2}{3} = y$ $\quad \frac{1}{3}(x-2) = y$

$y = \frac{x-2}{3}$ $\quad y = \frac{1}{3}x - \frac{2}{3}$

$f^{-1}(x) = \frac{x-2}{3}$ $\quad f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$

switch x + y
reflected over y=x

Figure 1



Example 4. To verify that two functions are inverses, we show that $\boxed{f\left(f^{-1}(x)\right) = x}$ and that $\boxed{f^{-1}\left(f(x)\right) = x.}$ Do this for the previous example.

$$f(x) = 3x+2 \qquad f^{-1}(x) = \frac{x-2}{3}$$

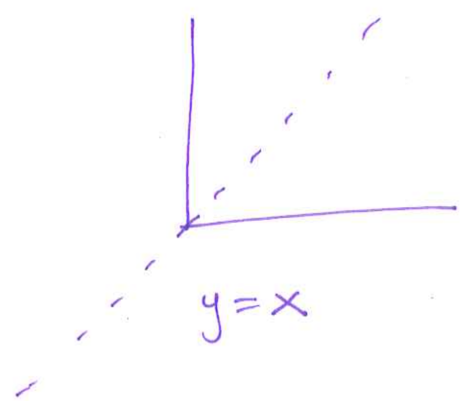
$$\begin{aligned} f(f^{-1}(x)) &= f\left(\frac{x-2}{3}\right) \\ &= \cancel{3}\left(\frac{x-2}{\cancel{3}}\right)+2 \\ &= x-\cancel{2}+\cancel{2} \\ &= x \end{aligned}$$

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}(3x+2) \\ &= \frac{3x+\cancel{2}-\cancel{2}}{3} \\ &= \frac{\cancel{3}x}{\cancel{3}} \\ &= x \end{aligned}$$

Functions that are inverses

$$y = x^2 \qquad y = \sqrt{x}$$

$$y = \sqrt[3]{x} \qquad y = x^3$$



Example 5. The function f defined by $f(x) = -\dfrac{2x}{x-1}$ is one-to-one. Find the inverse function. Confirm that the inverse function you found is correct by showing $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

$$y = -\frac{2x}{x-1}$$ switch $x + y$

$$(y-1)x = -\frac{2y}{\cancel{(y-1)}}\cancel{(y-1)}$$

$$xy - x = -2y$$ get the y's on one side

$$2y + xy = x$$

$$\frac{y(2+x)}{2+x} = \frac{x}{2+x}$$

$$y = \frac{x}{2+x}$$

$$f^{-1}(x) = \frac{x}{2+x}$$

$$\boxed{y = \frac{\cancel{x}}{\cancel{x}(x+2)}}$$ ok to cancel

$$f(f^{-1}(x)) = f\left(\frac{x}{2+x}\right)$$

$$= -\frac{2\left(\frac{x}{\cancel{2+x}}\right)}{\left(\left(\frac{x}{2+x}\right)-1\right)}\frac{\left(\frac{\cancel{2+x}}{1}\right)}{\left(\frac{2+x}{1}\right)}$$

$$= -\frac{2x}{\frac{x}{\cancel{2+x}}\frac{(\cancel{2+x})}{1} - 1\left(\frac{2+x}{1}\right)} = \frac{-2x}{\cancel{x}-2-\cancel{x}} = \frac{-2x}{-2} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(-\frac{2x}{x-1}\right)$$

$$= \frac{\frac{-2x}{\cancel{x-1}}\cdot\left(\frac{\cancel{x-1}}{1}\right)}{\left(2+\left(\frac{-2x}{x-1}\right)\right)\left(\frac{x-1}{1}\right)}$$

$$= \frac{-2x}{2(x-1)-2x}$$

$$= \frac{-2x}{\cancel{2x}-2-\cancel{2x}}$$

$$= \frac{-2x}{-2}$$

$$= x$$

State the domain and range of each f and f^{-1}.

$f(x)$:
D: $\{x \mid x \neq 1\}$
R: $\{x \mid x \neq 2\}$

$f^{-1}(x)$:
D: $\{x \mid x \neq -2\}$
R: $\{x \mid x \neq 1\}$

The domain of f is the range of f^{-1}. Similarly, the range of f is the domain of f^{-1}.

