D3-D4: Apportionment, Gerrymandering and the Efficiency Gap - SOLUTIONS

Group Activity

D3: Apportionment - SOLUTIONS

It's very important to acknowledge that when Hamilton, Jefferson and others were making the rules for apportionment, black people were owned and enslaved in the United States. The 3/5 rule counted the black population as only 3/5 of the white population. This is often omitted from textbooks, but we need to remember why we still have racism and systemic inequality now and continue to make our systems fair for everyone.

1. A college offers tutoring in Math, English, Chemistry, and Biology. The number of students enrolled in each subject is listed below. If the college can only afford to hire 15 tutors, determine how many tutors should be assigned to each subject. Examples adapted from David Lippman, http://www.opentextbookstore.com/mathinsociety/index.html

a. Hamilton's Method

Subject	Students	÷53 Standard Quota	Cut off decimal	Give Extra to highest decimal
Math	330	**6.23**	**6**	**6**
English	265	**5**	**5**	**5**
Chemistry	130	**2.45**	**2 + 1**	**3**
Biology	70	**1.32**	**1**	**1**
Total	**795**		**14**	**15**

add 1 to subject with highest decimal

Divisor: $795 \div 15 = 53$

b. Jefferson's Method

Subject	Students	÷53 Standard Quota	Cut off Decimal	÷45 Use New Divisor	Cut off Decimal
Math	330	**6.23**	**6**	**7.3**	**7**
English	265	**5**	**5**	**5.82**	**5**
Chemistry	130	**2.45**	**2**	**2.89**	**2**
Biology	70	**1.32**	**1**	**1.56**	**1**
Total	**795**		**14**		**15**

Lower divisor until the total is 15. Try 45

Divisor: $795 \div 15 = 53$

c. Webster's Method

Subject	Students	Standard Quota ($\div 53$)	Rounded Decimal	Use New Divisor ($\div 52$)	Rounded Decimal
Math	330	**6.23**	**6**	**6.35**	**6**
English	265	**5**	**5**	**5.10**	**5**
Chemistry	130	**2.45**	**2**	**2.5**	**3**
Biology	70	**1.32**	**1**	**1.35**	**1**
Total	**795**		**14**		**15**

Lower divisor until they round to a total of 15

Divisor: $795 \div 15 = 53$

d. Hill-Huntington Method

Subject	Students	Standard Quota ($\div 53$)	Geometric Mean	Rounded Decimal if above Geometric Mean
Math	330	**6.23**	$\sqrt{6 \cdot 7} = 6.48$	**6**
English	265	**5**	$\sqrt{5 \cdot 6} = 5.48$	**5**
Chemistry	130	**2.45**	$\sqrt{2 \cdot 3} = 2.45$	**3**
Biology	70	**1.32**	$\sqrt{1 \cdot 2} = 1.41$	**1**
Total	**795**			**15**

Lower divisor if needed until they round to a total of 15

Divisor: $795 \div 15 = 53$

Quota Rule
The Quota Rule says that the final number of representatives a state gets should be within one of that state's quota. Since we're dealing with whole numbers for our final answers, that means that each state should either go up to the next whole number above its quota, or down to the next whole number below its quota.

Do any of our examples violate the quota rule? **No. All representatives are either up or down to the next whole number from the quota.**

D4 Gerrymandering and Solutions - SOLUTIONS

Azavea, a data analytics organization, has calculated the efficiency gap for all 50 states. We will first look at the infographics together.
https://www.azavea.com/blog/2017/07/19/gerrymandered-states-ranked-efficiency-gap-seat-advantage/

2. a. You have just been hired as consultants to your state legislature in the re-districting of the state. To assess the current map below, calculate the efficiency gap.

