Math 111 - Tues, 5/31

go over quizzes
Questions on 3.4 packet
Finish 3.4 packet
New Material: 3.5 packet

Checkpoint 8 on Thurs (3.4 + 3.5)
Mission 4 due on Thurs
Last bonus + final review handed out Thurs
Class Party on Thursday! Bring a snack to share if you wish

Final Boss Thursday, 6/9 8am

Math 111 Lecture Notes

Section 3.5: Graphing Rational Functions

A **rational function** is of the form $R(x) = \dfrac{p(x)}{q(x)}$ where p and q are polynomial functions.

The **zeros** of a rational function are the values of x for which $p(x) = 0$, as the function's value is zero where the value of the numerator is zero. Most of the time, the zeros will occur at a when the factor $(x - a)$ is in the numerator of R.

A rational function is undefined where $q(x) = 0$, as this would cause division by zero.

A **vertical asymptote** occurs when the denominator of the *simplified* form of R is equal to zero. Most of the time, the vertical asymptote $x = b$ will occur when the factor $(x - b)$ is in the denominator of the *simplified* form of R.

A **hole** occurs when both the numerator and denominator equal zero for some value of x. We will identify a zero at c when the linear factor $(x - c)$ occurs in both the numerator and denominator of a rational function. Note that during simplification this factor cancels and results in a domain restriction for R.

The **long run behavior** and **horizontal asymptote** of R can be determined by the ratio of leading terms of p and q.

http://changestartsintheheart.wordpress.com/tag/asymptote/
http://www.theoildrum.com/node/5110

Example 1. Graph the rational function $R(x) = \frac{2x-6}{x+4}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

$R(x) = \frac{2(x-3)}{(x+4)}$

Domain: $\{x \mid x \neq -4\}$ ← V.A. $x=-4$, multiplicity 1

No holes

H.A. $\frac{2\cancel{x}}{\cancel{x}} = 2$ $y=2$ as $x \to \infty$, $y \to 2$
as $x \to -\infty$, $y \to 2$

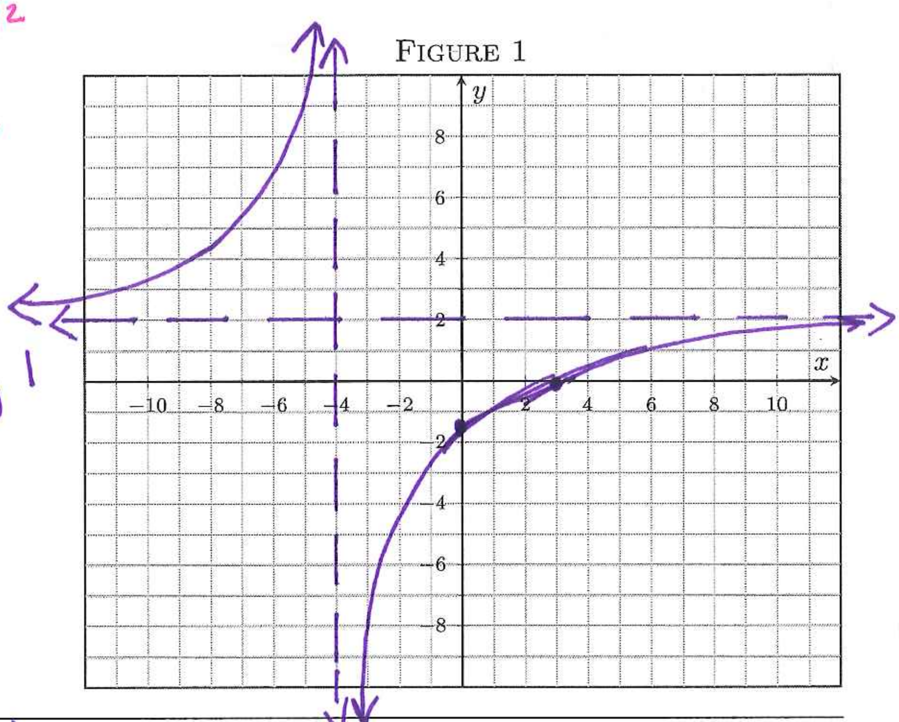
y-int: $R(0) = \frac{2(0-3)}{0+4}$
$= \frac{\cancel{2}^{1}(-3)}{\cancel{4}_{2}}$
$= -\frac{3}{2}$

