

D3: Apportionment - SOLUTIONS

Group Activity

1. A college offers tutoring in Math, English, Chemistry, and Biology. The number of students enrolled in each subject is listed below. If the college can only afford to hire 15 tutors, determine how many tutors should be assigned to each subject. Examples adapted from David Lippman, <http://www.opentextbookstore.com/mathinsociety/index.html>

a. Hamilton's Method

<u>Subject</u>	<u>Students</u>	$\div 53$ <u>Standard Quota</u>	<u>Cut off decimal</u>	<u>Give Extra to highest decimal</u>
Math	330	6.23	6	6
English	265	5	5	5
Chemistry	130	2.45	2 + 1	3
Biology	<u>70</u>	1.32	<u>1</u>	<u>1</u>
Total	795		14	15

add 1 to subject with highest decimal

Divisor: $795 \div 15 = 53$

b. Jefferson's Method

<u>Subject</u>	<u>Students</u>	$\div 53$ <u>Standard Quota</u>	<u>Cut off Decimal</u>	$\div 45$ <u>Use New Divisor</u>	<u>Cut off Decimal</u>
Math	330	6.23	6	7.3	7
English	265	5	5	5.82	5
Chemistry	130	2.45	2	2.89	2
Biology	<u>70</u>	1.32	<u>1</u>	1.56	<u>1</u>
Total	795		14		15

Lower divisor until the total is 15. Try 45

Divisor: $795 \div 15 = 53$

c. Webster's Method

<u>Subject</u>	<u>Students</u>	$\div 53$ <u>Standard Quota</u>	<u>Rounded Decimal</u>	$\div 52$ <u>Use New Divisor</u>	<u>Rounded Decimal</u>
Math	330	6.23	6	6.35	6
English	265	5	5	5.10	5
Chemistry	130	2.45	2	2.5	3
Biology	<u>70</u>	1.32	<u>1</u>	1.35	<u>1</u>
Total	795		14		15

Lower divisor until they round to a total of 15

Divisor: $795 \div 15 = 53$

d. Hill-Huntington Method

<u>Subject</u>	<u>Students</u>	$\div 53$ <u>Standard Quota</u>	<u>Geometric Mean</u>	<u>Rounded Decimal if above Geometric Mean</u>
Math	330	6.23	$\sqrt{6 \cdot 7} = 6.48$	6
English	265	5	$\sqrt{5 \cdot 6} = 5.48$	5
Chemistry	130	2.45	$\sqrt{2 \cdot 3} = 2.45$	3
Biology	<u>70</u>	1.32	$\sqrt{1 \cdot 2} = 1.41$	<u>1</u>
Total	795			15

Lower divisor if needed until they round to a total of 15

Divisor: $795 \div 15 = 53$

2. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

- If the legislature has 10 seats, use Hamilton's method to apportion the seats.
- If the legislature grows to 11 seats, use Hamilton's method to apportion the seats.
- Does the new apportionment seem fair? Why or why not?

<u>State</u>	<u>Population</u>	$\div 1,400$ <u>Standard Quota</u>		$\div 1,272.73$ <u>Standard Quota</u>	
A	6,000	4.29	4	4.71	4 + 1
B	6,000	4.29	4	4.71	4 + 1
C	<u>2,000</u>	1.43	<u>1</u> + 1	1.57	1
Total	14,000		9		11
Divisor	$14,000 \div 10 = 1,400$			For 11 seats: $14,000 \div 11 = 1,272.73$	

This is not fair because C lost a representative and both A and B gained a representative. This is one of the problems with the Hamilton Method.

3. Repeat problem 2 using Jefferson’s method. A small country consists of three states, whose populations are listed below.

A: 6,000 B: 6,000 C: 2,000

- a. If the legislature has 10 seats, use Jefferson’s method to apportion the seats. What happens?
- b. If the legislature grows to 11 seats, use Jefferson’s method to apportion the seats
- c. Does the new apportionment seem fair? Why or why not?

<u>State</u>	<u>Population</u>	$\div 1,300$	<u>Standard Quota</u>	$\div 1,200$	
A	6,000	4.61	4	5.0	5
B	6,000	4.61	4	5.0	5
C	2,000	1.53	<u>1</u>	1.67	<u>1</u>
Total	14,000		9		11 (Fails)
Divisor	$14,000 \div 10 = 1,400$			For 11 seats: $14,000 \div 11 = 1,272.73$	

Jefferson’s method does not work in this case because A and B will get another representative before C does so you can’t get 10 representatives. Similar to Hamilton’s method, it doesn’t seem fair for A and B to have 5 reps and C only has 1.

