Math 111, Tues, 4/5

Please turn in Team Mission 1

Hand back checkpoints - Talk about formatting

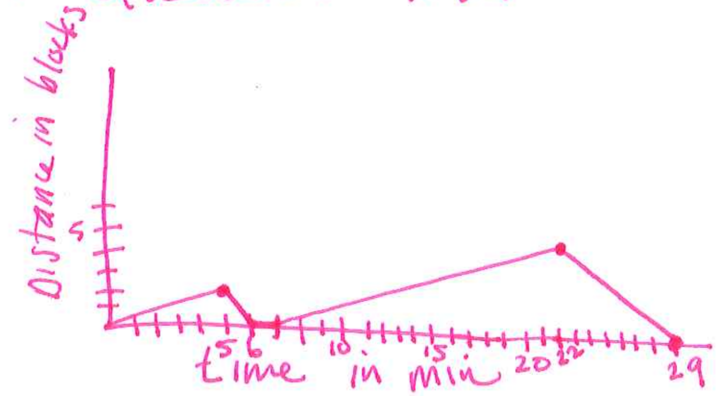
Q's on 1.2

New Material: 1.3 + supplement

Checkpoint 2 on Thursday (covers 1.2 and the beginning of 1.3 (even/odd)

Homework Questions 1.2

43.



27. $f(x) = \dfrac{2x^2}{x^4+1}$ Domain?

$x^4 + 1 \neq 0$

$\sqrt[4]{x^4} \neq \sqrt[4]{-1}$

$x \neq \sqrt[4]{-1}$ not a real number

There is no real number we can't have

D: $\mathbb{R}$

$(-\infty, \infty)$

$f(x) = \dfrac{2x^2}{x^4+1}$

For each function find and simplify $f(-x)$. What patterns do you notice?

a. $f(x) = 3x^4 - 6x^3 - 10x^2 + x - 3$

$$(-x)^4 = (-x)(-x)(-x)(-x) = x^4$$

$$f(-x) = 3(-x)^4 - 6(-x)^3 - 10(-x)^2 + (-x) - 3$$
$$= 3x^4 - 6(-x^3) - 10x^2 - x - 3$$
$$= 3x^4 + 6x^3 - 10x^2 - x - 3$$

$$(-x)^3 = (-x)(-x)(-x) = -x^3$$

neither even nor odd

b. $f(x) = -2x^4 - 7x^2 - 3$

$$f(-x) = -2(-x)^4 - 7(-x)^2 - 3$$
$$= -2x^4 - 7x^2 - 3$$
$$= f(x)$$

even function

c. $f(x) = 5x^3 + 4x$

$$f(-x) = 5(-x)^3 + 4(-x)^1$$
$$= -5x^3 - 4x$$
$$= -(5x^3 + 4x)$$
$$= -f(x)$$

odd function

Math 111 Lecture Notes

Section 1.3: Properties of Functions

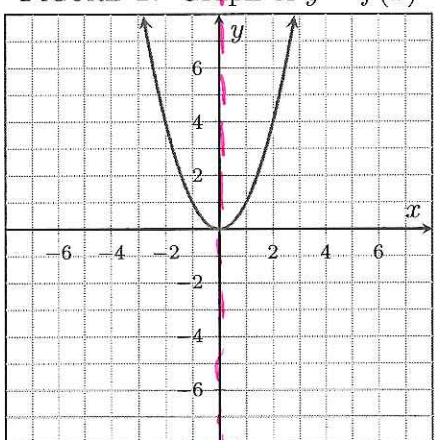
A function f is **even** if for every x in the domain of f it holds that $f(-x) = f(x)$. Visually, an even function is *symmetric about the y-axis.*

A function f is **odd** if for every x in the domain of f it holds that $f(-x) = -f(x)$. Visually, an odd function is *symmetric about the origin.*

Example 1. Two classic examples of even and odd functions are $f(x) = x^2$ and $g(x) = x^3$, respectively, as shown in Figures 1 and 2 below.

