Math 111 - Tuesday, 4/19

Please turn in Mission 2!

Hand back checkpoints

Questions on 1.5 + supplement

New material: Section 4.1

Checkpoint 4 on Thursday (1.5)

1st Boss/Midterm next ~~Tuesday~~ Thursday - Study guide Handout

Next Tuesday is an Inservice Day - No Classes before 4pm

Questions on 1.5

$\left(\frac{b}{2}\right)^2$ or $\left(\frac{1}{2}b\right)^2$

$$f(x) = \underbrace{x^2 + 6x} - 7$$
$$= x^2 + 6x + \underline{3^2} - 7 - \underline{9}$$
$$= (x+3)^2 - 16$$

$$f(x) = (x+3)^2 - 16$$

$\left(\frac{4}{2}\right)^2$

$$f(x) = \underbrace{2x^2 + 8x} + 3$$

$2 \cdot \underline{4}$

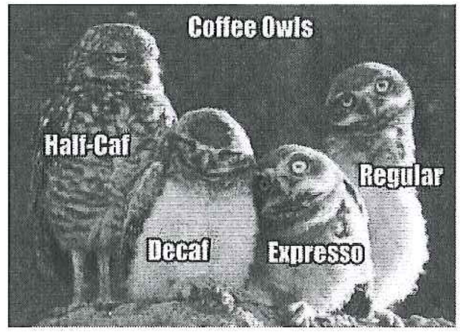
$$= 2(x^2 + 4x + \underline{2^2}) + 3 - \underline{8}$$
$$= 2(x+2)^2 - 5$$

$$f(x) = 2(2x+2)^2 - 5$$
$$= 2(2(x+1))^2 - 5$$

Math 111 Lecture Notes

Section 4.1: Function Composition

When a person consumes caffeine, it is absorbed into their blood. Over time, the amount of caffeine in the bloodstream decreases (assuming they stop consuming caffeine). The result of caffeine being in the bloodstream is that the person's heart rate is elevated. This "chain reaction" is a simple example of a *composite function.* The person's heart rate depends on the amount of caffeine in their bloodstream, which depends on the amount of time since it was consumed. It makes sense then that we should be able to combine these two functions and determine person's heart rate at a given time.



Example 1. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

$$g(x) = -10x + 90, \qquad h(y) = 3y - 90$$

(a) Find and interpret $g(3)$.

$g(3) = -10(3)+90$
$= -30+90$
$= 60$ mg.

After 3 hours you still have 60 mg of caffeine

(b) Find and interpret $h(60)$.

$h(60) = 3(60)-90$
$= 180-90$
$= 90$ beats per minute

With 60 mg of caffeine your heart rate is 90 bpm

(c) Find and interpret $h(g(3))$.

$h(g(3)) = h(-10(3)+90)$
$= h(-30+90)$
$= h(60)$
$= 3(60)-90$
$= 180-90$
$= 90$ beats per minute

After 3 hours your heart rate is 90 bpm.

" or

$(h \circ g)(3)$

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read "f composed with g" or "f of g") is defined by

$$(f \circ g)(x) = f(g(x))$$

inside

The function g is referred to as the *inside function* and the function f is referred to as the *outside function.*

In determining the domain for the composite function, the domain of the inside function and the domain for the resultant composite function must be accounted for.

Example 2. Use the functions f and g given in Table 1 to determine the following.

TABLE 1

x	-2	-1	0	1	2
$f(x)$	5	4	-3	2	0
$g(x)$	0	-2	6	9	-1

(a) $(g \circ f)(2)$

$f(2)=0$

$g(f(2)) = g(0)$
$= 6$

(c) $(g \circ g)(-1)$

$g(g(-1)) = g(-2)$
$= 0$

(b) $(f \circ g)(2)$

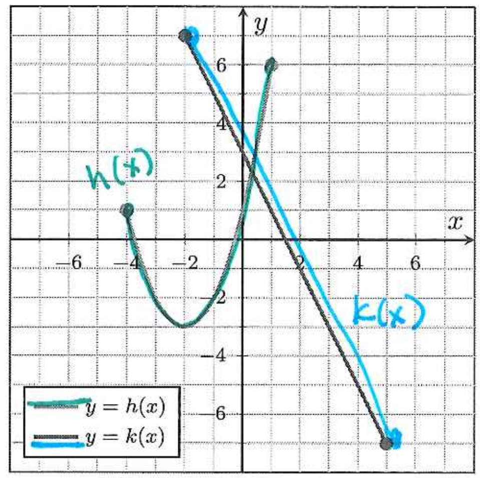
$f(g(2)) = f(-1)$
$= 4$

(d) $(f \circ f)(-2)$

$f(f(-2)) = f(5)$
$=$ undefined

Example 3. Use Figure 1 to complete the following, if they exist.

