D1-D4 Chapter Review – Extra Practice

D1 Voting Methods

1. Consider a city council election where there are two Democrats: Don and Key, and two Republicans, Elle and Fant. Answer the questions below and determine the winner under each type of voting. Adapted from David Lippman, http://www.opentextbookstore.com/mathinsociety/index.html

	300	219	281	210
1st choice	Elle	Don	Key	Fant
2nd choice	Fant	Key	Don	Don
3rd choice	Don	Elle	Fant	Elle
4th choice	Key	Fant	Elle	Key

a. How many people voted? 1,010 people

b. How many votes are needed for a majority?
$1,010 \div 2 = 505$ 505 + 1 = 506 for majority

c. How many votes are needed for plurality win (at a minimum)?
$1,010 \div 4 = 252.5$ 253 votes needed for plurality

d. Find the winner of the Plurality Method.   E=300   D=219   K=281 F=210
                                                    Elle wins under plurality.

e. Find the winner of the Instant Runoff Method.
        E=300   D=219   K=281   F=210
                    +210
                     429   Don wins the Instant Runoff Method.
                    +281
                     710
f. Find the winner of the Point System or Borda Count Method.
$E = 4 \cdot 300 + 2 \cdot 219 + 1 \cdot 281 + 2 \cdot 210 = 2339$

$D = 2 \cdot 300 + 4 \cdot 219 + 3 \cdot 281 + 3 \cdot 210 = 2949$

$K = 1 \cdot 300 + 3 \cdot 219 + 4 \cdot 281 + 1 \cdot 210 = 2291$ Don wins the Borda Count Method.

$F = 3 \cdot 300 + 1 \cdot 219 + 2 \cdot 281 + 4 \cdot 210 = 2521$

g. Find the winner of the Condorcet Method or Pairwise Comparisons Method.

    E 300, D 710   D 729, K 281   K 500, F 510
    E 510, K 500   D 500, F 510
    E 519, F 491

    This method results in a tie between Elle, Don and Fant because they each have two pairwise wins.

D2 Voting Power

2. Consider a hypothetical country with 6 states. The population and number of electoral votes are shown in the table below.

State	Population (millions)	Number of electors	Electoral Power Electors per million people
Utopia	4.5	11	$11 \div 4.5 = 2.4$
Verity	10.5	19	$19 \div 10.5 = 1.8$
Windfall	3.2	9	$9 \div 3.2 = 2.8$
Xanadu	0.5	3	$3 \div 0.5 = 6$
Yorkshire	8.0	17	$17 \div 8.0 = 2.1$
Zodiac	1.5	5	$5 \div 1.5 = 3.3$

a. Calculate the voting power per state (Electors per millions of people) to complete the table.

See table above

b. Which state or states have the most electoral power?

Xanadu has the most electoral power. It is the smallest state.

c. Which state or states have the least electoral power?

Verity has the least electoral power. It is the largest state.

d. Assume that each state gives either all or none of its electoral votes to the winning candidate in their state. What is the fewest number of states (and which states) must a candidate carry in order to win the election? Explain.

There is a total of 64 electoral votes, so a majority would be 33 votes (64/2=32+1=33)

Starting with the biggest states: 19 + 17 = 36. Only 2 states would be needed to win: Verity and Yorkshire.

D3 Apportionment

Webster's and the Hill-Huntington Methods will not be on the final.

2. A small country has four states whose populations are listed below. Their legislature has 116 seats. Determine the number of seats that each state would get using the methods below. Adapted from David Lippman, http://www.opentextbookstore.com/mathinsociety/index.html

a. Hamilton's Method

State	Population	÷7100 Standard Quota	Initial or Minimum	Final
North	33,700	4.75	4 +1	5
South	559,500	78.80	78 +1	79
East	141,300	19.90	19 +1	20
West	89,100	12.55	12	12
Total	823,600		113	116

Divisor $823,600 \div 116 = 7100$

b. Jefferson's Method

State	Population	÷7100 Standard Quota	Initial	(trials) ÷7000	÷6980	Final
North	33,700	4.75	4	4.81	4.83	4
South	559,500	78.80	78	79.93 80.16	80	
East	141,300	19.90	19	20.19 20.24	20	
West	89,100	12.55	12	12.73	12.77	12
Total	823,600		113			116

Divisor $823,600 \div 116 = 7100$

D4 Gerrymandering

3. A State map is shown below. Tally the voters and use the information to answer the questions below.

a. Calculate the results of an election and the efficiency gap.

Election Results:

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	3	4	3	4-4=0
2	3	4	3	4-4=0
3	3	4	3	4-4=0
4	6	1	6-4=2	1
5	3	4	3	4-4=0
6	3	4	3	4-4=0
Total	21	21	17	1

Democrats win

____1____ seats

Republicans win

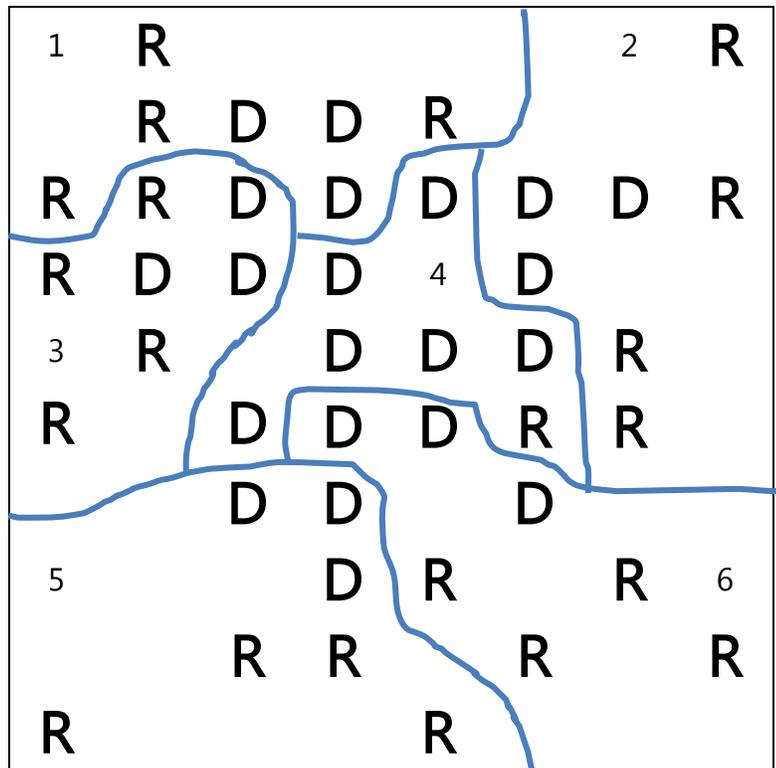___5_____ seats

Efficiency Gap

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$= \frac{17-1}{42} = 0.38 \text{ or } 38\%$$

b. Calculate the percentage of voters that each seat represents.

$$= \frac{100\%}{6 \text{ districts}} = 0.17 \text{ or } 17\%$$

c. Compare the efficiency gap with the percentage for each seat. How many seats is the efficiency gap worth?

The efficiency gap of 38% is worth more than two full seats.

d. Is this a fair map? Why or why not?

No, this map is not fair because the population is 50-50 but the seats are 5-1.