# Math 111 – Thurs, 4/21

Q's on 1.5
Checkpoint 4 (on 1.5)
Q's on 4.1
New material: 4.2 + supplement
inverse matching activity

No class on Tuesday (Inservice Day until 4pm)

1st Boss/midterm on Thursday
(1.1 – 1.5, 4.1, 4.2)

8 am – 10:20 – You may leave when finished

Part 1 – No Calculator
Part 2 – Graphing Calculator Needed

Review problems in the book
Review packet – solutions will be posted by Friday

# 1.5 Questions - Completing the square

$\left(\frac{1}{2}b\right)^2$ or $\left(\frac{b}{2}\right)^2$

$y = \underbrace{x^2 + 4x} - 1$

$= x^2 + 4x + \underline{2^2} - 1 - \underline{4}$

$= (x+2)^2 - 5$

$y = \underbrace{3x^2 - 9x} + 1$ $\quad \left(\frac{3}{2}\right)^2$

$= 3\left(x^2 - 3x + \underline{\left(\tfrac{3}{2}\right)^2}\right) + 1 - \frac{3 \cdot 9}{1 \cdot 4}$

$= 3\left(x - \frac{3}{2}\right)^2 + 1 - \frac{27}{4}$

$= 3\left(x - \frac{3}{2}\right)^2 + \frac{4}{4} - \frac{27}{4}$

$= 3\left(x - \frac{3}{2}\right)^2 - \frac{23}{4}$

# Math 111 Lecture Notes

## Section 4.2: Inverse Functions

**Example 1.** Temperature in degrees Fahrenheit, $F$, can be written as a function of temperature in degrees Celsius, $C$. This relationship is given by $F = g(C) = \frac{9}{5}C + 32$.

(a) Find and interpret $g(100)$.

$$g(100^\circ C) = \frac{9}{\cancel{5}_1}\left(\frac{\cancel{100}^{20}}{1}\right) + 32$$
$$= 180 + 32$$
$$= 212^\circ F$$

100°C is equivalent to 212°F. (boiling point of water)

(b) Solve and interpret the solution to $g(C) = 32$.

$$g(C) = \frac{9}{5}C + 32$$
$$32 = \frac{9}{5}C + 32$$
$$-32 \qquad -32$$
$$\frac{5}{9} \cdot 0 = \frac{\cancel{9}}{\cancel{5}}C\frac{\cancel{5}}{\cancel{9}}$$
$$0^\circ C = C$$

0°C is equivalent to 32°F (Freezing point of water)

(c) Solve the equation $F = \frac{9}{5}C + 32$ for $C$.

$$F = \frac{9}{5}C + 32$$
$$\frac{5}{9}(F - 32) = \frac{\cancel{9}}{\cancel{5}}C\left(\frac{\cancel{5}}{\cancel{9}}\right)$$
$$\frac{5}{9}(F - 32) = C$$

output ↙ ↙ input

$$C = \frac{5}{9}(F - 32)$$ Inverse functions

$$F = \frac{9}{5}C + 32$$

↑ output ↑ input

A function $f$ is said to be **one-to-one** if for every $y$-value in the range of $f$ there is exactly one $x$-value in the domain of $f$.

A function must be one-to-one in order to have an inverse. The **inverse function of** $f$ reverses the process of the original function. In other words, the input and output switch roles. The original function is given by $y = f(x)$. The inverse function is given by $x = f^{-1}(y)$. If we want to graph both of these functions in the $(x, y)$-plane, then we use $\boxed{y = f^{-1}(x)}$. "f inverse of x"

The inverse function of $f$ is denoted by $f^{-1}$. It is important to note that this notation *is not* denoting a reciprocal. That is, $f^{-1}(x) \neq \frac{1}{f(x)}$.

$$5^{-1} = \frac{1}{5}$$

**Example 2.** Write the definition for $g^{-1}(F)$ for Example 1.

$$C = \frac{5}{9}(F-32)$$

$$g^{-1}(F) = \frac{5}{9}(F-32)$$

**Example 3.** The function $f$ defined by $f(x) = 3x + 2$ is one-to-one. Find its inverse. Then graph $y = f(x)$ and $y = f^{-1}(x)$ in Figure 1. Include the graph of $y = x$ also.