> The **horizontal line test** is a way of determining if a function is one-to-one. It states that if every horizontal line passes through a graph at most once, then the function is one-to-one.
> In the same way that the vertical line test verifies if a graph represents a function, the horizontal line test verifies if the graph of a function is one-to-one (and thus invertible).

Example 6. Graph $y = f(x)$ for $f(x) = x^2$ in Figure 2. Then graph $y = g(x)$ for $g(x) = x^2, x \geq 0$ in Figure 3. Is either function invertible? Why or why not?

Figure 2. Graph of $y = f(x)$

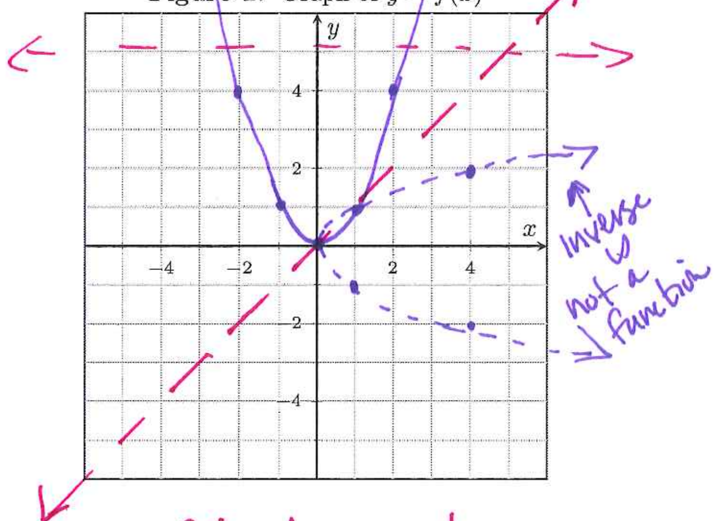
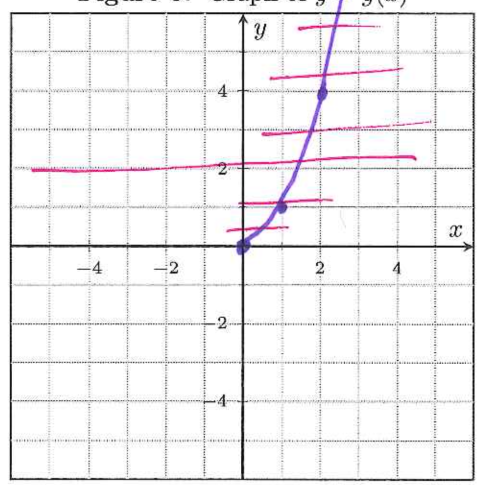


Figure 3. Graph of $y = g(x)$

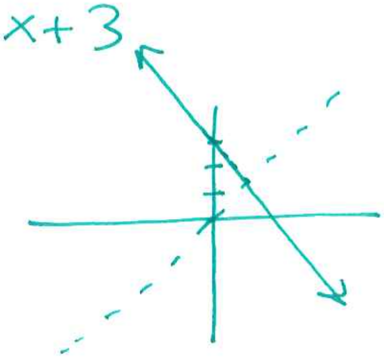


This does not pass the horizontal line test, so it is not invertible

This does pass the horizontal line test so it is one-to-one and invertible.

$f(x) = -x+3 \qquad g(x) = -x+3$

$f(g(x)) = f(-x+3)$
$= -(-x+3)+3$
$= x-3+3$
$= x$



Example 7. The function g defined by $g(x) = \sqrt[3]{x+8}$ is one-to-one. Find the inverse function and confirm that it is the inverse by showing $g\left(g^{-1}(x)\right) = x$ and $g^{-1}\left(g(x)\right) = x$. In Figure 4, use transformations to sketch $y = g(x)$, $y = g^{-1}(x)$ and $y = x$.

