12A,B: Voting Methods and Voting Power - SOLUTIONS

Group Activity

1. Ranked Choice Voting Election. Our class will elect the best candy out of the three that are running for office: **R=Reese's Peanut Butter Cups, S=Starburst, K=KitKat**

Please get ballot forms for your group and fill them out anonymously and turn them in. When all the results are tabulated, make a preference schedule.

Preference Schedule

Number of Voters						
1st choice						
2nd choice						
3rd choice						

a. How many voters voted in this election?

b. How many votes are needed for a majority?

c. How many votes are needed for a plurality win?

d. Find the winner under the plurality method.

e. Find the winner under the Instant Runoff Voting method.

f. Find the winner under the Borda Count method.

g. Find the winner under the Pairwise Comparisons method.

h. Which method do you think is the most fair in this situation and why?

2. A homeowners' association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options have been proposed. The votes are:

Number of voters	**8**	**9**	**11**	**7**	**7**	**5**
1st choice	B	A	D	A	B	C
2nd choice	C	D	B	B	A	D
3rd choice	A	C	C	D	C	A
4th choice	D	B	A	C	D	B

a. How many voters voted in this election? $8+9+11+7+7+5=47$

b. How many votes are needed for a majority? $47\div 2=23.5$
24 votes are needed for a majority

c. How many votes are needed for a plurality win? $47\div 4=11.75$
12 votes are needed for plurality

d. Find the winner under the plurality method.
A=16 B=15 C=5 D=11
A is the plurality winner.

e. Find the winner under the Instant Runoff Voting method.
A=16 ~~B=15~~ ~~C=5~~ D=11
+8 +5
+7 16
31
A is the instant runoff winner.

f. Find the winner under the Borda Count method.
$A=2\cdot 8+4\cdot 9+1\cdot 11+4\cdot 7+3\cdot 7+2\cdot 5=122$
$B=4\cdot 8+1\cdot 9+3\cdot 11+3\cdot 7+4\cdot 7+1\cdot 5=128$
$C=3\cdot 8+2\cdot 9+2\cdot 11+1\cdot 7+2\cdot 7+4\cdot 5=105$
$D=1\cdot 8+3\cdot 9+4\cdot 11+2\cdot 7+1\cdot 7+3\cdot 5=115$
B is the winner. Note: The total is 470 which is (1+2+3+4)(47)

g. Find the winner under the Pairwise Comparisons method.
A 21, **B 26** **B 33**, C 14 C 20, **D 27**
A 23, **C 24** B 22, **D 25**
A 31, D 16
Both B and D have two pairwise wins, so it is not clear who would win. Who do you think should win?

h. Which method do you think is the most fair in this situation and why?

12C Apportionment

It's very important to acknowledge that when Hamilton, Jefferson and others were making the rules for apportionment, black people were owned and enslaved in the United States. The 3/5 rule counted the black population as only 3/5 of white people. This is often omitted from textbooks, but we need to remember why we still have racism and systemic inequality now and continue to make our systems fair for everyone.

3. A college offers tutoring in Math, English, Chemistry, and Biology. The number of students enrolled in each subject is listed below. If the college can only afford to hire 15 tutors, determine how many tutors should be assigned to each subject. Examples adapted from David Lippman, http://www.opentextbookstore.com/mathinsociety/index.html

a. Hamilton's Method

Subject	Students	÷53 Standard Quota	Cut off decimal	Give Extra to highest decimal
Math	330	**6.23**	**6**	**6**
English	265	**5**	**5**	**5**
Chemistry	130	**2.45**	**2 + 1**	**3**
Biology	70	**1.32**	**1**	**1**
Total	**795**		**14**	**15**

add 1 to subject with highest decimal

Divisor: $795 \div 15 = 53$

b. Jefferson's Method

Subject	Students	÷53 Standard Quota	Cut off Decimal	÷45 Use New Divisor	Cut off Decimal
Math	330	**6.23**	**6**	**7.3**	**7**
English	265	**5**	**5**	**5.82**	**5**
Chemistry	130	**2.45**	**2**	**2.89**	**2**
Biology	70	**1.32**	**1**	**1.56**	**1**
Total	**795**		**14**		**15**

Lower divisor until the total is 15. Try 45

Divisor: $795 \div 15 = 53$

4. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

a. If the legislature has 10 seats, use Hamilton's method to apportion the seats.
b. If the legislature grows to 11 seats, use Hamilton's method to apportion the seats
c. Does the new apportionment seem fair? Why or why not?