Election Results:

Democrats win

__3_____ seats

Republicans win

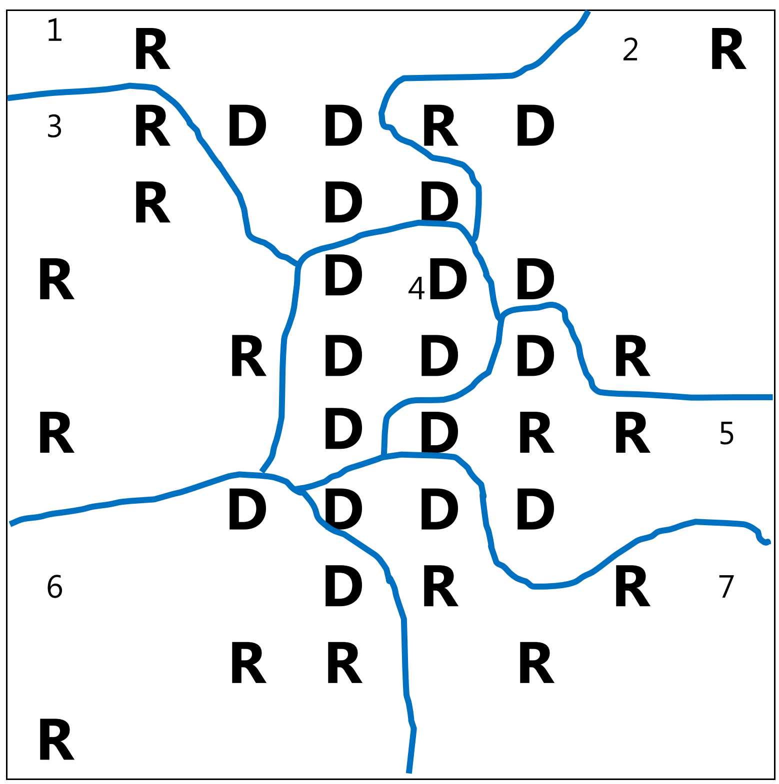
__4_____ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	4	1	4-3=1	1
2	2	3	2	3-3=0
3	0	5	0	5-3=2
4	5	0	5-3=2	0
5	3	2	3-3=0	2
6	2	3	2	3-3=0
7	2	3	2	3-3=0
Total	18	17	9	5

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$= \frac{4}{35} \approx 0.114$$ **or 11.4%**



b. Calculate the percentage of voters that each seat represents.

$$100\% \div 7 \approx 14.3\%$$

c. Is the efficiency gap worth one seat or more? How many seats?

The gap is worth just less than one seat.

d. Is this a fair map? Why or why not?

3. Now it is time for re-districting and you get to draw the lines. There are three rules:

Rules
1. All legislative districts must contain the same number of people.
2. Districts must not be drawn according to race or ethnicity.
3. District must be contiguous – no split districts allowed

a. Use packing and cracking to win as many seats as possible for the **Democrats** and calculate the efficiency gap.

Election Results:

Democrats win

6 seats

Republicans win

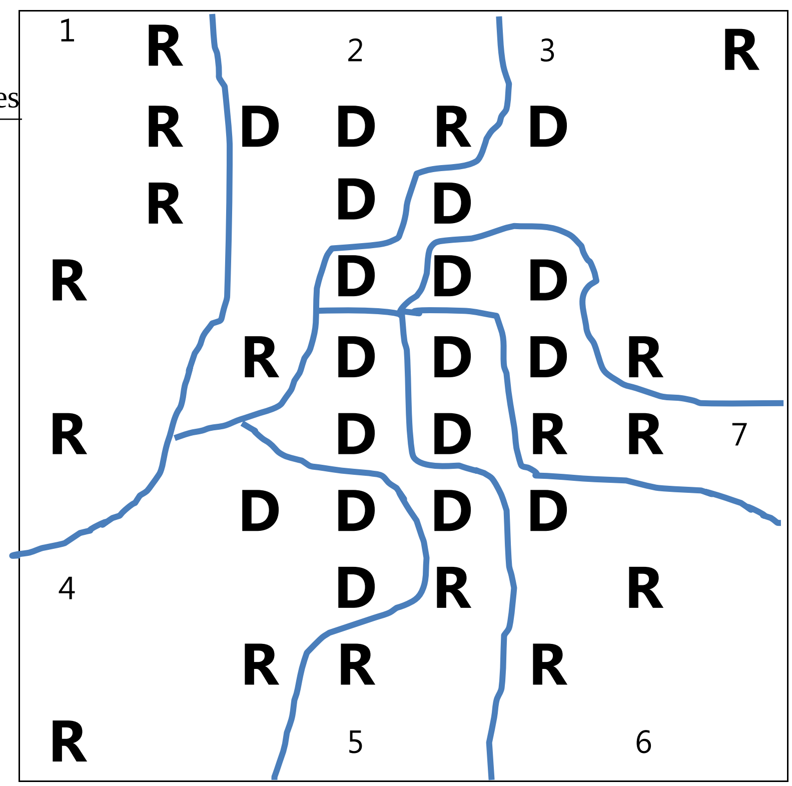
1 seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	0	5	0	5-3=2
2	3	2	3-3=0	2
3	3	2	3-3=0	2
4	3	2	3-3=0	2
5	3	2	3-3=0	2
6	3	2	3-3=0	2
7	3	2	3-3=0	2
Total	18	17	0	14