$f(x)=x^2$

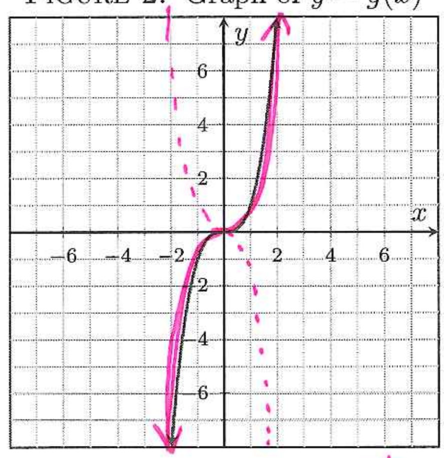
FIGURE 1. Graph of $y = f(x)$



symmetric about the y-axis

$f(x)=x^3$

FIGURE 2. Graph of $y = g(x)$



symmetry about the origin flip across the x-axis and the y-axis.

Algebraically verify that f is an even function and that g is an odd function.

Example 2. Algebraically determine if the following functions are even, odd or neither.

(a) $h(x) = x^3 - x^1$

$h(-x) = (-x)^3 - (-x)$
$= -x^3 + x$
$= -h(x)$
odd

(b) $g(t) = \frac{1}{2}t^4 - 1x^0$ ← even

$g(-t) = \frac{1}{2}(-t)^4 - 1$
$= \frac{1}{2}t^4 - 1$
even

(c) $f(t) = t^3 + 1x^0$ ← even

$f(-t) = (-t)^3 + 1$
$= -t^3 + 1$
neither

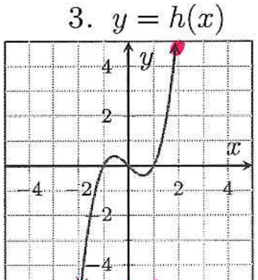
(d) $f(x) = |x| - 4$

$f(-x) = |-x| - 4$

$|-x| = |x|$

(a)

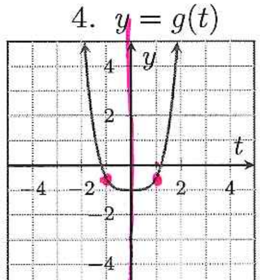
FIGURE 3. $y = h(x)$



odd

(b)

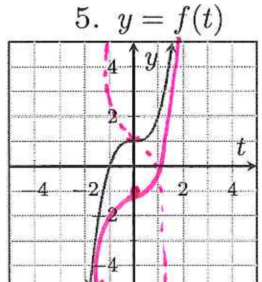
FIGURE 4. $y = g(t)$



even

(c)

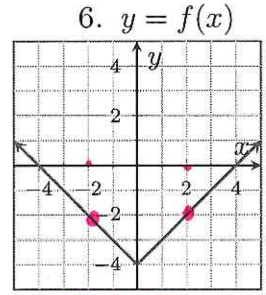
FIGURE 5. $y = f(t)$



neither

(d)

FIGURE 6. $y = f(x)$



even

find $f(-x)$

Example 3. Algebraically determine if the function f defined by $f(x) = -\dfrac{2x^3 - x}{3x^4 + 5x^2}$ is even, odd or neither.

$$f(-x) = -\frac{2(-x)^3 - (-x)}{3(-x)^4 + 5(-x)^2}$$

$$= -\left(\frac{-2x^3 + x}{3x^4 + 5x^2}\right)$$

$$= \frac{2x^3 - x}{3x^4 + 5x^2}$$

$$= -f(x)$$

odd

$$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$$

$$\frac{-1}{-2} = \frac{1}{2}$$

Group Work 1. Determine if the following functions are even, odd or neither.

(a) $g(x) = \dfrac{x^2}{x^4 + 5}$

$$g(-x) = \frac{(-x)^2}{(-x)^4 + 5}$$

$$= \frac{x^2}{x^4 + 5}$$

$$= g(x)$$

even

(b) $f(x) = 5x^3 + 3x^2$

$$f(-x) = 5(-x)^3 + 3(-x)^2$$

$$= -5x^3 + 3x^2$$

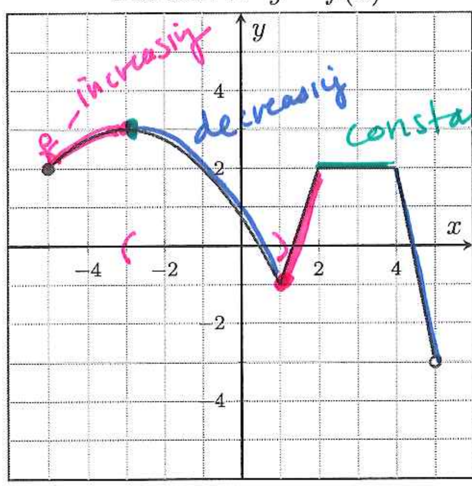
neither

A function f is **increasing** on an open interval I if for every x_1 and x_2 in I with $x_1 < x_2$ we have $f(x_2) > f(x_1)$.
A function f is **decreasing** on an open interval I if for every x_1 and x_2 in I with $x_1 < x_2$ we have $f(x_2) < f(x_1)$.
A function f is **constant** on an open interval I if for every x_1 and x_2 in I with $x_1 < x_2$ we have $f(x_2) = f(x_1)$.