State	Population	$\div 1,400$ Standard Quota			$\div 1,272.73$ Standard Quota		
A	6,000	**4.29**	**4**	**4**	**4.71**	**4 +1**	**5**
B	6,000	**4.29**	**4**	**4**	**4.71**	**4 +1**	**5**
C	2,000	**1.43**	**1 +1**	**2**	**1.57**	**1**	**1**
Total	**14,000**		**9**	**10**			**11**
Divisor	$14,000 \div 10 = 1,400$				For 11 seats: $14,000 \div 11 = 1,272.73$		

This is not fair because C lost a representative and both A and B gained a representative. This is one of the problems with the Hamilton Method.

5. Repeat the problem above using Jefferson's method. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

a. If the legislature has 10 seats, use Jefferson's method to apportion the seats. What happens?
b. If the legislature grows to 11 seats, use Jefferson's method to apportion the seats
c. Does the new apportionment seem fair? Why or why not?

State	Population	$\div 1,300$ Standard Quota		$\div 1,200$	
A	6,000	**4.61**	**4**	**5.0**	**5**
B	6,000	**4.61**	**4**	**5.0**	**5**
C	2,000	**1.53**	**1**	**1.67**	**1**
Total	**14,000**		**9**		**11 (Fails)**
Divisor	$14,000 \div 10 = 1,400$			For 11 seats: $14,000 \div 11 = 1,272.73$	

Jefferson's method does not work in this case because A and B will get another representative before C does so you can't get 10 representatives. Similar to Hamilton's method, it doesn't seem fair for A and B to have 5 reps and C only has 1.

Quota Rule
The Quota Rule says that the final number of representatives a state gets should be within one of that state's quota. Since we're dealing with whole numbers for our final answers, that means that each state should either go up to the next whole number above its quota, or down to the next whole number below its quota.

Do any of our examples violate the quota rule? **No.**

12D Gerrymandering and Solutions

Azavea, a data analytics organization, has calculated the efficiency gap for all 50 states. At a good time we will look at the infographics together.
https://www.azavea.com/blog/2017/07/19/gerrymandered-states-ranked-efficiency-gap-seat-advantage/

6 a. You have just been hired as consultants to your state legislature in the re-districting of the state. To assess the current map below, calculate the efficiency gap.

Election Results:

Democrats win

__3_____ seats

Republicans win

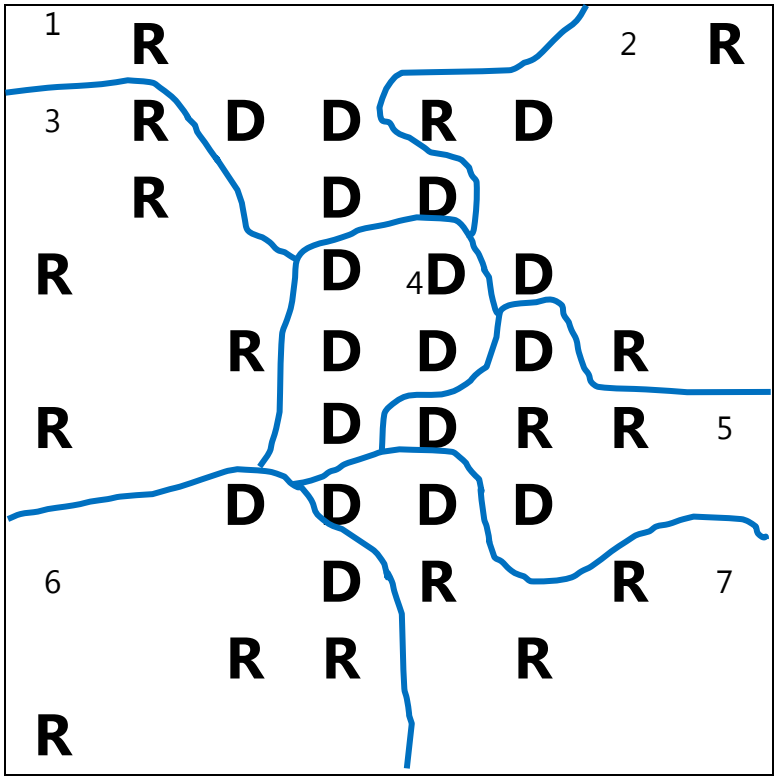
__4_____ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	4	1	4-3=1	1
2	2	3	2	3-3=0
3	0	5	0	5-3=2
4	5	0	5-3=2	0
5	3	2	3-3=0	2
6	2	3	2	3-3=0
7	2	3	2	3-3=0
Total	18	17	9	5

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$= \frac{4}{35} \approx 0.114 \text{ or } 11.4\%$$



b. Calculate the percentage of voters that each seat represents.

$$100\% \div 7 \approx 14.3\%$$

c. Is the efficiency gap worth one seat or more? How many seats?

