Math 111, Thurs, 4/7

Q's on 1.3 + supplement (and 1.2)
Checkpoint 2
New material: 1.4

1.2 Questions

25. $f(x) = \frac{x+2}{x-6}$ ← equation

c. $(x-6)2 = \left(\frac{x+2}{x-6}\right)(x-6)$

$2x - 12 = x + 2$

$x - 12 = 2$

$x = 14$

x is 14

(14, 2)

$\left\{ \begin{array}{l} x = 2 \\ f(x) = 2 \end{array} \right.$

expression $\frac{x+2}{x-6} + \frac{x-1}{x+5}$

1.3 Even, odd, Neither

$f(x) = x^2 - 5$

$f(-x) = (-x)^2 - 5$

$= x^2 - 5$

$= f(x)$

even

$f(x) = x^2 - 5x$

$f(-x) = (-x)^2 - 5(-x)$

$= x^2 + 5x$

neither

$f(x) = -x^3 - 5x$

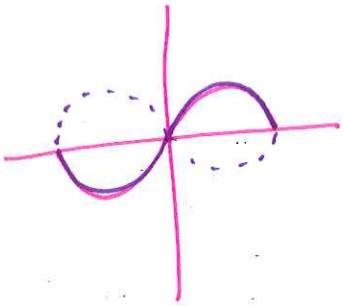
$f(-x) = -(-x)^3 - 5(-x)$

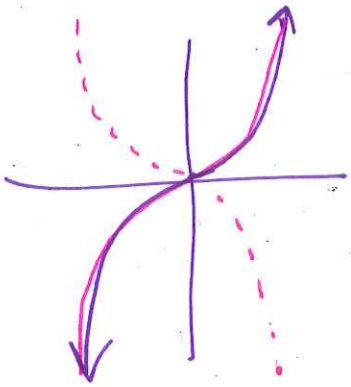
$= -(-x^3) + 5x$

$= x^3 + 5x$

$= -f(x)$

odd





x-intercepts

3, 6, 2

(3,0) (6,0) (2,0)

Solve ~ write a solution set

Math 111 Lecture Notes

Section 1.4: Piecewise-defined Functions

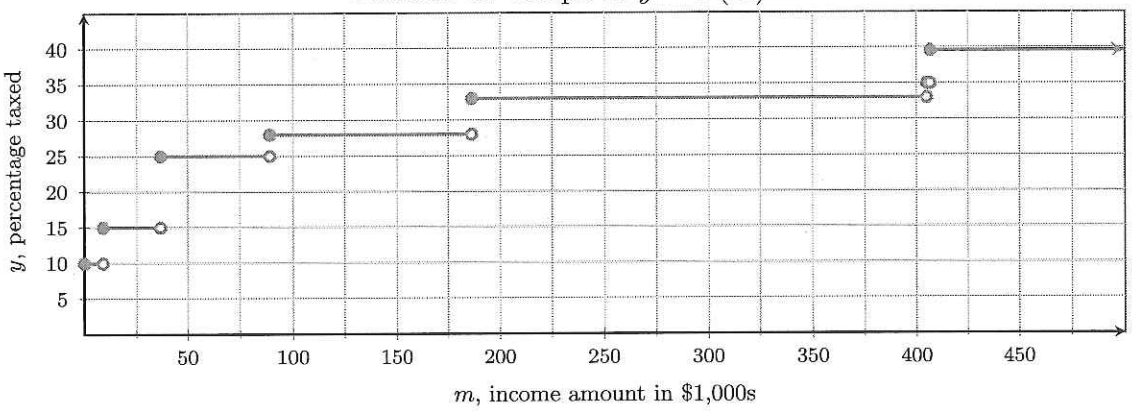
In Table 1, the 2014 federal income tax rates[1] for 2014 are shown.

Table 1. Federal Income Tax Percentage Rates for 2014 (Single Filing Status)

Income Amount (m)	Percentage of Income Taxed ($P(m)$, in %)
$0 \leq m < 9075$	10
$9075 \leq m < 36900$	15
$36900 \leq m < 89350$	25
$89350 \leq m < 186350$	28
$186350 \leq m < 405100$	33
$405100 \leq m < 406750$	35
$m \geq 406750$	39.6

Notice that for each interval, the percentage of income taxed as a function of income is *constant*. If we graph each *piece* over its respective interval, we obtain the following:

Figure 1. Graph of $y = P(m)$



[1] http://taxfoundation.org/article/2014-tax-brackets

A function that is defined by different formulas on different parts of its domain is a **piecewise-defined function**.

