

D1-D2: Voting Methods and the Electoral College vs. Popular Vote SOLUTIONS

Group Activity

D1: Voting Methods

1. A homeowners' association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options have been proposed. The votes are:

Number of voters	8	9	11	7	7	5
1st choice	B	A	D	A	B	C
2nd choice	C	D	B	B	A	D
3rd choice	A	C	C	D	C	A
4th choice	D	B	A	C	D	B

- a. How many voters voted in this election? $8+9+11+7+7+5=47$
- b. How many votes are needed for a majority? $47 \div 2 = 23.5$
24 votes are needed for a majority
- c. How many votes are needed for a plurality win? $47 \div 4 = 11.75$
12 votes are needed for plurality
- d. Find the winner under the plurality method.
A=16 B=15 C=5 D=11
A is the plurality winner.
- e. Find the winner under the Instant Runoff Voting method.
A=16 B=15 C=5 D=11

$$\begin{array}{r} +8 \\ +7 \\ \hline 31 \end{array} \qquad \begin{array}{r} +5 \\ \hline 16 \end{array}$$

A is the instant runoff winner.
- f. Find the winner under the Borda Count method.
 $A = 1 \cdot 11 + 2 \cdot 13 + 3 \cdot 7 + 4 \cdot 16 = 122$
 $B = 1 \cdot 14 + 2 \cdot 0 + 3 \cdot 18 + 4 \cdot 15 = 128$
 $C = 1 \cdot 7 + 2 \cdot 27 + 3 \cdot 8 + 4 \cdot 5 = 105$
 $D = 1 \cdot 15 + 2 \cdot 7 + 3 \cdot 14 + 4 \cdot 11 = 115$
B is the Borda Count winner. Note: The total is 470 which is $(1+2+3+4)(47)$
- g. Find the winner under the Pairwise Comparisons method.
A 21, **B 26** **B 33**, C 14 C 20, **D 27**
A 23, **C 24** B 22, **D 25**
A 31, D 16

Both B and D have two pairwise wins, so it is not clear who would win. Who do you think should win?

h. Which method do you think is the most fair in this situation and why?

Your opinion😊

2. In the election shown below under the plurality method, explain why voters in the third column may feel they cannot vote for their first choice. How could it affect the outcome of the election?

Number of voters	96	90	10
1st choice	A	B	C
2nd choice	B	A	B
3rd choice	C	C	A

a. How many voters voted in this election? $96 + 90 + 10 = 196$

b. How many votes are needed for a majority? $196 \div 2 = 98$, $98 + 1 = 99$
99 votes are needed for a majority

c. How many votes are needed for a plurality win? $196 \div 3 = 65.33$
A minimum of 66 votes are needed for plurality

d. Find the winner under the plurality method.
A=96 B=90 C=10
A is the plurality winner.

e. Under the plurality method, explain why voters in the third column may feel they cannot vote for their first choice. How could that affect the outcome of the election?
The 10 voters prefer C, but their candidate C has no chance of winning. If they vote for C, then A will win, who is their last choice. They may feel like they have to vote for B, their second choice. This is called voting insincerely.

f. Find the winner under the Instant Runoff Voting method.
A=96 B=90 C=10
 +10
 100 B is the winner with Instant Runoff Voting.

g. Find the winner under the Borda Count method.
 $A = 1 \cdot 10 + 2 \cdot 90 + 3 \cdot 96 = 478$
 $B = 2 \cdot 96 + 2 \cdot 10 + 3 \cdot 90 = 482$ **B is the winner with Borda Count.**
 $C = 1 \cdot 96 + 1 \cdot 90 + 3 \cdot 10 = 216$

h. Find the winner under the Pairwise Comparisons method.
A 96, B 100 B 186, C 10

A 186, C 10

B is the winner with the Condorcet Method

- i. Which method do you think is the most fair in this situation and why?
Every method except plurality seems fair because more people prefer B to A and C.

D2: Electoral College vs. Popular Vote and Voting Power

3. In the U.S., the Electoral College is used in presidential elections. Each state is awarded a number of electors equal to the number of representatives (based on population) and senators (2 per state) they have in congress.

Most states award the winner of the popular vote in their state all their state’s electoral votes, but not all. To explore how the Electoral College works, we’ll look at a mini-country with only 4 states. Here is the outcome of a hypothetical election:

State	Smalota	Medigan	Bigonia	Hugodo
Population	50,000	70,000	100,000	240,000
Votes for A	40,000	50,000	80,000	50,000
Votes for B	10,000	20,000	20,000	190,000

- a. If this country used a popular vote, which candidate would win the election?

Votes for A: $40,000 + 50,000 + 80,000 + 50,000 = 220,000$ votes

Votes for B: $10,000 + 20,000 + 20,000 + 190,000 = 240,000$ votes

Based on the popular vote, Candidate B would win.

- b. Now suppose this country uses an Electoral College where each state has one representative for every 10,000 people and 2 senators. The number of electoral votes is the total of the representatives and senators. Each state will give all of their electoral votes to the winner of the popular vote in the state.

Use this information with the table above to fill in this table.

State	Smalota	Medigan	Bigonia	Hugodo
Number of Representatives	5	7	10	24
Number of Senators	2	2	2	2
Number of Electoral Votes	7	9	12	26
Electoral Votes for A	7	9	12	0
Electoral Votes for B	0	0	0	26

c. Which candidate wins the election based on the electoral college?

Electoral Votes for A: $7 + 9 + 12 = 28$ electoral votes

Electoral Votes for B: 26 electoral votes

Based on the Electoral College, Candidate A would win.

d. Which method do you think is more fair and why? **Personal opinion**

Voting Power

4. a. Do you think each state has the same voting power? Why or why not?

b. Let's calculate the voting power of each state.

State	Smalota	Medigan	Bigonia	Hugodo
Population	50,000	70,000	100,000	240,000
Number of Electoral Votes	7	9	12	26
Number of Electoral Votes per 10,000 people	$\frac{7}{5}$ =1.4	$\frac{9}{7}$ =1.29	$\frac{12}{10}$ =1.2	$\frac{26}{26}$ =1.08

c. Which states have more voting power? Less voting power?

The smallest state has the most voting power and the largest state has the least.

d. Which states in the U.S.A. do you think have more voting power? Less voting power? Why?

The smallest states have the most electoral power.