$(0, -\frac{3}{2})$

zeros: $x=3$
multiplicity 1

Test point: $x=-5$
$R(-5) = \frac{2(-5)-6}{-5+4}$
$= \frac{-10-6}{-1}$
$= \frac{-16}{-1} = 16$

FIGURE 1



Example 2. Graph the rational function $R(x) = \dfrac{8}{x^2 - 4}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

$R(x) = \dfrac{8}{(x+2)(x-2)}$ Domain: $\{x \neq 2, -2\}$, no holes

Horizontal Asymptote : $\dfrac{8}{x^2} \to 0$ $y=0$ as $x \to \infty, y \to 0$; as $x \to -\infty, y \to 0$

$R(0) = \dfrac{8}{0^2-4} = \dfrac{8}{-4} = -2$

Zeros : none

Vertical Asymptotes : $x = -2, 2$, mult 1

Checkpoints:

$x=4$

$R(4) = \dfrac{8}{4^2-4}$

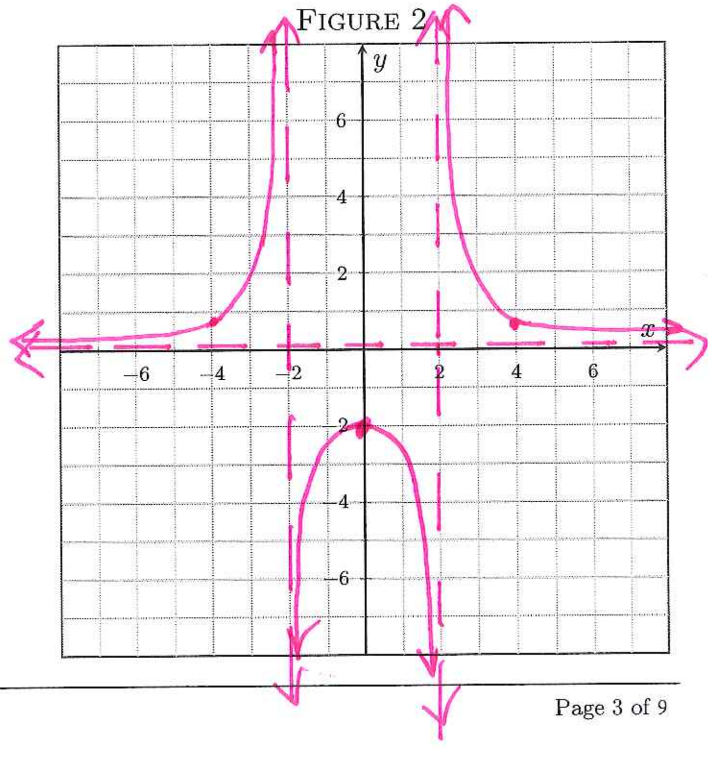
$= \dfrac{8}{12}$

$= \dfrac{2}{3}$

$R(-4) = \dfrac{8}{(-4)^2-4}$

$= \dfrac{2}{3}$

FIGURE 2



Example 3. Graph the rational function $R(x) = \dfrac{x^2 - 4x + 3}{x - 3}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

$R(x) = \dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}} = x-1, \ x \neq 3$ $\quad y=mx+b$

graph of a line with a hole in it

No V.A.

No H.A.