$g(x) = \sqrt[3]{x+8}$

$y = \sqrt[3]{x+8}$ switch x + y

$(x)^3 = \left(\sqrt[3]{y+8}\right)^3$ solve for y

$x^3 = y + 8$

$x^3 - 8 = y$

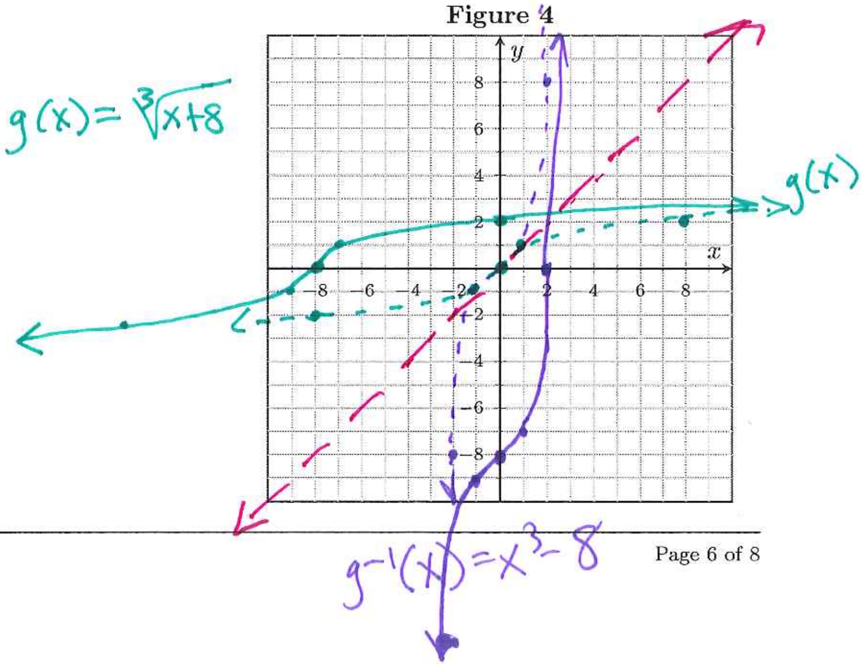
$y = x^3 - 8$

$g^{-1}(x) = x^3 - 8$

$$\begin{aligned} g(g^{-1}(x)) &= g(x^3-8) \\ &= \sqrt[3]{(x^3\cancel{-8})\cancel{+8}} \\ &= \sqrt[3]{x^{3}} \\ &= x \end{aligned}$$

$$\begin{aligned} g^{-1}(g(x)) &= g^{-1}\left(\sqrt[3]{x+8}\right) \\ &= \left(\left(\sqrt[3]{x+8}\right)^3 - 8\right) \\ &= x + 8 - 8 \\ &= x \end{aligned}$$

Figure 4



Example 8. Use the functions f and g given in Table 1 to determine the following.

reverse the x + y

x	7	2	0	-2	9
$g^{-1}(x)$	-2	-1	0	1	2

Table 1

x	-2	-1	0	1	2
$f(x)$	5	4	2	-1	1
$g(x)$	7	2	0	-2	9

x	5	4	2	-1	1
$f^{-1}(x)$	-2	-1	0	1	2

(a) $g^{-1}(-2) = 1$

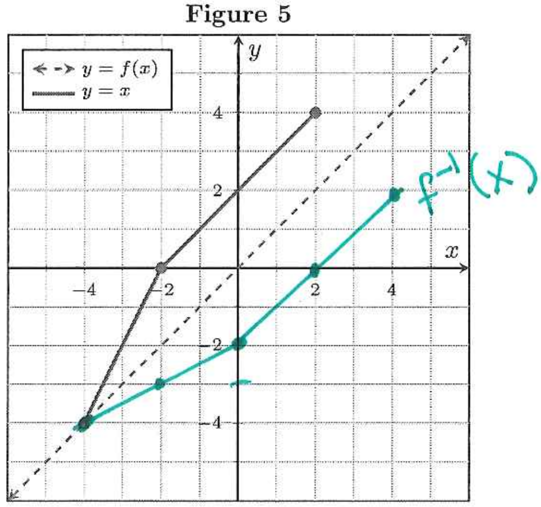
(b) $f^{-1}(2) = 0$

(c) $f^{-1}(0) =$ undef

(d) $f(g^{-1}(0))$
$= f(0)$
$= 2$

Example 9. Graph the inverse function of f in Figure 5. Then use your sketch to find the values of f^{-1} below.