Example 1. Use the piecewise-defined function f defined below to answer the following.

$$f(x) = \begin{cases} \dfrac{3}{x-4} & \text{if } x \le -2 \\ 7x - 8 & \text{if } -2 < x \le 5 \\ -11 & \text{if } x > 5 \end{cases}$$

-6 is in here

0 is in here

(a) $f(0) = 7(0) - 8 = 0 - 8 = -8$

(b) $f(2) = 7(2) - 8 = 14 - 8 = 6$

(c) $f(-6) = \dfrac{3}{(-6)-4} = \dfrac{3}{-10} = -\dfrac{3}{10}$

(d) $f(8)$

$f(8) = -11$

(e) $f(-2) = \dfrac{3}{-2-4} = \dfrac{3}{-6} = -\dfrac{1}{2}$

(f) $f(5)$

$f(5) = 7(5) - 8 = 35 - 8 = 27$

Example 2. As a prelude to graphing piecewise functions, let's graph just a few of the "pieces."

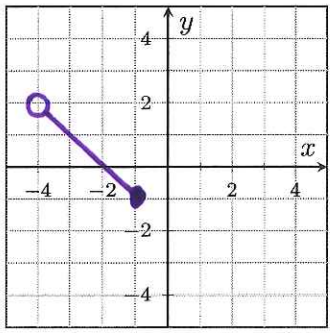
$f(-4) = -(-4) - 2 = 2$

$f(-1) = -(-1) - 2 = -1$

x	y
-4	2
-1	-1

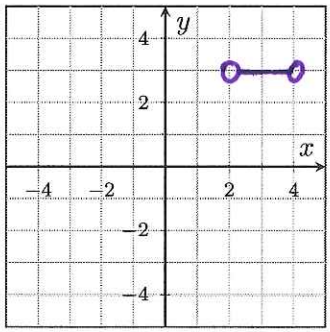
- Graph the linear function defined by $f(x) = -x - 2$ for values of x where $-4 < x \le -1$.

FIGURE 2



- Graph the constant function defined by $f(x) = 3$ for values of x where $2 < x < 4$.

FIGURE 3

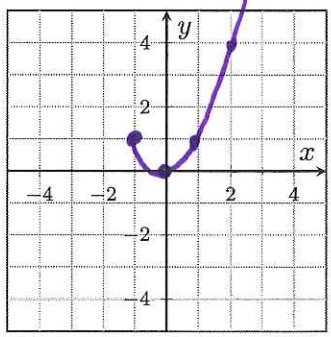


- Graph the linear function defined by $f(x) = x^2$ for values of x where $x \ge -1$.

parabola

x	y
-1	1
0	0
1	1
2	4
3	9

FIGURE 4



Example 3. Graph $y = g(x)$ in Figure 5 for the piecewise-defined function g given below.

$$g(x) = \begin{cases} -3x - 2 & \text{if } x < -1 \\ 4 & \text{if } -1 \le x < 2 \\ \frac{3}{2}x - 4 & \text{if } 2 < x \le 4 \end{cases}$$

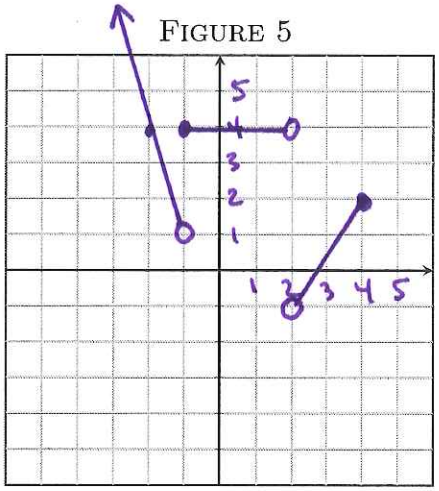
$g(-3) = 9 - 2 = 7$
$g(-2) = 6 - 2 = 4$
$g(-1) = 3 - 2 = 1$

x	y
-3	7
-2	4
-1	1

x	y
2	-1
4	2

$g(2) = \frac{3}{2} \cdot 2 - 4 = 3 - 4 = -1$
$g(4) = \frac{3}{2} \cdot 4 - 4 = 6 - 4 = 2$

FIGURE 5



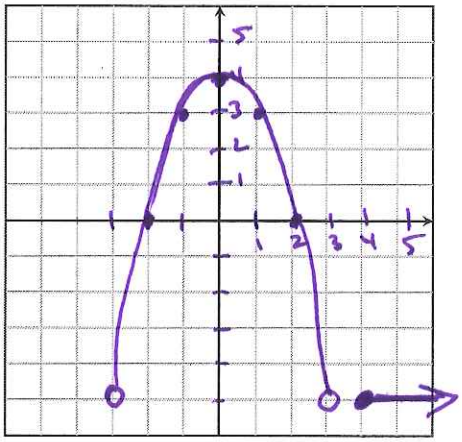
Example 4. Graph $y = h(x)$ in Figure 6 for the piecewise-defined function h given below.

