7A,B: Fundamentals of Probability and Combining Probabilities - SOLUTIONS

Coin Toss. In the video we looked at the theoretical probabilities for flipping a quarter, dime and nickel. Now we will do a class experiment to find empirical probabilities.

1a. Empirical Probability . Get a quarter, nickel and dime for your group. Take turns tossing them for a total of 10 trials. Record H or T for each coin in each trial.

Trial	1	2	3	4	5	6	7	8	9	10
Quarter										
Nickel										
Dime										

b. From your 10 trials, count the number of times you got 0 heads, 1 head, 2 heads and 3 heads. Write the number in each column. They should add up to 10 trials.

Number of Heads	0	1	2	3
Group Count				

c. Combining the Class Data. Record your totals on the class sheet on the document camera. Once all the data is added, write the totals in the next table. Number of trials_____

Number of Heads	0	1	2	3
Total Class Count				

d. **Empirical Probability Model.** Using the class totals, calculate the empirical probability of each outcome.

Number of Heads	0	1	2	3
Empirical Probability				

e. Compare these numbers to the theoretical outcomes on your notes. How do they compare?

f. What would you expect if we repeated this experiment for 1000 trials?
We would expect the empirical probabilities to be close to the theoretical probabilities. The more trials we do, the closer they should get.

Theoretical Probability

2. Using the prize wheel below, make a theoretical probability model and then use it to find the probabilities below.

	Sub	Drink	Cookies	Chips	BOGO	Mystery Prize
Probability	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

3. If you spin the wheel once, what's the probability that you get

a. chips or a drink?

$$P(\text{chips or drink}) = \frac{4}{13} + \frac{2}{13} = \frac{6}{13}$$

b. not the mystery prize?

$$P(\text{not mystery}) = 1 - P(\text{mystery}) = 1 - \frac{1}{13} = \frac{12}{13}$$

c. a drink or not BOGO?

$$P(\text{drink or not BOGO}) = \frac{2}{13} + \frac{9}{13} = \frac{11}{13}$$

Be careful not to double count the drinks!

4. Find the following odds:

a. The odds of winning the mystery prize.

The odds of winning the mystery prize are 1:12

b. The odds against winning the mystery prize.

The odds against winning the mystery prize are 12:1

c. The odds on winning a sandwich.

The odds against winning a sandwich are 11:2

5. If you get to spin the wheel repeatedly, would that be like drawing with or without replacement? **With replacement because the wheel is the same every time. That makes the spins independent.**

a. If you get to spin 3 times, what is the chance you would get 3 bags of chips?

$$P(\text{chips and chips and chips}) = \frac{4}{13} \cdot \frac{4}{13} \cdot \frac{4}{13} = \frac{64}{2197}$$

b. If you get to spin twice, what is the chance you will get two BOGO's?

$$P(\text{BOGO and BOGO}) = \frac{2}{13} \cdot \frac{2}{13} = \frac{4}{169}$$

Subjective Probability

6. Make up an example of a subjective probability.

I think there is a 90% chance that I will go to the beach this summer.

7. Dinner combinations:

Starter—Caesar salad, mozzarella sticks, steamer clams, chicken skewers, calamari

Protein—Alaskan king crab, prime rib, grilled chicken, pork ribs, rainbow trout

Side—baked potato, french fries, garlic mashed potatoes, steamed broccoli, garlic toast

Dessert—apple pie, carrot cake, marionberry cobbler, caramel sundae

a. If a meal is made from one choice in each category, find the total number of different meals.
   5 x 5 x 5 x 4 = 500
   There are 500 possible meals

b. How many meals include a Caesar salad?
   1 x 5 x 5 x 4
   = 100
   There are 100 meals that include a Caesar salad

c. What is the probability that a meal includes a Caesar salad?
   $\frac{100}{500} = \frac{1}{5}$ or 0.2 or 20%

8. If you can use capital letters, lowercase letters, the numbers 0-9 and 8 special characters (!,@,#, etc.), how many 8-character passwords could you make?

26 + 26 + 10 + 8 = 70 possible characters

For an 8-character password:
70 x 70 x 70 x 70 x 70 x 70 x 70 x 70 = 70^8 =
576,480,100,000,000

There are over 5.76 trillion password combinations

9. The t-shirts for your school group just arrived: 5 red small, 5 orange small, 10 red medium, 10 orange medium, 15 red large, 15 orange large, 10 red extra large, 10 orange extra large.

If you grab one t-shirt at random, what is the probability that

a. it is a small or an extra large

Disjoint

$$P(\text{small or xlarge}) = \frac{10}{80} + \frac{20}{80} = \frac{30}{80} = \frac{3}{8}$$

b. it is extra large or orange?

Overlapping

$$P(\text{xlarge or orange}) = \frac{20}{80} + \frac{30}{80} = \frac{50}{80} = \frac{5}{8}$$

Be careful not to double count orange XL's

c. it is not small or medium?