Figure 5



(a) $f^{-1}(-4) = -4$

(c) $f^{-1}(0)$ -2

(e) $f^{-1}(4) = 2$

(b) $f^{-1}(-2) = -3$

(d) $f^{-1}(2)$ 0

Example 10. The diameter of a Window-Pane oyster, d (in mm), as a function of its weight, w (in grams) can be modeled by

$$d = f(w) = 25 + 20w^{1/3}$$

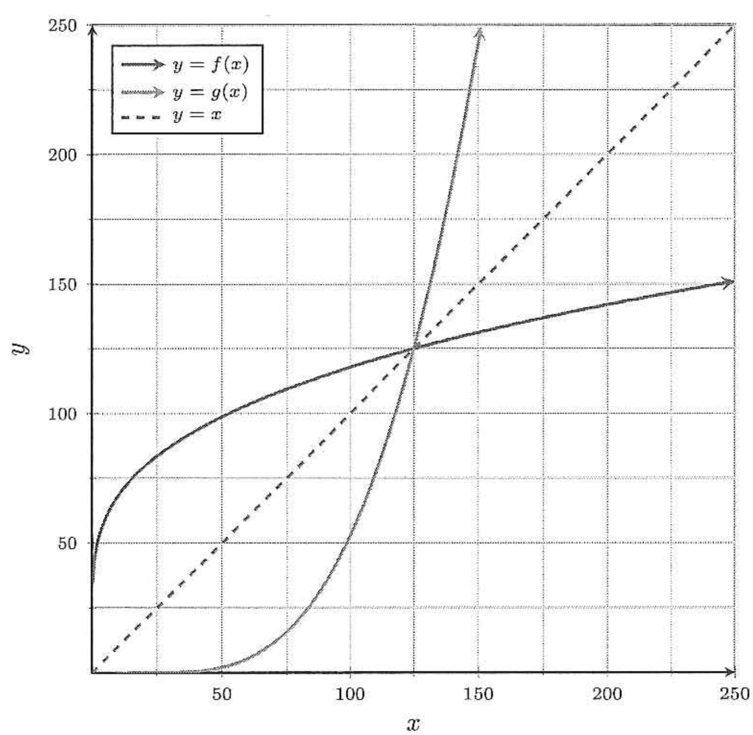
Find the inverse function by solving $d = 25 + 20w^{1/3}$ for w. Write this inverse function as $g(d)$.

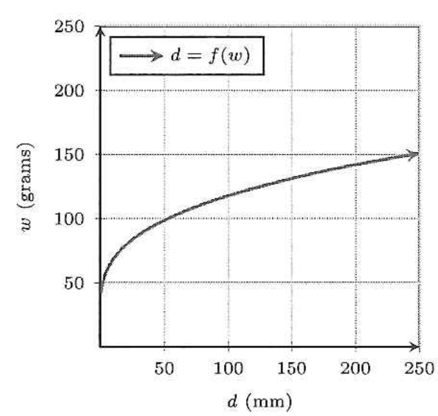
$d = 25 + 20\sqrt[3]{w}$ solve for w

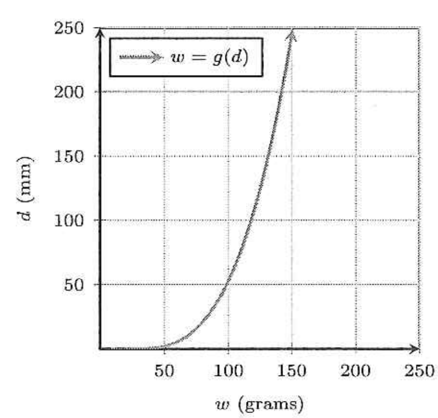
$$\frac{d-25}{20} = \frac{20\sqrt[3]{w}}{20}$$

$$\left(\frac{d-25}{20}\right)^3 = \left(\sqrt[3]{w}\right)^3$$

$$\left(\frac{d-25}{20}\right)^3 = w$$







$-x+3$

Math 111 Inverse Matching Activity
Section 4.2

1. Find the person that has the inverse to your function. Complete this sheet together.

2. Write down your functions.

Your Function	The Inverse Function
$g(x) = \frac{(2x-4)^3}{4}$	$f(x) = \frac{4+\sqrt[3]{4x}}{2}$
$\frac{1}{4}(2x-4)^3$	$= 2 + \frac{1}{2}\sqrt[3]{4x}$