Example 4. Determine the following for the function f graphed in Figure 7. State each using interval notation.

use open intervals ( )

FIGURE 7. $y = f(x)$



(a) Increasing: $(-5,-3) \cup (1,2)$

(b) Decreasing: $(-3,1) \cup (4,5)$

(c) Constant: $(2,4)$

(d) Domain of f: $[-5,5)$

(e) Range of f: $(-3,3]$

local peaks not endpoints

A function has a **local maximum** at c if there exists an open interval I containing c so that for all x not equal to c in I, it holds that $f(x) < f(c)$. The output $f(c)$ is referred to as the **local maximum** of f.

local valleys

A function has a **local minimum** at c if there exists an open interval I containing c so that for all x not equal to c in I, it holds that $f(x) > f(c)$. The output $f(c)$ is referred to as the **local minimum** of f.

Example 5. Use Figure 7 to answer the following:

(a) Identify all local maximum values of f and state where they occur.
y x

There is a local max of 3 at $x = -3$

(b) Identify all local minimum values of f and state where they occur.

There is a local min of -1 at $x = 1$

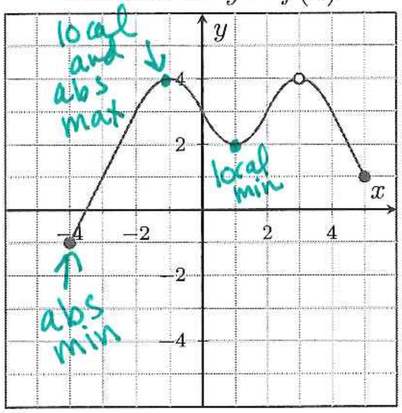
or global

Highest or lowest point on the whole graph

Let f be a function defined on an interval I.
A function has an **absolute maximum** at u if it holds that $f(x) \leq f(u)$ for all x in the interval I. The output $f(u)$ is referred to as the **absolute maximum** of f.
A function has an **absolute minimum** at u if it holds that $f(x) \geq f(u)$ for all x in the interval I. The output $f(u)$ is referred to as the **absolute minimum** of f.

Example 6. Use Figure 8 to answer the following:

FIGURE 8. $y = f(x)$



(a) Identify all absolute maximum values of f and state where they occur.

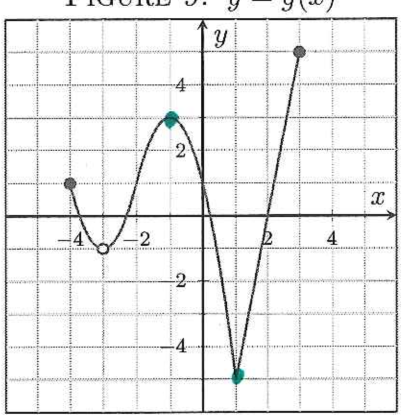
Absolute max of 4 at $x=-1$

(b) Identify all absolute minimum values of f and state where they occur.

Absolute minimum of -1 at $x=-4$

Group Work 2. Use Figure 9 to answer the following:

FIGURE 9. $y = g(x)$



(a) Identify all local maximum values of g and state where they occur.

Local max of 3 at $x=-1$

(b) Identify all local minimum values of g and state where they occur.

Local min of -5 at $x=1$

(c) Identify all absolute maximum values of g and state where they occur.

Absolute max of 5 at $x=3$

(d) Identify all absolute minimum values of g and state where they occur.

Absolute min of -5 at $x=1$

Concavity

So far, we have looked at where a function is increasing and decreasing and where it attains maximum and minimum values. We will now study the concept of *concavity*. This concept involves looking at the rate at which a function increases or decreases.

The graph of a function f whose rate of change increases (becomes less negative or more positive as you move left to right) over an interval is **concave up** on that interval. Visually, the graph "bends upward."