parabola

$$h(x) = \begin{cases} -x^2 + 4 & \text{if } -3 < x < 3 \\ -5 & \text{if } x \ge 4 \end{cases}$$

x	y
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5

FIGURE 6

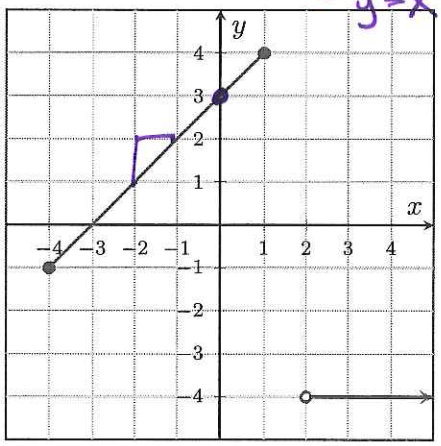


Example 5. Find the formula for the piecewise-defined function f graphed in Figure 7 below.

$$f(x) = \begin{cases} x + 3 & \text{if } -4 \le x \le 1 \\ -4 & \text{if } x > 2 \end{cases}$$

$y = mx + b$
$y = x + 3$

FIGURE 7



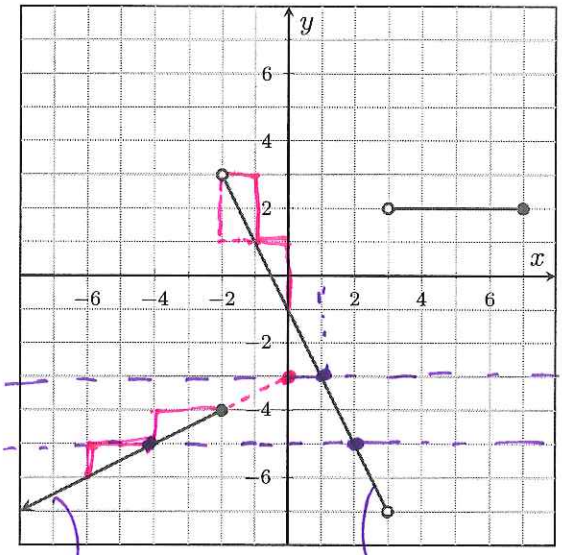
Example 6. The graph of a piecewise function g is graphed in Figure 8.

(a) State the domain and range of g.

D: $(-\infty, 3) \cup (3, 7]$

R: $(-\infty, 3)$

FIGURE 8



(b) Evaluate $g(6)$.

$g(6) = 2$

(c) Evaluate $g(-2)$.

$g(-2) = -4$

(d) Solve $g(x) = -3$. ← y

$\{1\}$

(e) Solve $g(x) = -5$. ← y

$\{-4, 2\}$ or $\{x \mid x = -4 \text{ or } 2\}$

(f) Write the formula for the function g.

$$g(x) = \begin{cases} \frac{1}{2}x - 3 & \text{if } x \le -2 \\ -2x - 1 & \text{if } -2 < x < 3 \\ 2 & \text{if } 3 < x \le 7 \end{cases}$$

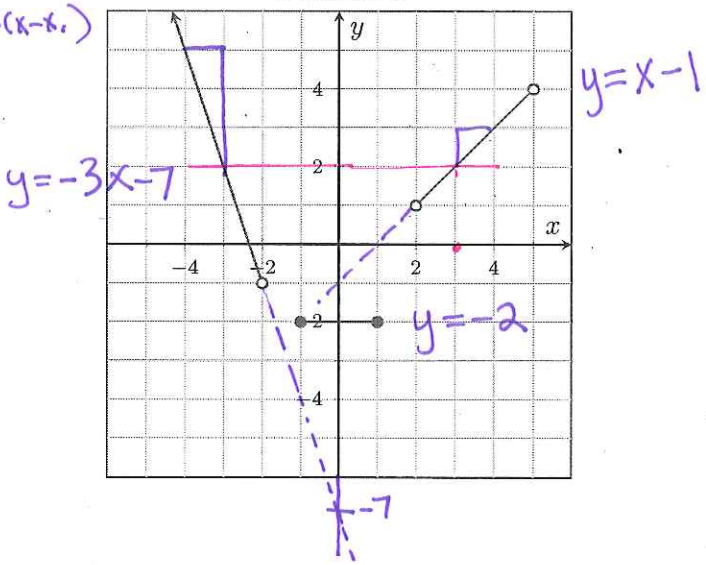
Group Work 1. The graph of the piecewise-defined function f is shown in Figure 9.

(a) Find the formula for this function.

$$f(x)=\begin{cases} -3x-7 & \text{if } x<-2 \\ -2 & \text{if } -1\le x\le 1 \\ x-1 & \text{if } 2<x<5 \end{cases}$$

$y-y_1=m(x-x_1)$

FIGURE 9



(b) Find $f(1)$.