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$\frac{14-0}{35} = \frac{14}{35} = \mathbf{40\%}$$



b. Use packing and cracking to win as many seats as possible for the **Republicans** and calculate the efficiency gap.

Election Results:

Democrats win

__2______ seats

Republicans win

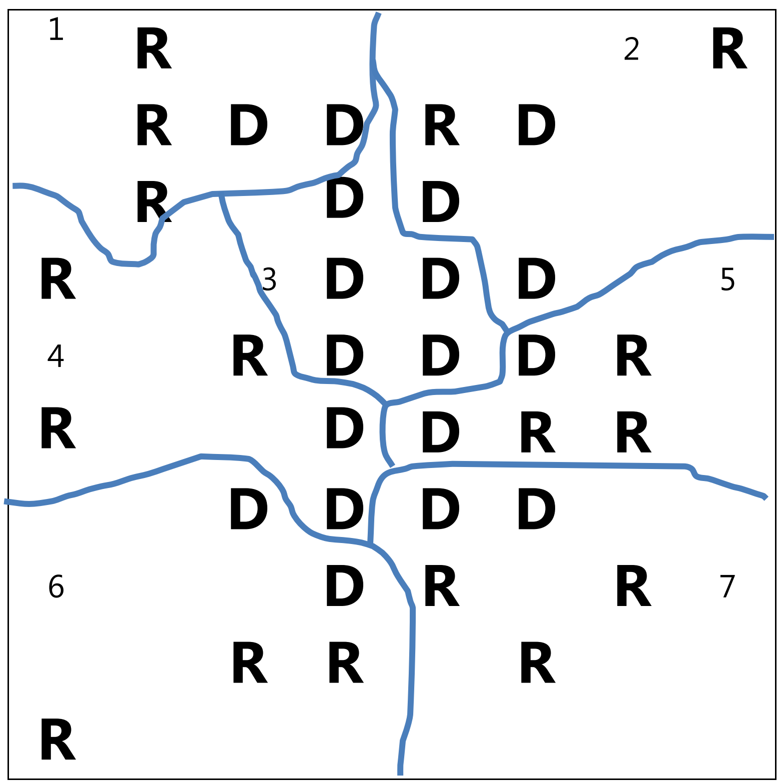
__5______ seats

District	D Votes	R Votes	D Surplus or Wasted Votes	R Surplus or Wasted Votes
1	2	3	2	3-3=0
2	3	2	3-3=0	2
3	5	0	5-3=2	0
4	2	3	2	3-3=0
5	2	3	2	3-3=0
6	2	3	2	3-3=0
7	2	3	2	3-3=0
Total	18	17	12	2

Efficiency Gap =

$$\frac{\text{Party A Wasted Votes} - \text{Party B Wasted Votes}}{\text{Total Votes}}$$

$$\frac{12-2}{35} = \frac{10}{35} = \textbf{29\%}$$



More Practice

4. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

a. If the legislature has 10 seats, use Hamilton's method to apportion the seats.
b. If the legislature grows to 11 seats, use Hamilton's method to apportion the seats
c. Does the new apportionment seem fair? Why or why not?

State	Population	Standard Quota ($\div 1,400$)			Standard Quota ($\div 1,272.73$)		
A	6,000	**4.29**	**4**	**4**	**4.71**	**4 +1**	**5**
B	6,000	**4.29**	**4**	**4**	**4.71**	**4 +1**	**5**
C	2,000	**1.43**	**1 +1**	**2**	**1.57**	**1**	**1**
Total	**14,000**		**9**	**10**			**11**

Divisor $14,000 \div 10 = 1,400$ For 11 seats: $14,000 \div 11 = 1,272.73$

This is not fair because C lost a representative and both A and B gained a representative. This is one of the problems with the Hamilton Method.

