

4.2: Fundamentals of Probability

Class Prep Assignment

Due at the beginning of next class

Three Types of Probabilities, Theoretical, Empirical, Subjective

Theoretical Probability– Based on counting equally likely outcomes

Example 1. You have a quarter, a dime, and a nickel. You toss them in the air, and record whether they land on heads or tails.

a. Draw a tree to see the possible outcomes.

b. Can you think of a way to multiply to find the number of outcomes? (Multiplication Principle)

c. What is the probability of getting 3 heads?

d. exactly 1 head?

Theoretical Probability Model: We are interested in the number of heads, so we will list the possible outcomes for the number of heads, along with the probability of getting each.

Number of Heads				
Probability				

e. What is the probability of getting 1 or 2 heads?

f. What is the probability of getting fewer than 2 heads?

g. What is the probability of not getting 2 heads (**Complement**)?

h. What is the probability of getting **at least one** head? (**Complement of none**)

Empirical Probability (Relative Frequency) – Based on observations or an experiment
 15% of the parts sampled were found to be defective, so there is a 15% chance that a randomly selected part will be defective.

Subjective Probability – Based on intuition, experience or feeling
 What’s the chance you will go to the party on Friday?

Relationship between Odds and Probability

Example 2. A team is given odds of winning of 4:7. What is the chance they will win? Lose?

Gambling Odds: “odds on” means “odds against”

Example 3. At a horse race, the odds on My Little Pony are given as 8 to 1. What is the probability of My Little Pony winning and losing? The 8 to 1 odds mean that for every \$1 you bet on My Little Pony, you get \$8 if you win. If you bet \$10 and My Little Pony wins, how much do you win?

Combining Probabilities

Example 4. You have 10 prizes in a bag and people are going to draw them at random. Two are yellow erasers, one is a yellow calculator, three are green calculators and four are red erasers. First, let’s find the individual probabilities:

P(YE)= P(YC)= P(GC)= P(RE)=
 P(Calculator)= P(Eraser)= P(Red)= P(Yellow)= P(Green)=

“Or” Events (Single Draw)		“And” Events (Multiple Draws)	
Add $P(A \text{ or } B) = P(A) + P(B)$	Add Be careful not to double count the intersection	Multiply $P(A \text{ and } B) = P(A) \cdot P(B)$	Multiply Change the probability for each draw

“Or” Events

- a. What is the probability of drawing a yellow or green item?

- b. What is the probability of drawing a red item or an eraser?

“And” Events

- c. If we put the items back in each time, (draw with replacement), what is the probability of drawing three red erasers in a row?

- d. If we do not put the marbles back in each time, (draw without replacement), what is the probability of drawing three red erasers in a row?

4.3: Expected Value

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The Law of Large Numbers: In an experiment with independent trials, as you increase the number of trials the relative frequency gets closer to the theoretical probability.

Example 1. Theo rolled a standard 6-sided die 1,000 times and recorded that the number 2 came up 100 times. He suspects that the die is not fair. Is he correct?

Gambler’s Fallacy: The mistaken belief that a streak or run of bad luck will make the opposite outcome more likely.

Expected Value: The long run average or mean value for many repeated samples.

The expected value is an _____ that is weighted by the _____.

Example 2. A lottery ticket has five possible prize amounts and the chances of winning each are shown in the probability model.

Prize	\$1	\$10	\$50	\$1000	0
Probability	$\frac{1}{5}$	$\frac{1}{50}$	$\frac{1}{100}$	$\frac{1}{2000}$	$\frac{1539}{2000} = 0.7695$

a. Find the expected value for this lottery ticket.

b. If the cost of the ticket is \$2, what are your expected winnings?

Example 3. Primo Insurance sells an annual car insurance policy for \$1,350. Based on past data collected, an average of 1 in 50 policyholders will file a \$6,000 claim, an average of 1 in 100 policyholders will file a \$15,000 claim, and an average of 1 in 300 policyholders will file a \$33,000 claim.

Insurance Payout				
Probability				

a. Find the expected value for the amount that Primo will pay per policy.

b. What is the expected profit or loss per policy?