Quota Rule

The Quota Rule says that the final number of representatives a state gets should be within one of that state’s quota. Since we’re dealing with whole numbers for our final answers, that means that each state should either go up to the next whole number above its quota, or down to the next whole number below its quota.

Do any of our examples violate the quota rule? **No. All representatives are either up or down to the next whole number from the quota.**

Class Activity**The Three-Fifths Compromise and 1790 Census Data**

4. In 1787, there was a Constitutional Convention in Philadelphia. The 55 delegates debated many issues and two of the most important were **slavery** and **representation**. Under the **Great Compromise**, the number of representatives per state would be determined by population size. But should **enslaved African people, who had no rights in the United States**, count as part of the population? Southern states said yes. Northern states said no. We will explore why.

The 1790 Census Data – The First US Census

State	1 Total Population	2 Free Persons	3 Slave Population	4 3/5 Slave Population	5 Compromise Pop Total	6 Number of Reps to House of Reps
Vermont	85,539	85,539	0	0	85,539	2
New Hampshire	141,885	141,727	158	95	141,822	4
Maine	96,540	96,540	0	0	9,6540	3
Massachusetts	378,787	378,787	0	0	378,787	11
Rhode Island	68,825	67,877	948	569	68,446	2
Connecticut	237,946	235,182	2,764	1,658	236,840	7
New York	340,120	318,796	21,324	12,794	331,590	10
New Jersey	184,139	172,716	11,423	6,854	179,570	5
Pennsylvania	434,373	430,636	3,737	2,242	432,878	13
Delaware	59,094	50,207	8,887	5,332	55,539	1
Maryland	319,728	216,692	103,036	61,822	278,514	8
Virginia	747,610	454,983	292,627	175,576	630,559	19
Kentucky	73,677	61,247	12,430	7,458	68,705	2
North Carolina	393,751	293,179	100,572	60,343	353,522	10
South Carolina	249,073	141,979	107,094	64,256	206,235	6
Georgia	82,348	53,284	29,264	17,558	70,842	2

Counting the Total Population

a. Look at the **total population** (Column 1) of Massachusetts and North Carolina in the 1790 Census Data. Use a divisor of 32,150 people (for Jefferson's Method) to determine the number of representatives that Massachusetts and North Carolina would have.

Massachusetts: 11 Representatives North Carolina: 12 Representatives

Counting only Free Persons

b. Look at the number of **free persons** (Column 2) for Massachusetts and North Carolina. Use a divisor of 32,150 people (for Jefferson's Method) to determine the number of representatives that Massachusetts and North Carolina would have.

Massachusetts: 11 Representatives North Carolina: 9 Representatives

c. Why might including enslaved people as part of a state's population – even though they had no freedom or rights – anger states that had few or no slaves?

It would give the South greater representation, but the slaves were not represented, only the wealthy white male landowners were really represented.

To break the deadlock between the states, the delegates agreed to count only **3/5** of enslaved African people towards representation. This agreement was known as the *Three-Fifths Compromise*

The Results of the 3/5 Compromise

d. Calculate the missing values in Columns 4 and 5 for Massachusetts and North Carolina.

e. Use a divisor of 32,150 people (for Jefferson's Method) to determine the number of representatives that Massachusetts and North Carolina would have under the compromise. Then complete column 6 using the compromise column. There were a total of 105 representatives

Massachusetts: 11 Representatives North Carolina: 10 Representatives

Your Thoughts:

f. Why do you think the South wanted to count enslaved people as part of their state's population? Why do you think the North did not?

The South wanted more representation that would give them more power, but the North did not think that was fair since enslaved people did not have any rights.

g. Why didn't the delegates end slavery and make the enslaved people full citizens?

I don't know. I think the North and South could not agree on this so they came up with a compromise.

h. Was the 3/5th compromise a fair resolution for slave states v. non-slave states? Why were the delegates so concerned about fairness when the enslavement of African people was not fair? Who else was not represented at the time?

It seems better that the South did not have more power than they did but slavery should not have existed in the first place. Native Americans and women were also not represented.

i. For further exploration you can look up the slave trade compromise that was also made at the 1787 Convention.

