

4.2: Theoretical Probability - SOLUTIONS

Group Activity

1. Using the prize wheel below, make a theoretical probability model and then use it to find the probabilities below.

	Sub	Drink	Cookies	Chips	BOGO	Mystery Prize
Probability	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

2. If you spin the wheel once, what's the probability that you get

a. chips or a drink?

$$P(\text{chips or drink}) = \frac{4}{13} + \frac{2}{13} = \frac{6}{13}$$

b. not the mystery prize?

$$P(\text{not mystery}) = 1 - P(\text{mystery}) = 1 - \frac{1}{13} = \frac{12}{13}$$

c. a drink or not BOGO?

$$P(\text{drink or not BOGO}) = \frac{2}{13} + \frac{9}{13} = \frac{11}{13}$$

Be careful not to double count the drinks!



3. Find the following odds:

a. The odds of winning the mystery prize.

The odds of winning the mystery prize are 1:12

b. The odds against winning the mystery prize.

The odds against winning the mystery prize are 12:1

c. The odds against winning a sandwich.

The odds against winning a sandwich are 11:2

4. If you get to spin the wheel repeatedly, would that be like drawing with or without replacement? **With replacement because the wheel is the same every time. That makes the spins independent.**

a. If you get to spin 3 times, what is the chance you would get 3 bags of chips?

$$P(\text{chips and chips and chips}) = \frac{4}{13} \cdot \frac{4}{13} \cdot \frac{4}{13} = \frac{64}{2197}$$

b. If you get to spin twice, what is the chance you will get two BOGO's?

$$P(\text{BOGO and BOGO}) = \frac{2}{13} \cdot \frac{2}{13} = \frac{4}{169}$$

5. The t-shirts for your school group just arrived: 5 red small, 5 orange small, 10 red medium, 10 orange medium, 15 red large, 15 orange large, 10 red extra large, 10 orange extra large.

If you grab one t-shirt at random, what is the probability that

a. it is a small or an extra large

Disjoint

$$P(\text{small or xlarge}) = \frac{10}{80} + \frac{20}{80} = \frac{30}{80} = \frac{3}{8}$$

b. it is extra large or orange?

Overlapping

$$P(\text{xlarge or orange}) = \frac{20}{80} + \frac{30}{80} = \frac{50}{80} = \frac{5}{8}$$

Be careful not to double count orange XL's

c. it is not small or medium?

Disjoint

$$P(\text{not (small or medium)}) \\ = 1 - \frac{30}{80} = \frac{50}{80} = \frac{5}{8}$$

d. it is not small or red? (not small & not red)

Overlapping

$$P(\text{not (small or red)}) = \frac{35}{80}$$

Be careful not to double count

6. If five people come up and you draw 5 shirts at random, what is the probability that

a. they are all red larges? **Drawing without replacement**

$$\frac{15}{80} \cdot \frac{14}{79} \cdot \frac{13}{78} \cdot \frac{12}{77} \cdot \frac{11}{76} = \frac{3}{24,016} \approx 0.00013$$

b. there is at least one orange extra large? **At least one is the complement of none**

$$1 - P(\text{no orange XL}) = 1 - \frac{70}{80} \cdot \frac{69}{79} \cdot \frac{68}{78} \cdot \frac{67}{77} \cdot \frac{66}{76} \approx 1 - 0.5035 \approx 0.4965$$

4.3: Expected Value

7. a. Calculate the expected value of the Subway prize wheel from activity 7A,B. Let's say the mystery prize is a \$20 gift card.

	Sub	Drink	Cookies	Chips	BOGO	Mystery Prize
Prize Value	\$4.25	\$1.60	\$1.30	\$0.99	\$4.25	\$20
Probability	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

$$\begin{aligned}
 & \$4.25\left(\frac{2}{13}\right) + 1.60\left(\frac{2}{13}\right) + 1.30\left(\frac{2}{13}\right) + 0.99\left(\frac{4}{13}\right) \\
 & + 4.25\left(\frac{2}{13}\right) + 20\left(\frac{1}{13}\right) \approx \$3.60
 \end{aligned}$$



b. What does the expected value mean in this example? Explain it in a complete sentence.

The expected value of \$3.60 means that Subway will give out an average of \$3.60 per customer who spins the wheel. They should probably be careful with that.

8. Based on historical data, an auto insurance company estimates that a particular customer has a 1.5% likelihood of having an accident in the next year, with the average insurance payout being \$10,000.

If the company charges this customer an annual premium of \$500, what is the company's expected value of this insurance policy?

a. Make a probability table.

Possibilities	Accident	No Accident
Payout	\$10,000	\$0
Probability	0.015	0.985

b. Calculate the expected value for the company.

$$\$10,000(0.015) + \$0(0.985) = \$150$$

$$\$500 - 150 = \$350$$

The company will gain an average of \$350 in profit per insurance policy.

9. A company estimates that 7% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of \$250.

If they want to offer a 2-year extended warranty, what price should they charge so that they'll break even (in other words, so the expected value will be 0)

a. Make a probability table.

Possibilities	Breaks during extended warranty	Does not break during extended warranty
Payout	\$250	\$0
Probability	0.07	0.93

b. Calculate the expected value and answer the question.

$$\$250(0.07) + \$0(0.93) = \$17.50$$

The company should charge \$17.50 for an extended warranty if they want to break even. (They would charge more to make a profit)

More Practice



Beginning in October, 2015, **Powerball**® became an even larger combined large jackpot game and cash game. Every Wednesday and Saturday night at 10:59 p.m. Eastern Time, we draw five white balls out of a drum with 69 balls and one red ball out of a drum with 26 red balls.

Source: http://www.powerball.com/powerball/pb_prizes.asp

Powerball - Prizes and Odds

Match	Prize	Odds
	Grand Prize	1 in 292,201,338.00
	\$1,000,000	1 in 11,688,053.52
	\$50,000	1 in 913,129.18
	\$100	1 in 36,525.17
	\$100	1 in 14,494.11
	\$7	1 in 579.76
	\$7	1 in 701.33
	\$4	1 in 91.98
	\$4	1 in 38.32

The overall odds of winning a prize are 1 in 24.87.
The odds presented here are based on a \$2 play (rounded to two decimal places).

1.a. If the current Powerball grand prize amount is \$90 million, calculate the expected winnings per ticket:

$$\begin{aligned}
 & \$90,000,000 \left(\frac{1}{292,201,338} \right) + 1,000,000 \left(\frac{1}{11,688,053.52} \right) + 50,000 \left(\frac{1}{913,129.18} \right) + 100 \left(\frac{1}{36,525.17} \right) \\
 & + 100 \left(\frac{1}{14,494.11} \right) + 7 \left(\frac{1}{579.76} \right) + 7 \left(\frac{1}{701.33} \right) + 4 \left(\frac{1}{91.98} \right) + 4 \left(\frac{1}{38.32} \right) \approx \$0.63
 \end{aligned}$$

The expected winnings are \$0.63 per ticket.

b. Calculate the expected profit or loss for the ticket-holder per Powerball ticket:

$$\$0.63 - \$2.00 = \$-1.37.$$

On average, customers will lose \$1.37 per ticket.