The graph of a function f whose rate of change decreases (becomes less positive or more negative as you move left to right) over an interval is **concave down** on that interval. Visually, the graph "bends downward."

FIGURE 10. Concave UP ☺

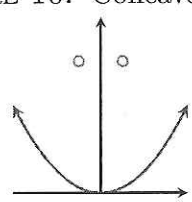
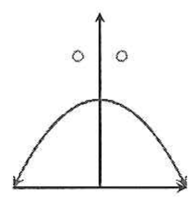
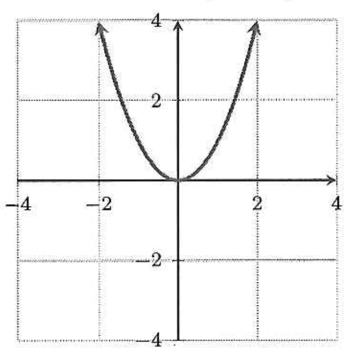


FIGURE 11. Concave DOWN ☹



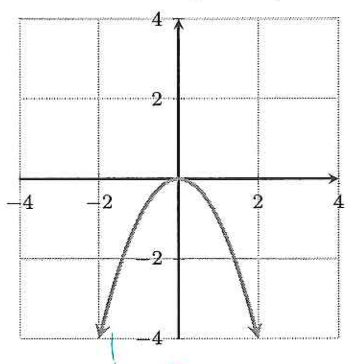
Example 7. The function defined by $f(x) = x^2$ is concave up on its entire domain. Notice that it is **decreasing** on the interval $(-\infty, 0)$ and **increasing** on the interval $(0, \infty)$. The function defined by $f(x) = -x^2$ is concave down on its entire domain. Notice that it is **increasing** on the interval $(-\infty, 0)$ and **decreasing** on the interval $(0, \infty)$.

FIGURE 12. Graph of $y = x^2$



$(-\infty, \infty)$ concave up

FIGURE 13. Graph of $y = -x^2$



$(-\infty, \infty)$ concave down

EXAMPLE 3: Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

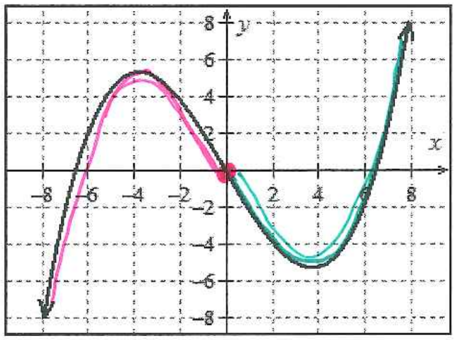


Figure 7: $y = f(x)$

Concave down: $(-\infty, 0)$
concave up $(0, \infty)$

b.

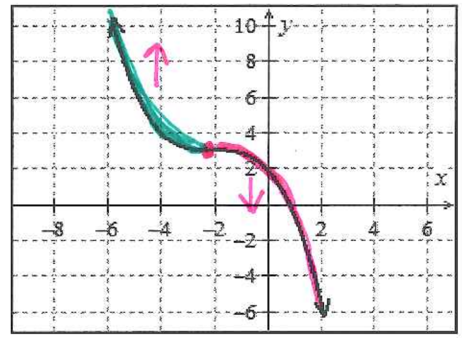


Figure 8: $y = g(x)$

Concave up: $(-\infty, -2)$
concave down: $(-2, \infty)$

Solution: **a.** f is concave up on the interval $(0, \infty)$ and concave down on the interval $(-\infty, 0)$.

b. g is concave up on the interval $(-\infty, -2)$ and concave down on the interval $(-2, \infty)$.

EXERCISES:

1. Determine the interval(s) on which the functions graphed below are concave up or concave down.

a.

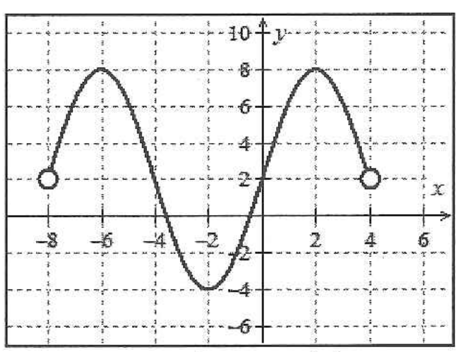


Figure 9: $y = r(x)$

b.