The gap is worth just less than one seat.

d. Is this a fair map? Why or why not?

7. Now it is time for re-districting and you get to draw the lines. There are three rules:

Rules
1. All legislative districts must contain the same number of people.
2. Districts must not be drawn according to race or ethnicity.
3. District must be contiguous – no split districts allowed

a. Use packing and cracking to win as many seats as possible for the **Democrats** and calculate the efficiency gap.

Election Results:

Democrats win

___6_____ seats

Republicans win

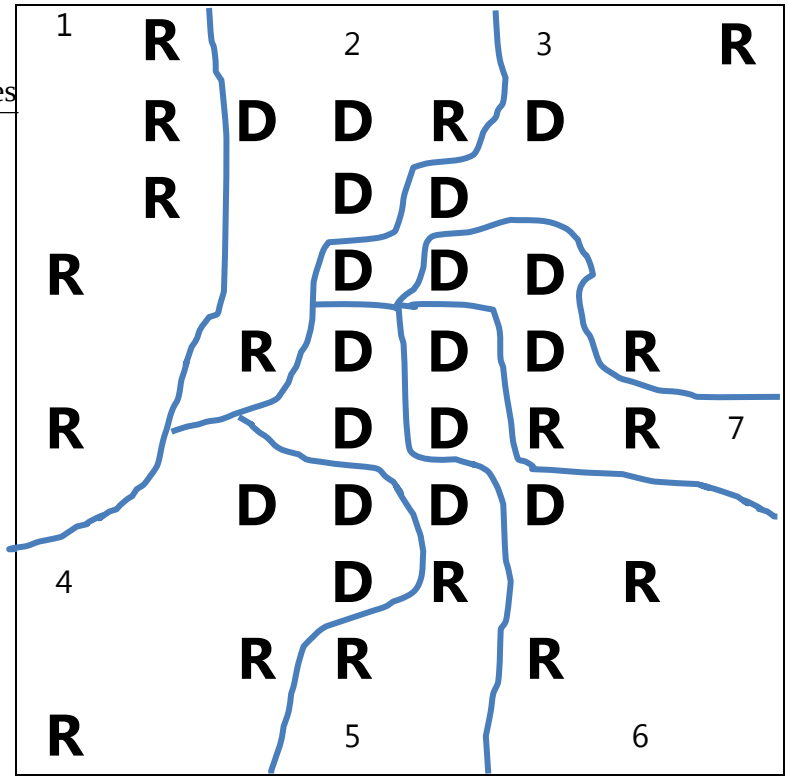
___1_____ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	0	5	0	5-3=2
2	3	2	3-3=0	2
3	3	2	3-3=0	2
4	3	2	3-3=0	2
5	3	2	3-3=0	2
6	3	2	3-3=0	2
7	3	2	3-3=0	2
Total	18	17	0	14

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$\frac{14-0}{35} = \frac{14}{35} =$ **40%**



b. Use packing and cracking to win as many seats as possible for the **Republicans** and calculate the efficiency gap.

Election Results:

Democrats win

__2__ seats

Republicans win

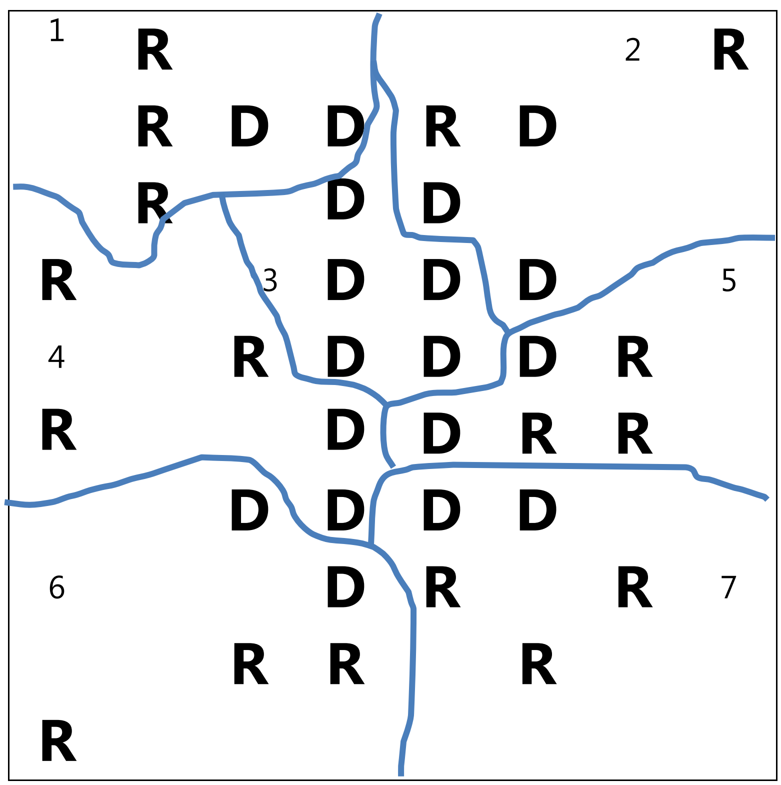
__5__ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	2	3	2	3-3=0
2	3	2	3-3=0	2
3	5	0	5-3=2	0
4	2	3	2	3-3=0
5	2	3	2	3-3=0
6	2	3	2	3-3=0
7	2	3	2	3-3=0
Total	18	17	12	2

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$\frac{12-2}{35} = \frac{10}{35} = \mathbf{29\%}$$



Extra Gerrymandering Problem – Use this to practice for the final because gerrymandering is not in MyOpenMath.

8. A State map is shown below. Tally the voters and use the information to answer the questions below.

a. Calculate the results of an election and the efficiency gap.

Election Results:

Democrats win

____**1**____ seats

Republicans win

___**5**____ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	3	4	3	4-4=0
2	3	4	3	4-4=0
3	3	4	3	4-4=0
4	6	1	6-4=2	1
5	3	4	3	4-4=0
6	3	4	3	4-4=0
Total	21	21	17	1

Efficiency Gap

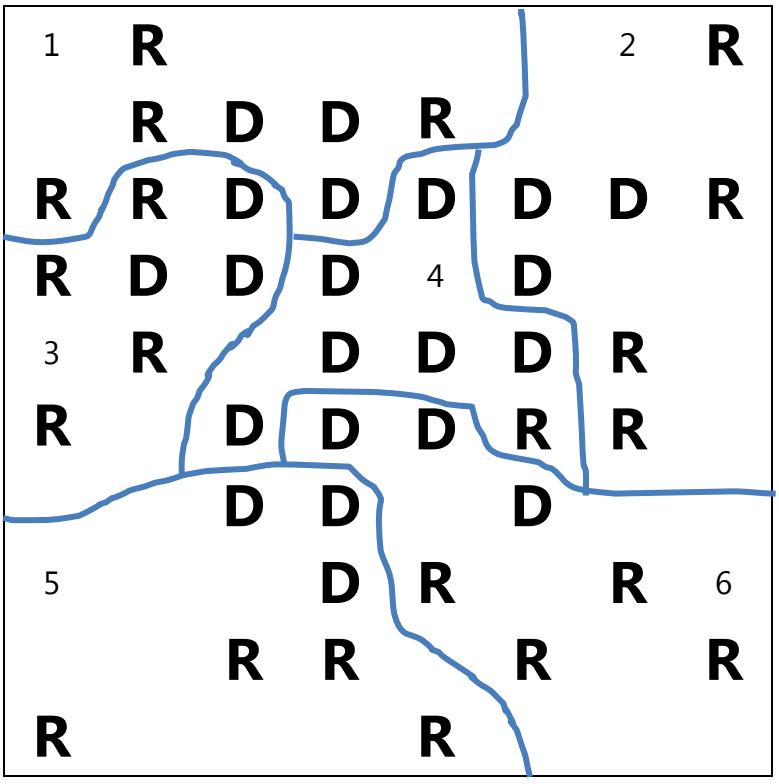
$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$= \frac{17-1}{42}$ **=0.38 or 38%**

b. Calculate the percentage of voters that each seat represents.

$= \frac{100\%}{6 \text{ districts}}$ **=0.17 or 17%**

c. Compare the efficiency gap with the percentage for each seat. How many seats is the efficiency gap worth?



The efficiency gap of 38% is worth more than two full seats.

d. Is this a fair map? Why or why not?

No, this map is not fair because the population is 50-50 but the seats are 5-1.

Using the graph below, estimate the number of extra seats held by the majority party in the current House of Representatives.

Source: https://www.brookings.edu/blog/fixgov/2017/02/22/misrepresentation-in-the-house/