$\frac{1}{4}(2(x-2))^3$

3. Verify that your functions are inverses of each other.

$f(g(x)) = \cancel{4+4}\ f\left(\frac{(2x-4)^3}{4}\right)$

$= \frac{4+\sqrt[3]{\cancel{4}\frac{(2x-4)^{\cancel{3}}}{\cancel{4}}}}{2}$

$= \frac{\cancel{4}+2x-\cancel{4}}{2}$

$= \frac{2x}{2}$

$= x$

$g(f(x)) =$

$g\left(\frac{4+\sqrt[3]{4x}}{2}\right)$

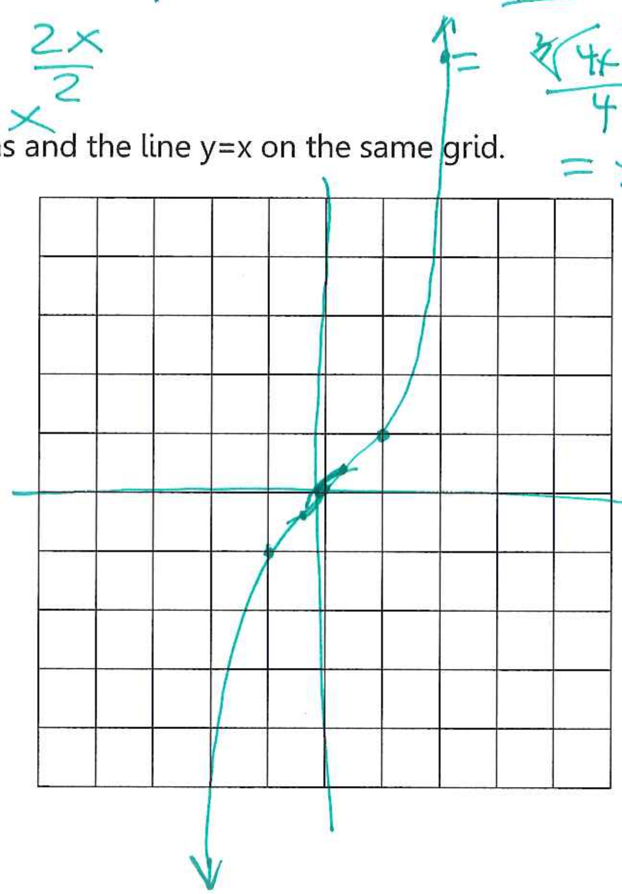
$= \frac{\left(\cancel{2}\left(\frac{4+\sqrt[3]{4x}}{\cancel{2}}\right)-4\right)^3}{4}$

$= \frac{(\cancel{4}+\sqrt[3]{4x}-\cancel{4})^3}{4}$

$= \frac{\sqrt[3]{4x}^3}{4}$

$= x$

4. Use transformations to graph both functions and the line y=x on the same grid.



Cara Lee

Key to all functions:

1. $f(x) = \frac{-16+x}{4}$	Q. $g(x) = 4n + 16$
2. $f(x) = \frac{4+\sqrt[3]{4x}}{2}$	P. $g(x) = \frac{(2x-4)^3}{4}$
3. $f(x) = -\frac{2}{x} - 1$	O. $g(x) = -\frac{2}{x+1}$
4. $f(x) = \sqrt[3]{x} - 3$	N. $g(x) = (x+3)^3$
5. $f(x) = \frac{1}{x} - 2$	M. $g(x) = \frac{1}{x+2}$
6. $f(x) = 2x^3 + 3$	L. $g(x) = \sqrt[3]{\frac{x-3}{2}}$
7. $f(x) = -4x + 1$	K. $g(x) = -\frac{1}{4}x + \frac{1}{4}$
8. $f(x) = \frac{7x+18}{2}$	J. $g(x) = \frac{2x-18}{7}$
9. $f(x) = -x + 3$	I. $g(x) = -x + 3$
10. $f(x) = x + 3$	H. $g(x) = x - 3$
11. $f(x) = 4x$	G. $g(x) = \frac{x}{4}$
12. $f(x) = -\frac{1}{5}x - 1$	F. $g(x) = -5x - 5$
13. $f(x) = \frac{1}{x-1}$	E. $g(x) = \frac{1}{x} + 1$
14. $f(x) = -2x^3 + 1$	D. $g(x) = \sqrt[3]{\frac{-x+1}{2}}$
15. $f(x) = \frac{-x-5}{3}$	C. $g(x) = -3x - 5$
16. $f(x) = 3 - 2x$	B. $g(x) = -\frac{1}{2}(x-3)$
17. $f(x) = 2x + 6$	A. $g(x) = \frac{1}{2}x - 3$