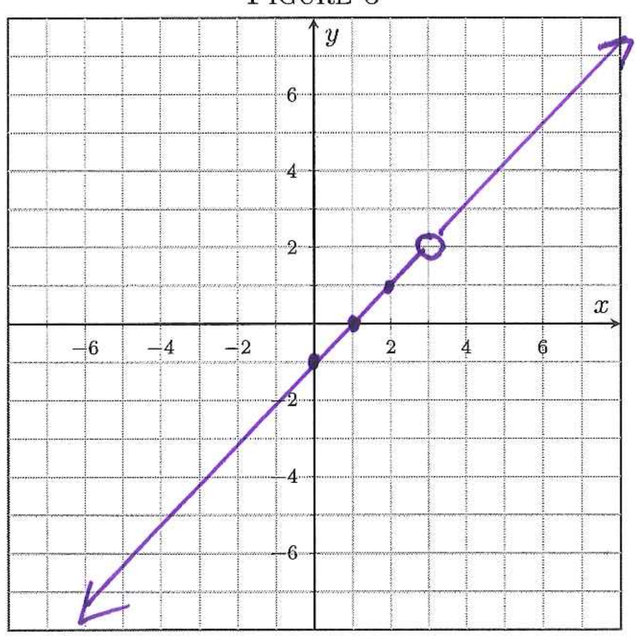
$y = x-1, \ x \neq 3$

zero: $x=1$

Could write as a piecewise function

$P(x) = \begin{cases} x-1, & x<3 \\ x-1, & x>3 \end{cases}$

FIGURE 3



Example 4. Graph the rational function $R(x) = \dfrac{3x-6}{x^2+x-6}$ by completing the following:

- Factor and simplify $R(x)$. State the domain and any holes.
- State the long-run behavior and any horizontal asymptote.
- Find the vertical intercept.
- Find any zeros and find any vertical asymptotes. State the behavior of the function around the zeros and vertical asymptotes (preferably by making a table).

$$R(x) = \frac{3\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{3}{x+3}, \ x \neq 2$$

Hole at $x=2$ $\quad R(2) = \frac{3}{2+3} = \frac{3}{5}$ $\quad$ Hole at $(2, \frac{3}{5})$

V.A. at $x=-3$, multiplicity 1

Domain: $\{x \mid x \neq -3, 2\}$

Long Run behavior

Horizontal Asymptote

H.A. $y=0$

$$\frac{3x}{x^2} = \frac{3}{x} \text{ or } \frac{3}{\infty} \to 0$$

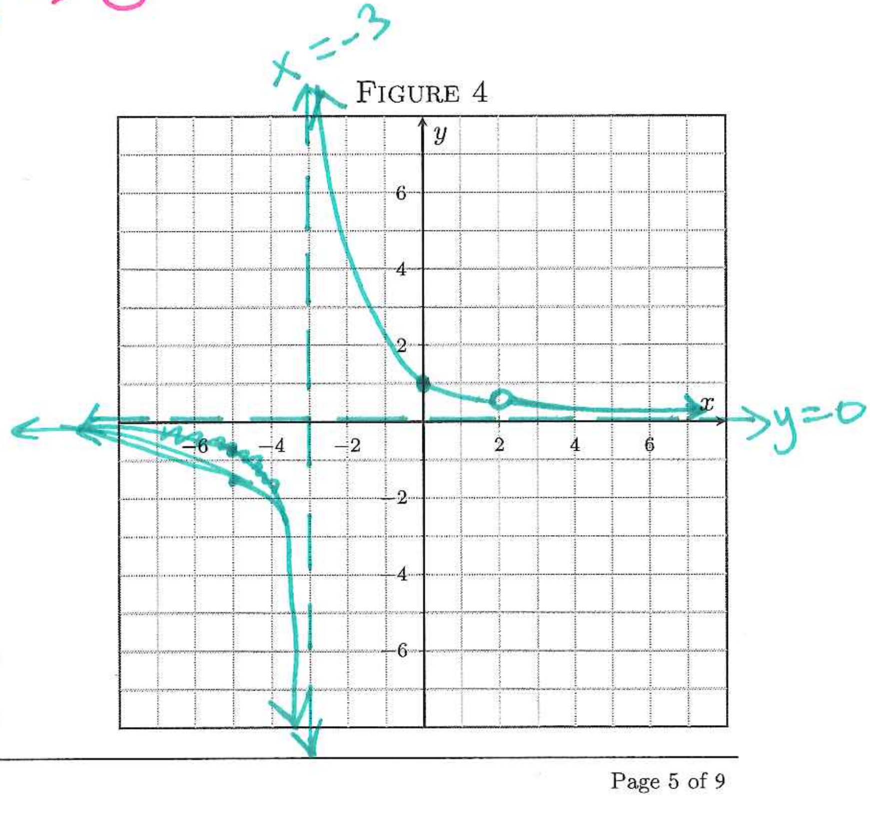
$$R(0) = \frac{3\cdot 0 - 6}{0^2+0-6} = \frac{-6}{-6} = 1$$