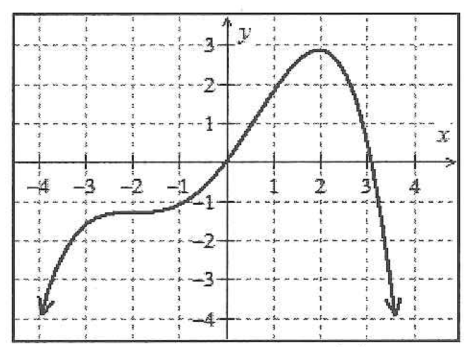


Figure 10: $y = s(x)$

c.

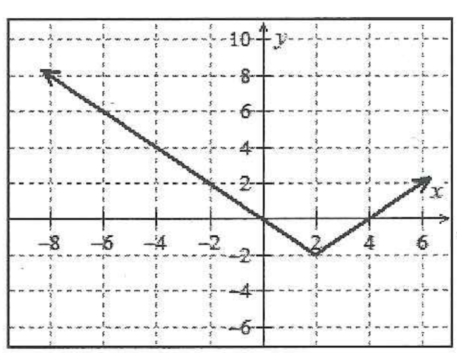


Figure 11: $y = t(x)$

d.

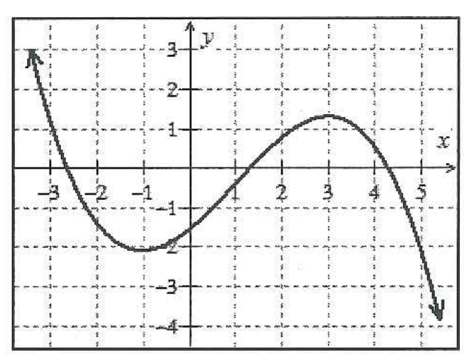
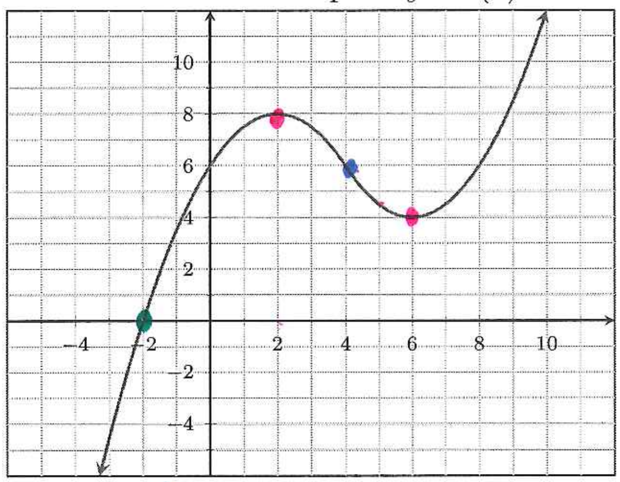


Figure 12: $y = w(x)$

Example 8. The graph of $y = h(x)$ is shown in Figure 14. Use this to answer the following.

FIGURE 14. Graph of $y = h(x)$



(a) State the interval(s) where h is positive.

$(-2, \infty)$

(b) State the interval(s) where h is negative.

$(-\infty, -2)$

(c) State the interval(s) where h is increasing.

$(-\infty, 2) \cup (6, \infty)$

(d) State the interval(s) where h is decreasing.

$(2, 6)$

(e) State the interval(s) where h is concave up.

$(4, \infty)$

(f) State the interval(s) where h is concave down.

$(-\infty, 4)$

(g) State any absolute maximum or absolute minimum values for h and where they occur.

none

(h) State any local maximum or local minimum values for h and where they occur.

local max of 8 at 2
local min of 4 at 6

Example 9. Graph the function defined by $k(x) = 2x^4 - 6x^3 - 6x^2 + 22x + 2$ on your calculator.

(a) Determine an appropriate window that shows the important features (such as the x-intercept(s), y-intercept, and any local maxima or minima).

Xmin = -5 ymin = -20
Xmax = 5 ymax = 20
X scl = 1 y scl = 5

(b) Use the MAXIMUM and MINIMUM features to find any local maxima and minima and where they occur.

local min of -18.612 at x = -1.147
3.651 at x = 2.397

local max of 14 at x = 1

(c) (Review) Use the ZERO feature and the VALUE feature to determine the x-intercepts and y-intercept.

zeros of -1.817
and -.0889

y-intercept of 2