State	1 Total Population	2 Free Persons	3 Slave Population	4 3/5 Slave Population	5 Compromise Pop Total	6 Number of Reps to House of Reps
Vermont	85,539	85,539	0	0	85,539	2
New Hampshire	141,885	141,727	158	95	141,822	4
Maine	96,540	96,540	0	0	96,540	3
<i>Massachusetts</i>	378,787	378,787	0	0	378,787	11
Rhode Island	68,825	67,877	948	569	68,446	2
Connecticut	237,946	235,182	2,764	1,658	236,840	7
New York	340,120	318,796	21,324	12,794	331,590	10
New Jersey	184,139	172,716	11,423	6,854	179,570	5
Pennsylvania	434,373	430,636	3,737	2,242	432,878	13
Delaware	59,094	50,207	8,887	5,332	55,539	1
Maryland	319,728	216,692	103,036	61,822	278,514	8
Virginia	747,610	454,983	292,627	175,576	630,559	19
Kentucky	73,677	61,247	12,430	7,458	68,705	2
<i>North Carolina</i>	393,751	293,179	100,572	60,343.2	353,522	10
South Carolina	249,073	141,979	107,094	64,256	206,235	6
Georgia	82,348	53,284	29,264	17,558	70,842	2
Total					3,615,928	105
Divisor					34437.41143	
Modified Divisor					32150	

Note: Maine was considered part of Massachusetts at the time, but their representatives were calculated separately.

Sources: https://www.census.gov/history/pdf/1790_Apportionment.pdf
https://en.wikipedia.org/wiki/1790_United_States_Census
https://en.wikipedia.org/wiki/United_States_congressional_apportionment

STATUTE I.

CHAP. XXIII.—*An Act for apportioning Representatives among the several States, according to the first enumeration.* April 14, 1792.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

APPROVED, April 14, 1792.

[Obsolete.]
Apportionment of representatives to Congress according to first enumeration.
1791, ch. 9.
1802, ch. 1.
1811, ch. 9.
1820, ch. 37.
1832, ch. 91.
1842, ch. 25.

(a) The provisions of the acts of Congress relating to the assignment of the circuits to the justices of the Supreme Court, have been: Act of April 13, 1792, sec. 3; act of March 2, 1793; act of April 29, 1802, sec. 5; act of March 3, 1803; act of March 3, 1837.

Y

More Practice

1. A small country consists of six states, whose populations are listed below. If the legislature has 200 seats, apportion the seats using each method.

A: 3,411 B: 2,421 C: 11,586 D: 4,494 E: 3,126 F: 4,962

a. Hamilton's Method

<u>State</u>	<u>Population</u>	$\div 150$ <u>Standard Quota</u>	<u>Cut off decimal</u>		<u>Give extra to highest decimals</u>
A	3,411	22.74	22	+1	23
B	2,421	16.14	16		16
C	11,586	77.24	77		77
D	4,494	29.96	29	+1	30
E	3,126	20.84	20	+1	21
F	4,962	33.08	33		33
Total	30,000		197		200

Divisor $30,000 \div 200 = 150$

b. Jefferson's Method

<u>State</u>	<u>Population</u>	$\div 150$ <u>Standard Quota</u>	<u>Cut off decimal</u>	$\div 145$ <u>Lower Divisor</u>	$\div 149$ <u>Raise Divisor</u>	$\div 148.5$ <u>Lower Divisor</u>
A	3,411	22.74	22	23	22	22
B	2,421	16.14	16	16	16	16
C	11,586	77.24	77	79	77	78
D	4,494	29.96	29	30	30	30
E	3,126	20.84	20	21	20	21
F	4,962	33.08	33	34	33	33
Total	30,000		197	203	198	200

Too high! Too Low! Just Right!

Divisor $30,000 \div 200 = 150$

Trial and error to find the right divisor. A divisor of 149 gives too few representatives and 148 gives too many. Try 148.5

c. Webster's Method

<u>State</u>	<u>Population</u>	$\div 150$ <u>Standard Quota</u>	<u>Round Decimal</u>
A	3,411	22.74	23
B	2,421	16.14	16
C	11,586	77.24	77
D	4,494	29.96	30
E	3,126	20.84	21
F	<u>4,962</u>	33.08	<u>33</u>
Total	30,000		200

Divisor $30,000 \div 200 = 150$

This came out the same as Hamilton's method because there were exactly 3 states with decimals that rounded up. This is not always the case.

d. Hill-Huntington Method

<u>State</u>	<u>Population</u>	$\div 150$ <u>Standard Quota</u>	<u>Geometric Mean</u>	<u>Rounded Decimal if above Geometric Mean</u>
A	3,411	22.74	$\sqrt{22 \cdot 23} = 22.49$	23
B	2,421	16.14	$\sqrt{16 \cdot 17} = 16.49$	16
C	11,586	77.24	$\sqrt{77 \cdot 78} = 77.50$	77
D	4,494	29.96	$\sqrt{29 \cdot 30} = 29.50$	30
E	3,126	20.84	$\sqrt{20 \cdot 21} = 20.49$	21
F	4,962	33.08	$\sqrt{33 \cdot 34} = 33.50$	<u>33</u>
Total	30,000			200

Divisor $30,000 \div 200 = 150$

This gives a slight advantage to smaller states because they are more likely to round up.