FIGURE 1



(a) $(h \circ k)(2)$

$h(k(2))$
$= h(-1)$
$= -2$

(c) $(h \circ h)(1)$

$h(h(1))$
$= h(6)$
$=$ undefined

(b) $(k \circ h)(-3)$

$k(h(-3))$
$= k(-2)$
$= 7$

(d) $(k \circ k)(-1)$

$k(k(-1))$
$= k(5)$
$= -7$

Example 4. Let $f(x) = \dfrac{\sqrt{x+4}}{3x-6}$ and $\boxed{g(x) = |2x - 8|.}$ Compute the following:

(a) $(f \circ g)(-2)$

$$f(g(-2)) = f(|2(-2)-8|)$$
$$= f(|-4-8|)$$
$$= f(|-12|)$$
$$= f(12)$$
$$= \frac{\sqrt{12+4}}{3\cdot 12-6}$$
$$= \frac{\sqrt{16}}{36-6}$$
$$= \frac{4}{30}$$
$$= \frac{2}{15}$$

(b) $(f \circ f)(-4)$

$$f(f(-4)) = f\left(\frac{\sqrt{-4+4}}{3(-4)-6}\right)$$
$$= f\left(\frac{\sqrt{0}}{-12-6}\right)$$
$$= f\left(\frac{0}{-18}\right)$$
$$= f(0)$$
$$= \frac{\sqrt{0+4}}{3(0)-6}$$
$$= \frac{\sqrt{4}}{0-6}$$
$$= \frac{2}{-6}$$
$$= -\frac{1}{3}$$

input

Example 5. Let $j(x) = 5\boxed{x}^2 + 3\boxed{x} - 1$ and $k(x) = 2\boxed{x} + 7$. Find and fully simplify each of the following:

(a) $(k \circ j)(x)$

$= k(j(x)) = k(5x^2+3x-1)$

$= 2(5x^2+3x-1)+7$

$= 10x^2+6x-2+7$

$= 10x^2+6x+5$

(b) $(k \circ k)(x) = k(k(x))$

$= k(\boxed{2x+7})$

$= 2(2x+7)+7$

$= 4x+14+7$

$= 4x+21$

(c) $(j \circ k)(x) = j(k(x))$

$= j(2x+7)$

$= 5(2x+7)^2+3(2x+7)-1$

$= 5(2x+7)(2x+7)+6x+21-1$

$= 5(4x^2+14x+14x+49)+6x+20$

$= 5(4x^2+28x+49)+6x+20$

$= 20x^2+140x+245+6x+20$

$= 20x^2+146x+265$

What is the domain of $k \circ j$? All real numbers

$\mathbb{R}$

Example 6. Find $(g \circ f)(x)$ if $f(x) = \dfrac{7}{x+4}$ and $g(x) = \dfrac{3\boxed{x}}{2\boxed{x}-5}$. $\boxed{\text{State the domain of } g \circ f.}$

$$g(f(x)) = g\left(\frac{7}{x+4}\right)$$

$$= \frac{\frac{3}{1}\left(\frac{7}{x+4}\right)}{\frac{2}{1}\left(\frac{7}{x+4}\right)-5}$$

$$= \frac{\frac{21}{x+4}}{\left(\frac{14}{x+4}-5\right)} \frac{\left(\frac{x+4}{1}\right)}{\left(\frac{x+4}{1}\right)}$$

$$= \frac{\left(\frac{21}{\cancel{x+4}}\right)\left(\frac{\cancel{x+4}}{1}\right)}{\left(\frac{14}{\cancel{x+4}}\right)\left(\frac{\cancel{x+4}}{1}\right) - \frac{5}{1}\left(\frac{x+4}{1}\right)}$$

$$= \frac{21}{14-5(x+4)}$$

$$= \frac{21}{14-5x-20} = \frac{21}{-5x-6}$$

$$g(f(x)) = \frac{21}{-5x-6}$$

Domain:

$-5x-6 \neq 0$

$\frac{-5x}{-5} \neq \frac{6}{-5}$

$x \neq -\frac{6}{5}$

Domain of inside function $\frac{7}{x+4}$

$x+4 \neq 0$

$x \neq -4$

use both

$$\boxed{\text{Domain of } g \circ f \quad \{x \mid x \neq -4 \text{ or } -\tfrac{6}{5}\}}$$

Example 7. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

$$g(x) = -10x + 90, \qquad h(y) = 3\boxed{y} - 90$$

Write the composite function $(h \circ g)(x)$. What does this function represent?

$$h(g(x)) = h(-10x+90)$$

$$= 3(-10x+90)-90$$

$$= -30x+270-90$$

$$= -30x+180$$

$h \circ g$ represents your heart rate x hours after consuming caffeine.

