4.3: Expected Value - SOLUTIONS

<u>Group Activity</u>

Beginning in October, 2015, **Powerball**® became an even larger combined large jackpot game and cash game. Every Wednesday and Saturday night at 10:59 p.m. Eastern Time, we draw five white balls out of a drum with 69 balls and one red ball out of a drum with 26 red balls.
Source: http://www.powerball.com/powerball/pb_prizes.asp

Powerball - Prizes and Odds

Match	Prize	Odds
	Grand Prize	1 in 292,201,338.00
	$1,000,000	1 in 11,688,053.52
	$50,000	1 in 913,129.18
	$100	1 in 36,525.17
	$100	1 in 14,494.11
	$7	1 in 579.76
	$7	1 in 701.33
	$4	1 in 91.98
	$4	1 in 38.32

The overall odds of winning a prize are 1 in 24.87.
The odds presented here are based on a $2 play (rounded to two decimal places).

1.a. If the current Powerball grand prize amount is $90 million, calculate the expected winnings per ticket:

$$\$90,000,000\left(\frac{1}{292,201,338}\right)+1,000,000\left(\frac{1}{11,688,053.52}\right)+50,000\left(\frac{1}{913,129.18}\right)+100\left(\frac{1}{36,525.17}\right)$$

$$+100\left(\frac{1}{14,494.11}\right)+7\left(\frac{1}{579.76}\right)+7\left(\frac{1}{701.33}\right)+4\left(\frac{1}{91.98}\right)+4\left(\frac{1}{38.32}\right)\approx\$0.63$$

The expected winnings are $0.63 per ticket.

b. Calculate the expected profit or loss for the ticket-holder per Powerball ticket:

$0.63-$2.00 = $-1.37.

On average, customers will lose $1.37 per ticket.

2. a. Calculate the expected value of the Subway prize wheel from activity 7A,B. Let's say the mystery prize is a $20 gift card.

	Sub	Drink	Cookies	Chips	BOGO	Mystery Prize
Prize Value	$4.25	$1.60	$1.30	$0.99	$4.25	$20
Probability	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

$$\$4.25\left(\frac{2}{13}\right)+1.60\left(\frac{2}{13}\right)+1.30\left(\frac{2}{13}\right)+0.99\left(\frac{4}{13}\right)$$
$$+4.25\left(\frac{2}{13}\right)+20\left(\frac{1}{13}\right)\approx\$3.60$$

b. What does the expected value mean in this example? Explain it in a complete sentence.

The expected value of $3.60 means that Subway will give out an average of $3.60 per customer who spins the wheel. They should probably be careful with that.

3. Based on historical data, an auto insurance company estimates that a particular customer has a 1.5% likelihood of having an accident in the next year, with the average insurance payout being $10,000.

If the company charges this customer an annual premium of $500, what is the company's expected value of this insurance policy?

a. Make a probability table.

Possibilities	Accident	No Accident
Payout	**$10,000**	**$0**
Probability	**0.015**	**0.985**

b. Calculate the expected value for the company.

$$\$10,000(0.015)+\$0(0.985)=\$150$$

$$\$500-150=\$350$$

The company will gain an average of $350 in profit per insurance policy.

4. A company estimates that 7% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of $250.

If they want to offer a 2-year extended warranty, what price should they charge so that they'll break even (in other words, so the expected value will be 0)

a. Make a probability table.

Possibilities	Breaks during extended warranty	Does not break during extended warranty
Payout	**$250**	**$0**
Probability	**0.07**	**0.93**

b. Calculate the expected value and answer the question.
$$\$250(0.07)+\$0(0.93)=\$17.50$$

The company should charge $17.50 for an extended warranty if they want to break even. (They would charge more to make a profit)