$(0,1)$ y-int

Zeros: none

Test point $R(-5) = \dfrac{3}{-5+3} = \dfrac{3}{-2} = -\dfrac{3}{2}$ $\quad (-5, -\frac{3}{2})$

FIGURE 4



How to find a possible formula for a rational function:

- State any zeros. Use these to determine factors and the multiplicity of each factor that appears in the numerator.
- State any vertical asymptotes. Use these to determine factors and the multiplicity of each factor that appears in the denominator.
- If a "hole" appears at $x = a$, then put the factor $(x - a)$ in both the numerator and denominator.
- Use one other point to determine if there is a constant factor other than 1.

Example 5. Find a possible formula for the rational function graphed in Figure 5.

$$R(x) = \frac{k(x-5)}{(x-4)}$$

plug in $(3, 6)$

$$6 = \frac{k(3-5)}{3-4}$$

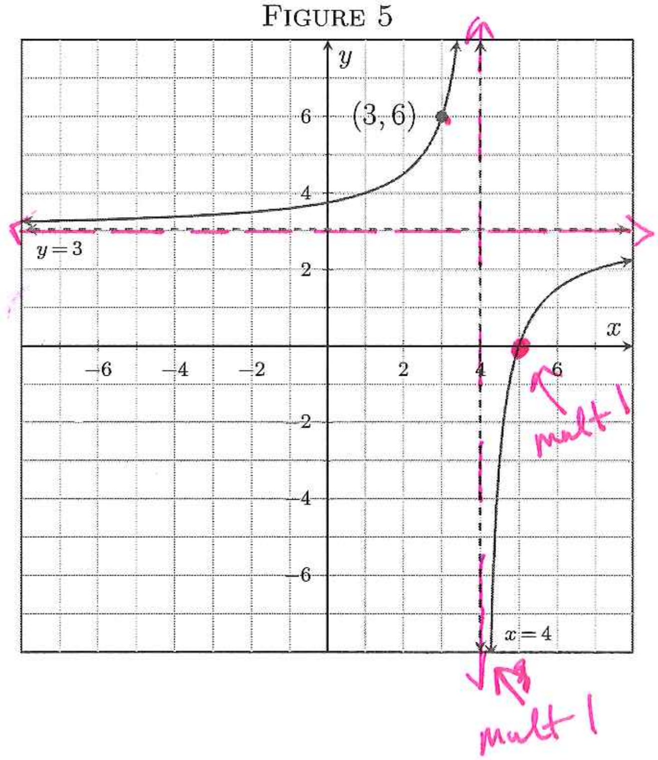
$$6 = \frac{k(-2)}{-1}$$

$$\frac{6}{2} = \frac{k \cdot 2}{2}$$

$$3 = k$$

$$R(x) = \frac{3(x-5)}{x-4}$$

FIGURE 5



