

3.3-3.4: Summary Statistics: Measures of Center and Spread - SOLUTIONS

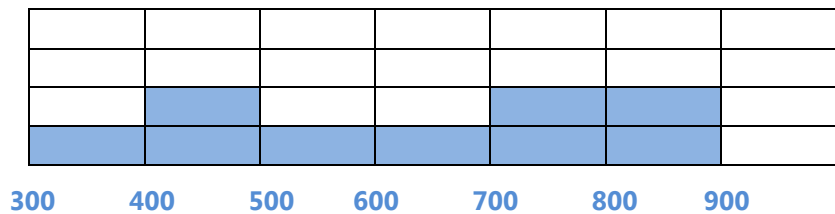
Group Activity

1. Halloween Data. A data scientist has been counting the number of trick-or-treaters that come to his house every year. We will analyze the shape, center and spread of this data.

Source: <https://www.dataplusscience.com/HalloweenData.html>

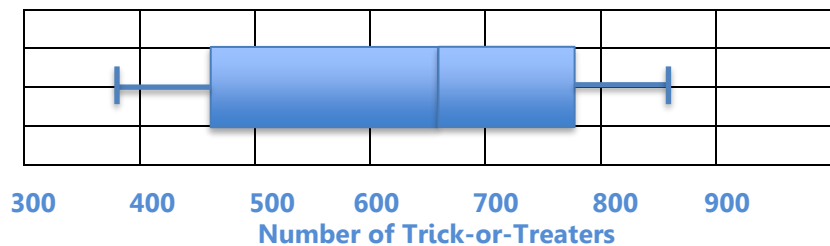
a. On the top grid, make a histogram of the number of trick-or-treaters using a bin-width of 100 trick-or-treaters. On the bottom grid you will make a boxplot, but not yet 😊. Follow the steps in order. Label your axes.

Histogram of Trick-or-Treaters



Year	Number of Trick-or-Treaters
2008	492
2009	542
2010	726
2011	869
2012	673
2013	391
2014	454
2015	747
2016	822

Boxplot of Trick-or-Treaters



b. What is the shape of the histogram? If you are not sure yet, compare the mean and the median first.

The Shape is bimodal and skewed to the left because the mean is less than the median.

c. Find the mean, median and mode, include units.

Put in order: 391, 454, 492, 542, 673, 726, 747, 822, 869

The mean is 635.1 trick-or-treaters. The median is 673 trick-or-treaters.

The modes are the centers of the highest peaks at 450 and 800 trick-or-treaters.

d. Find the 5-number summary, IQR and range, including units.

5-number summary: 391, 473, 673, 784.5, 869 trick-or-treaters

IQR = $Q3 - Q1 = 784.5 - 473 = 311.5$ trick-or-treaters

Range = $869 - 391 = 478$ trick-or-treaters

e. Use the 5-number summary to draw a boxplot on the second grid above. Make your horizontal scale match your histogram scale.

f. Do you think there are any outliers? Why or why not? **There do not seem to be any years that are outliers.**

Comparing Distributions

2. Below are fictitious student test scores from a Math 105 midterm in two different classes. You will be making a boxplot for each to compare their distributions.

Class 1: 72, 86, 65, 99, 86, 71, 55, 86, 92, 73, 95, 71 points

[55, 65, 71, 71, 72, 73], [86, 86, 86, 92, 95, 99]

Class 2: 75, 94, 82, 81, 69, 71, 85, 92, 88, 78, 73, 65, 66 points

[65, 66, 69, 71, 73, 75], 78, [81, 82, 85, 88, 92, 94]

a. Find the mean, 5-number summary, IQR and range for each class, including units.

Class 1:

Class 2:

Mean: 79.3 points

Mean: 78.4 points

**5-Number summary:
(55, 71, 79.5, 89, 99) points**

(65, 70, 78, 86.5, 94) points

IQR = Q3-Q1 = 89 – 71 = 18 points

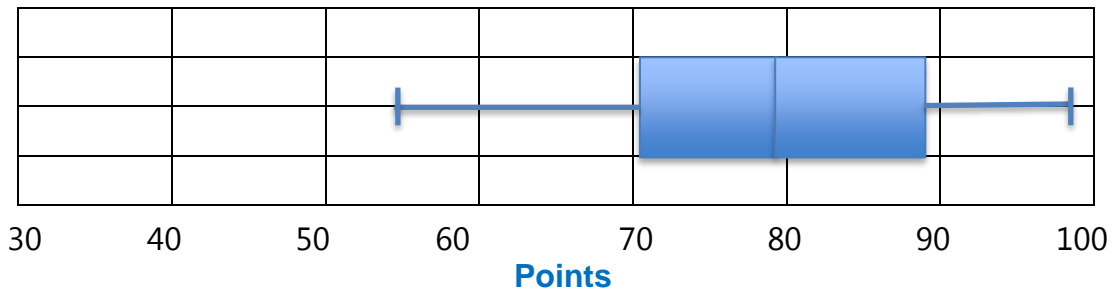
IQR = Q3-Q1 = 86.5 – 70 = 16.5 points

Range = Max – Min = 99 – 55 = 44 points

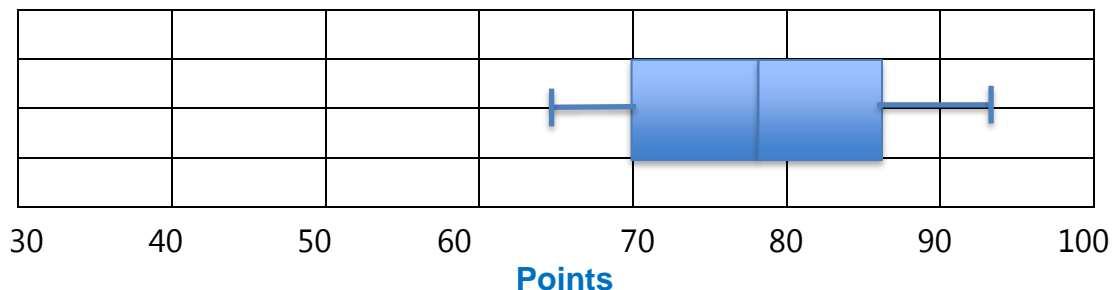
Max – Min = 94 – 65 = 29 points

b. Draw the boxplot for each class using the same scale.

Boxplot for Class 1



Boxplot for Class 2



c. What is the shape of the data for each class? How can you tell?

The shapes of both are approximately symmetric. The boxplots are fairly symmetric, and the medians are very close to the means for both classes.

Calculating Standard Deviation, s

d. Using your means rounded to one decimal place, find the standard deviation for each class, including units. The variable n refers to the number of data values.

Class 1:

Mean = 79.3 points, n = 12 students

Test Score (points)	Deviation from the mean	Squared deviation
72 – 79.3	– 7.3	53.29
86 – 79.3	6.7	44.89
65 – 79.3	– 14.3	204.49
99 – 79.3	19.7	388.09
86 – 79.3	6.7	44.89
71 – 79.3	– 8.3	68.89
55 – 79.3	– 24.3	590.49
86 – 79.3	6.7	44.89
92 – 79.3	12.7	161.29
73 – 79.3	– 6.3	39.69
95 – 79.3	15.7	246.49
71 – 79.3	– 8.3	68.89
Sum of the squared deviations (numerator)		1956.28

$$s = \sqrt{\frac{\sum (x - \text{mean})^2}{n - 1}}$$

$$= \sqrt{\frac{1956.28}{12 - 1}}$$

$$\approx 13.34 \text{ points}$$

75, 94, 82, 81, 69, 71, 85, 92, 88, 78, 73, 65, 66

Class 2:

Mean = 78.4 points, $n =$ 13 students

Test Score (points)	Deviation from the mean	Squared deviation
75 – 78.4	– 3.4	11.56
94 – 78.4	15.6	243.36
82 – 78.4	3.6	12.96
81 – 78.4	2.6	6.76
69 – 78.4	– 9.4	88.36
71 – 78.4	– 7.4	54.76
85 – 78.4	6.6	43.56
92 – 78.4	13.6	184.96
88 – 78.4	9.6	92.16
78 – 78.4	– 0.4	0.16
73 – 78.4	– 5.4	29.16
65 – 78.4	– 13.4	179.56
66 – 78.4	– 12.4	153.76
Sum of the squared deviations (numerator)		1101.08

$$s = \sqrt{\frac{\sum (x - \text{mean})^2}{n - 1}}$$

$$= \sqrt{\frac{1101.08}{13 - 1}}$$

≈ 9.58 points

e. Write a few complete sentences summarizing the four characteristics of the distribution of class 1.

The test scores for class 1 are approximately symmetric with a mean of 79.3 points and a standard deviation of 13.35 points. The student who scored 55 points may be an outlier.

f. Which class did better on the test? Use the vocabulary and values for center and spread in your answer.

Class 1 had a slightly higher mean (79.3 vs 78.4 points) and median (79.5 vs 78 points) and students scored higher, but some students also scored lower. It could be said that Class 1 did better.

It could also be said that Class 2 did better because they have a smaller range (29 vs 44 points) and standard deviation (9.58 vs 13.34 points), so their scores were less spread out and more consistent. The lowest score in class 2 was 65.

The lowest score in Class 1 was 55 points which may be an outlier due to a student having a bad day, illness or emergency, etc.