Domain $\{x \mid x \geq 0\}$ due to context

Group Work 1. Let $f(x) = 5x - 7$, $g(x) = \frac{2x}{x-3}$, and $h(x) = \sqrt{4x+8}$. Find and fully simplify each of the following. Also state the domain of $g \circ f$ and $f \circ h$.

(a) $(g \circ h)(2)$

$$g(h(2)) = g(\sqrt{4 \cdot 2 + 8}) = g(\sqrt{8+8}) = g(\sqrt{16}) = g(4) = \frac{2(4)}{4-3} = \frac{8}{1} = 8$$

(c) $(f \circ f)(-4)$

$$f(f(-4)) = f(5(-4)-7) = f(-20-7) = f(-27) = 5(-27)-7 = -135-7 = -142$$

(b) $(g \circ f)(x)$

$$g(f(x)) = g(5x-7) = \frac{2(5x-7)}{5x-7-3} = \frac{10x-14}{5x-10}$$

$g \circ f$: $5x - 10 \neq 0$, $5x \neq 10$, $x \neq 2$

no restriction on f.

Domain of $g \circ f$: $\{x \mid x \neq 2\}$

(d) $(f \circ h)(x)$

$$f(h(x)) = f(\sqrt{4x+8}) = 5(\sqrt{4x+8}) - 7 = 5\sqrt{4x+8} - 7 = 5\sqrt{4(x+2)} - 7 = 5 \cdot 2\sqrt{x+2} - 7 = 10\sqrt{x+2} - 7$$

$x + 2 \geq 0$

$x \geq -2$

Domain of $f \circ h$: $\{x \mid x \geq -2\}$

$4x + 8 \geq 0$

$\frac{4x}{4} \geq \frac{-8}{4}$

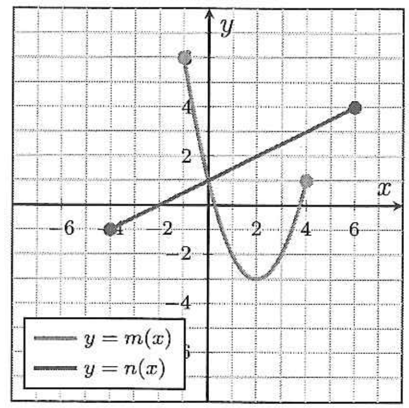
$x \geq -2$

Group Work 2. Use Table 2 and Figure 2 to complete the following, if they exist.

TABLE 2

x	-2	-1	0	1	2
$a(x)$	5	4	2	-1	1
$b(x)$	7	2	0	-9	-4

FIGURE 2



(a) $(a \circ m)(3) = a(m(3))$
$= a(-2)$
$= 5$

(b) $(m \circ m)(4)$
$= m(m(4))$
$= m(1)$
$= -2$

(c) $(b \circ a)(0) = b(a(0))$
$= b(2)$
$= -4$

(d) $(n \circ b)(1) = n(b(1))$
$= n(-9)$
$=$ undefined

Example 8. Let $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}(x - 5)$. Show that both $(f \circ g)(x) = x$ and that $(g \circ f)(x) = x$ for every x in the respective domains of $f \circ g$ and $g \circ f$.

$f(g(x)) = f\left(\frac{1}{3}(x-5)\right)$
$= \cancel{3}\left(\cancel{\frac{1}{3}}(x-5)\right) + 5$
$= x - 5 + 5$
$= x$

$g(f(x)) = g(3x+5)$
$= \frac{1}{3}(3x + \cancel{5} - \cancel{5})$
$= \cancel{\frac{1}{3}}(\cancel{3}x)$
$= x$

Example 9. For the following examples, find the functions f and g such that $H = f \circ g$. Do not choose $f(x) = x$ or $g(x) = x$.

inside

(a) $H(x) = \sqrt{3x+1}$

$\sqrt{\ }$ outside function

$3x+1$ inside function

$g(x) = 3x+1$

$f(x) = \sqrt{x}$

Check:

$f(g(x)) = f(3x+1)$

$= \sqrt{3x+1}$

(b) $H(x) = (5x-3)^2$

$f(x) = x^2$ (outside)

$g(x) = 5x-3$

Check

$f(g(x)) = f(5x-3)$

$= (5x-3)^2$

(c) $H(x) = \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}+1}$

inside $g(x) = \sqrt[3]{x}$

outside $f(x) = \dfrac{x}{x+1}$

(d) $H(x) = (x^2-1)^3$

inside $g(x) = x^2-1$

outside $f(x) = x^3$

(e) $H(x) = \dfrac{2}{x-3}$ inside

inside $g(x) = x-3$

outside $f(x) = \dfrac{2}{x}$