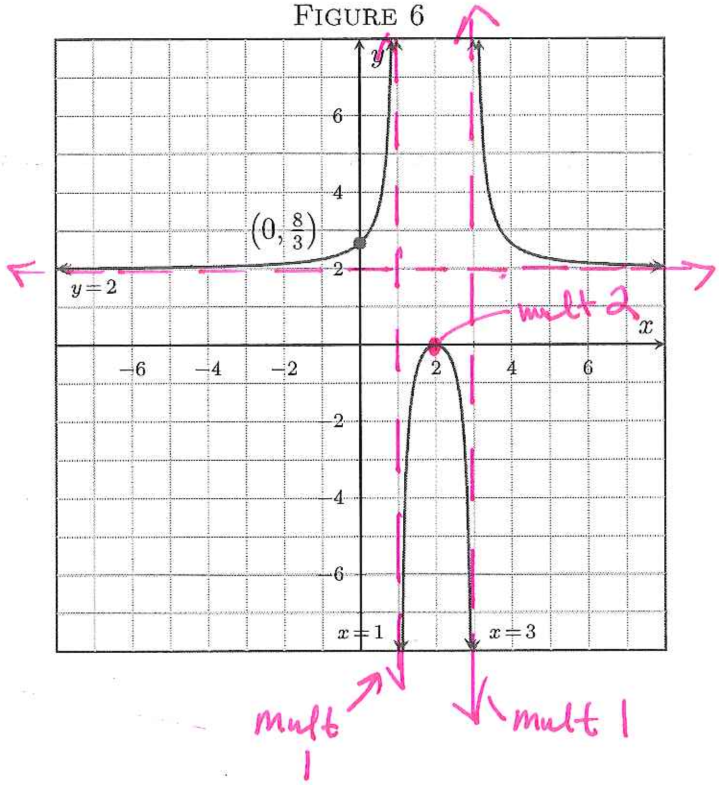
H.A. $\frac{3\cancel{x}}{\cancel{x}} = 3$ $y = 3$

Example 6. Find a possible formula for the rational function graphed in Figure 6.

$$R(x) = \frac{K(x-2)^2}{(x-1)(x-3)}$$

$K = 2$ because the top + bottom have the same degree and there is a H.A. at $y=2$

FIGURE 6



Check:

$$\frac{8}{3} = \frac{k(0-2)^2}{(0-1)(0-3)}$$

$$\frac{8}{3} = \frac{k(4)}{(-1)(-3)}$$

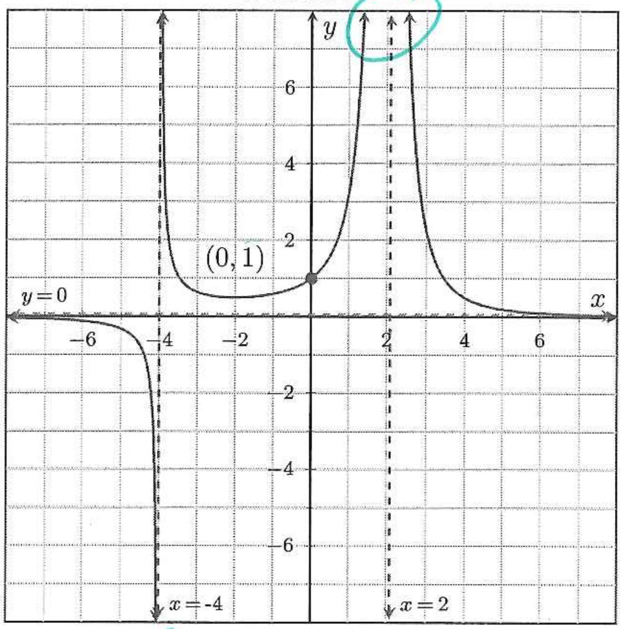
$$\frac{\cancel{3}}{4} \cdot \frac{8}{\cancel{3}} = k \cdot \frac{\cancel{4}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}}$$

$$2 = K$$

$$R(x) = \frac{2(x-2)^2}{(x-1)(x-3)}$$

Example 7. Find a possible formula for the rational function graphed in Figure 7.

FIGURE 7



mult 2

mult 1

$$R(x) = \frac{K}{(x+4)(x-2)^2}$$

Find K:

$$1 = \frac{K}{(0+4)(0-2)^2}$$

$$1 = \frac{K}{(4)(4)}$$

$$16 \cdot 1 = \frac{K}{\cancel{16}} \cdot \cancel{16}$$

$$16 = K$$

$$R(x) = \frac{16}{(x+4)(x-2)^2}$$

Check: H.A.