$f(1)=-2$

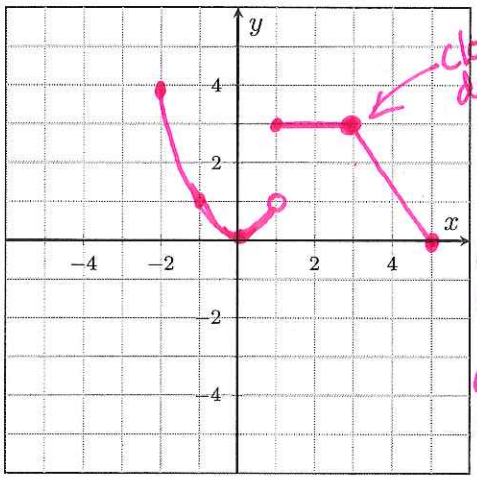
(c) Solve $f(x)=2$.

$\{-3,3\}$

Group Work 2. Graph the function h defined below and then complete the following.

$$h(x)=\begin{cases} x^2 & \text{if } -2\le x<1 \\ 3 & \text{if } 1\le x<3 \\ -\frac{3}{2}(x-5) & \text{if } 3\le x\le 5 \end{cases}$$

FIGURE 10



(a) State the domain and range of h.

x	y
3	3
5	0

D: $[-2,5]$

R: $[0,4]$

(b) State any horizontal and vertical intercepts.

Horizontal intercepts: $(0,0)$, $(5,0)$

Vertical intercepts: $(0,0)$

(c) State the absolute maximum of h and where it occurs. The absolute max is 4 at $x=-2$

Group Work 3. When calculating your electricity bill, PGE uses the follows rates: It costs 5.124 cents per kWh for the first 250 kWh used in a month. After the first 250 kWh, it costs 6.899 cents for each additional kWh used. Let $C(x)$ represent the monthly amount due (in dollars) for a PGE residential electricity bill where x kWh of energy were used that month.

(a) Write the formula for the piecewise-defined function C.

change to dollars and cents by moving the decimal over.

$$C(x) = \begin{cases} .05124x & \text{if } 0 \le x \le 250 \\ .05124(250) + .06899(x-250) & \text{if } x > 250 \end{cases}$$

1st 250 kWH

anything over 250

(b) Use that formula to determine the amount due (before taxes and other fees) when you use 325 kWh of electricity in a month.

$$\begin{aligned} C(325) &= .05124(250) + .06899(325-250) \\ &= 12.81 + .06899(75) \\ &= 12.81 + 5.17425 \\ &= \$17.98 \end{aligned}$$

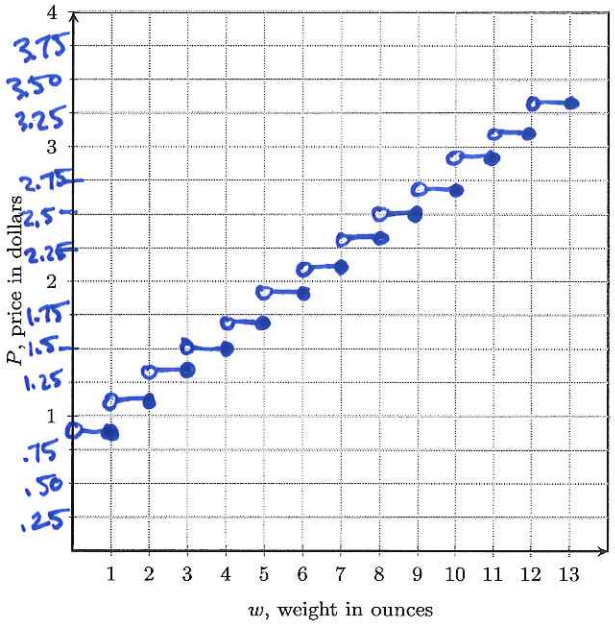
The cost for 325 kWH is $17.98

Group Work 4. The US Postal Service rates for large envelopes are given in Table 2, according to their weight.[2] Graph the cost P (in dollars) of mailing a large envelope as a function of the weight w (in ounces) in Figure 11.

TABLE 2. US Postal Service First-Class Mail Prices, Large Envelopes

Weight Not Over (in oz.)	Price (in $)
1	0.90
2	1.10
3	1.30
4	1.50
5	1.70
6	1.90
7	2.10
8	2.30
9	2.50
10	2.70
11	2.90
12	3.10
13	3.30

FIGURE 11. US Postal Service First-Class Mail Prices for Large Envelopes



[2] http://pe.usps.com/cpim/ftp/manuals/dmm300/Notice123.pdf