Disjoint

$$P\left(\text{not}\left(\text{small or medium}\right)\right)$$
$$= 1 - \frac{30}{80} = \frac{50}{80} = \frac{5}{8}$$

d. it is not small or red? (not small & not red)

Overlapping

$$P\left(\text{not}\left(\text{small or red}\right)\right) = \frac{35}{80}$$

Be careful not to double count

10. If five people come up and you draw 5 shirts at random, what is the probability that

a. they are all red larges? **Drawing without replacement**

$$\frac{15}{80} \cdot \frac{14}{79} \cdot \frac{13}{78} \cdot \frac{12}{77} \cdot \frac{11}{76} = \frac{3}{24{,}016} \approx 0.00013$$

b. there is at least one orange extra large? **At least one is the complement of none**

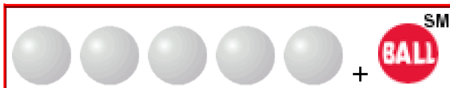
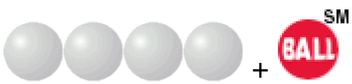
$$1 - P(\text{no orange XL}) = 1 - \frac{70}{80} \cdot \frac{69}{79} \cdot \frac{68}{78} \cdot \frac{67}{77} \cdot \frac{66}{76} \approx 1 - 0.5035 \approx 0.4965$$

7C: Expected Value and the Law of Large Numbers - SOLUTIONS

POWERBALL®

Beginning in October, 2015, **Powerball**® became an even larger combined large jackpot game and cash game. Every Wednesday and Saturday night at 10:59 p.m. Eastern Time, we draw five white balls out of a drum with 69 balls and one red ball out of a drum with 26 red balls.
Source: http://www.powerball.com/powerball/pb_prizes.asp

Powerball - Prizes and Odds

Match	Prize	Odds
	Grand Prize	1 in 292,201,338.00
	$1,000,000	1 in 11,688,053.52
	$50,000	1 in 913,129.18
	$100	1 in 36,525.17
	$100	1 in 14,494.11
	$7	1 in 579.76
	$7	1 in 701.33
	$4	1 in 91.98
	$4	1 in 38.32

The overall odds of winning a prize are 1 in 24.87.
The odds presented here are based on a $2 play (rounded to two decimal places).

11.a. If the current Powerball grand prize amount is $90 million, calculate the expected winnings per ticket:

$$\$90,000,000\left(\frac{1}{292,201,338}\right)+1,000,000\left(\frac{1}{11,688,053.52}\right)+50,000\left(\frac{1}{913,129.18}\right)+100\left(\frac{1}{36,525.17}\right)$$
$$+100\left(\frac{1}{14,494.11}\right)+7\left(\frac{1}{579.76}\right)+7\left(\frac{1}{701.33}\right)+4\left(\frac{1}{91.98}\right)+4\left(\frac{1}{38.32}\right)\approx \$0.63$$

The expected winnings are $0.63 per ticket.

b. Calculate the expected profit or loss for the ticket-holder per Powerball ticket:

$0.63-$2.00 = $-1.37.

On average, customers will lose $1.37 per ticket.

12. a. Calculate the expected value of the Subway prize wheel from activity 7A,B. Let's say the mystery prize is a $20 gift card.

	Sub	Drink	Cookies	Chips	BOGO	Mystery Prize
Prize Value	$4.25	$1.60	$1.30	$0.99	$4.25	$20
Probability	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{2}{13}$	$\frac{1}{13}$

$$\$4.25\left(\frac{2}{13}\right)+1.60\left(\frac{2}{13}\right)+1.30\left(\frac{2}{13}\right)+0.99\left(\frac{4}{13}\right)$$
$$+4.25\left(\frac{2}{13}\right)+20\left(\frac{1}{13}\right)\approx \$3.60$$

b. What does the expected value mean in this example? Explain it in a complete sentence.

The expected value of $3.60 means that Subway will give out an average of $3.60 per customer who spins the wheel. They should probably be careful with that.

13. Based on historical data, an auto insurance company estimates that a particular customer has a 1.5% likelihood of having an accident in the next year, with the average insurance payout being $10,000.

If the company charges this customer an annual premium of $500, what is the company's expected value of this insurance policy?

a. Make a probability table.

Possibilities	Accident	No Accident
Payout	**$10,000**	**$0**
Probability	**0.015**	**0.985**

b. Calculate the expected value for the company.

$\$10,000(0.015)+\$0(0.985)=\$150$

$\$500-150=\$350$

The company will gain an average of $350 in profit per insurance policy.

14. A company estimates that 7% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of $250.

If they want to offer a 2-year extended warranty, what price should they charge so that they'll break even (in other words, so the expected value will be 0)

a. Make a probability table.

Possibilities	Breaks during extended warranty	Does not break during extended warranty
Payout	**$250**	**$0**
Probability	**0.07**	**0.93**

b. Calculate the expected value and answer the question.

$\$250(0.07)+\$0(0.93)=\$17.50$

The company should charge $17.50 for an extended warranty if they want to break even. (They would charge more to make a profit)