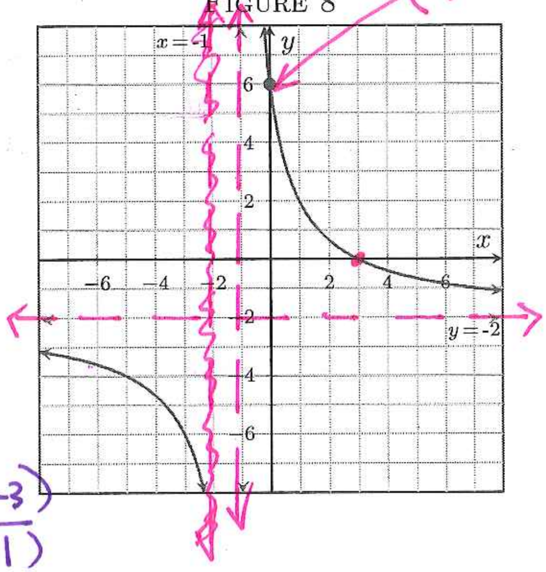
$\frac{16}{x^3}$ or $\frac{16}{\infty^3} \to 0$

as $x \to \infty$

$y=0$ is the H.A.

Group Work 1. Find a possible formula for the rational function graphed in Figure 8.

FIGURE 8



$$R(x) = \frac{k(x-3)}{(x+1)}$$

$$6 = \frac{k(0-3)}{(0+1)}$$

$$6 = \frac{k(-3)}{1}$$

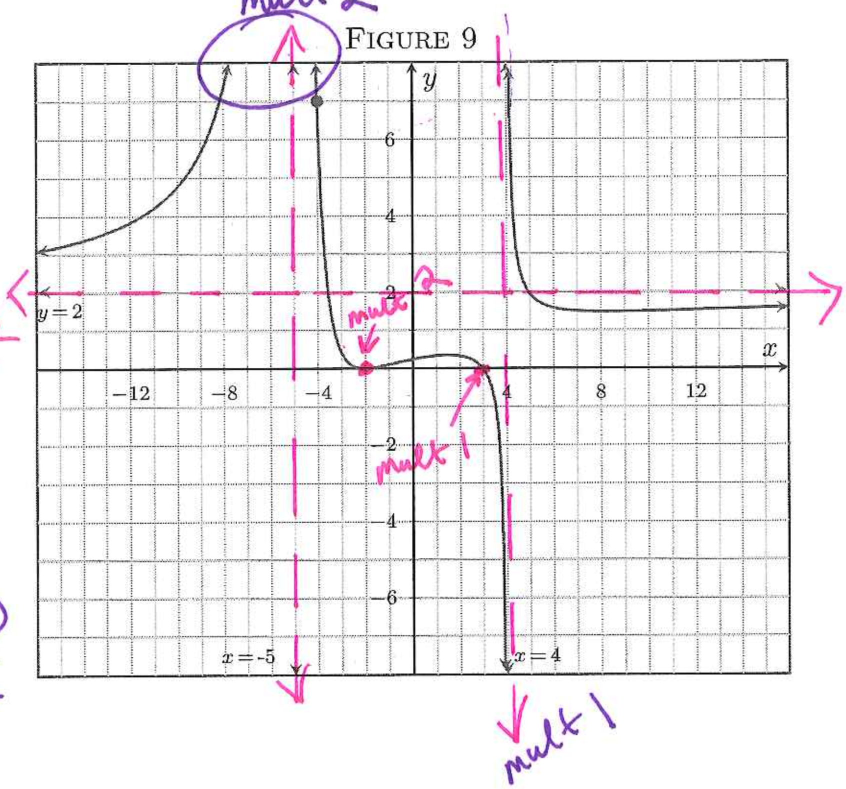
$$\frac{6}{-3} = \frac{k(-3)}{-3}$$

$$-2 = k$$

$$R(x) = \frac{-2(x-3)}{(x+1)}$$

Group Work 2. Find a possible formula for the rational function graphed in Figure 9.

FIGURE 9



$$R(x) = \frac{k(x+2)^2(x-3)}{(x-4)(x+5)^2}$$

$(-4, 7)$

$$7 = \frac{k(-4+2)^2(-4-3)}{(-4-4)(-4+5)^2}$$

$$7 = \frac{k(-2)^2(-7)}{(-8)(1)^2}$$

$$\frac{8}{28}\cdot 7 = k \cdot \frac{-28}{-8}\cdot\frac{8}{28}$$

$$2 = k$$

$$R(x) = \frac{2(x+2)^2(x-3)}{(x-4)(x+5)^2}$$