5. Repeat the previous problem above using Jefferson's method. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

a. If the legislature has 10 seats, use Jefferson's method to apportion the seats. What happens?
b. If the legislature grows to 11 seats, use Jefferson's method to apportion the seats
c. Does the new apportionment seem fair? Why or why not?

State	Population	Standard Quota ($\div 1,300$)		$\div 1,200$	
A	6,000	**4.61**	**4**	**5.0**	**5**
B	6,000	**4.61**	**4**	**5.0**	**5**
C	2,000	**1.53**	**1**	**1.67**	**1**
Total	**14,000**		**9**		**11 (Fails)**

Divisor $14,000 \div 10 = 1,400$ For 11 seats: $14,000 \div 11 = 1,272.73$

Jefferson's method does not work in this case because A and B will get another representative before C does so you can't get 10 representatives. Similar to Hamilton's method, it doesn't seem fair for A and B to have 5 reps and C only has 1.

6. A small country consists of six states, whose populations are listed below. If the legislature has 200 seats, apportion the seats using each method.

A: 3,411 B: 2,421 C: 11,586 D: 4,494 E: 3,126 F: 4,962

a. Hamilton's Method

State	Population	$\div 150$ Standard Quota	Cut off decimal		Give extra to highest decimals
A	3,411	**22.74**	**22**	**+1**	**23**
B	2,421	**16.14**	**16**		**16**
C	11,586	**77.24**	**77**		**77**
D	4,494	**29.96**	**29**	**+1**	**30**
E	3,126	**20.84**	**20**	**+1**	**21**
F	4,962	**33.08**	**33**		**33**
Total	**30,000**		**197**		**200**

Divisor $30,000 \div 200 = 150$

b. Jefferson's Method

State	Population	$\div 150$ Standard Quota	Cut off decimal	$\div 145$ Lower Divisor	$\div 149$ Raise Divisor	$\div 148.5$ Lower Divisor
A	3,411	**22.74**	**22**	**23**	**22**	**22**
B	2,421	**16.14**	**16**	**16**	**16**	**16**
C	11,586	**77.24**	**77**	**79**	**77**	**78**
D	4,494	**29.96**	**29**	**30**	**30**	**30**
E	3,126	**20.84**	**20**	**21**	**20**	**21**
F	4,962	**33.08**	**33**	**34**	**33**	**33**
Total	**30,000**		**197**	**203**	**198**	**200**
				Too high!	**Too Low!**	**Just Right!**

Divisor $30,000 \div 200 = 150$

Trial and error to find the right divisor. A divisor of 149 gives too few representatives and 148 gives too many. Try 148.5

c. Webster's Method

State	Population	Standard Quota ($\div 150$)	Round Decimal
A	3,411	**22.74**	**23**
B	2,421	**16.14**	**16**
C	11,586	**77.24**	**77**
D	4,494	**29.96**	**30**
E	3,126	**20.84**	**21**
F	4,962	**33.08**	**33**
Total	**30,000**		**200**

Divisor $30,000 \div 200 = 150$

This came out the same as Hamilton's method because there were exactly 3 states with decimals that rounded up. This is not always the case.

d. Hill-Huntington Method

State	Population	Standard Quota ($\div 150$)	Geometric Mean	Rounded Decimal if above Geometric Mean
A	3,411	**22.74**	$\sqrt{22 \cdot 23} = 22.49$	**23**
B	2,421	**16.14**	$\sqrt{16 \cdot 17} = 16.49$	**16**
C	11,586	**77.24**	$\sqrt{77 \cdot 78} = 77.50$	**77**
D	4,494	**29.96**	$\sqrt{29 \cdot 30} = 29.50$	**30**
E	3,126	**20.84**	$\sqrt{20 \cdot 21} = 20.49$	**21**
F	4,962	**33.08**	$\sqrt{33 \cdot 34} = 33.50$	**33**
Total	**30,000**			**200**

Divisor $30,000 \div 200 = 150$

This gives a slight advantage to smaller states because they are more likely to round up.