$$f(x) = 3x + 2$$

$y = 3x + 2$ switch x and y

$x = 3y + 2$ solve for y

$$\frac{x-2}{3} = \frac{3y}{3}$$

$$\frac{x-2}{3} = y$$

$$y = \frac{x-2}{3}$$

$$y = \frac{1}{3}x - \frac{2}{3}$$

$$f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$$

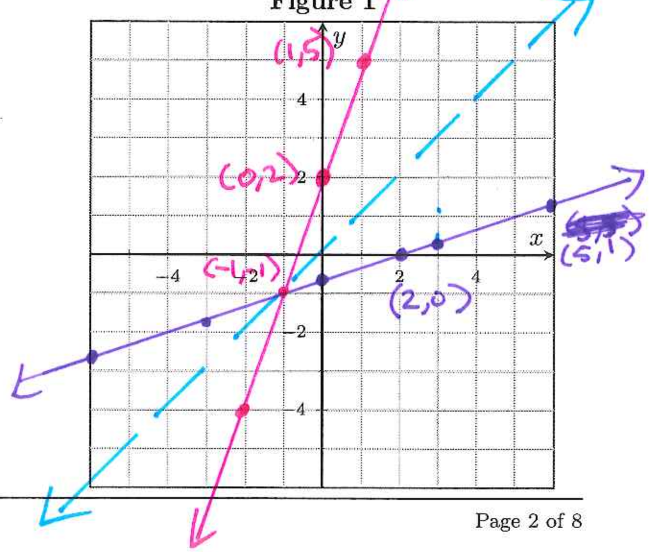
$y = mx + b$

$f(x) = 3x + 2$

$f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$

Reflection over the line $y = x$

**Figure 1**



**Example 4.** To verify that two functions are inverses, we show that $f\left(f^{-1}(x)\right) = x$ and that $f^{-1}\left(f(x)\right) = x$. Do this for the previous example.

$$f(x) = 3x + 2 \qquad f^{-1}(x) = \frac{\boxed{x}-2}{3}$$

f-inverse

$$f(f^{-1}(x)) = f\left(\frac{x-2}{3}\right)$$

$$= \frac{\cancel{3}}{1}\left(\frac{x-2}{\cancel{3}}\right) + 2$$

$$= x - \cancel{2} + \cancel{2}$$

$$= x \checkmark$$

$$f^{-1}(f(x)) = f^{-1}(3x+2)$$

$$= \frac{3x + \cancel{2} - \cancel{2}}{3}$$

$$= \frac{\cancel{3}x}{\cancel{3}}$$

$$= x \checkmark$$

**Example 5.** The function $f$ defined by $f(x) = -\dfrac{2x}{x-1}$ is one-to-one. Find the inverse function. Confirm that the inverse function you found is correct by showing $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

$y = -\dfrac{2x}{x-1}$ switch $x$ + $y$

$(y-1)x = -\dfrac{2y}{\cancel{(y-1)}}\cancel{(y-1)}$ Solve for $y$

$xy - x = -2y$
$-xy \qquad\qquad -xy$ put $y$'s on the same side and factor it out

$-x = -2y - xy$

$\dfrac{-x}{-1} = \dfrac{-y(2+x)}{-1}$

$\dfrac{x}{2+x} = \dfrac{y(2+x)}{2+x}$

$\dfrac{x}{2+x} = y$

$f^{-1}(x) = \dfrac{x}{2+x}$

check:

$f(f^{-1}(x)) = f\left(\frac{x}{2+x}\right)$

$= \dfrac{-2\left(\frac{x}{\cancel{2+x}}\right)}{\left(\left(\frac{x}{2+x}\right) - 1\right)} \cdot \dfrac{\left(\frac{\cancel{2+x}}{1}\right)}{\left(\frac{2+x}{1}\right)}$

$= \dfrac{-2x}{\left(\frac{x}{\cancel{2+x}}\right)\left(\frac{\cancel{2+x}}{1}\right) - \frac{1}{1}\left(\frac{2+x}{1}\right)}$

$= \dfrac{-2x}{\cancel{x} - 2 - \cancel{x}}$

$= \dfrac{-\cancel{2}x}{-\cancel{2}}$

$= x$

State the domain and range of each $f$ and $f^{-1}$.

Domain of $f: \{x \mid x \neq 1\}$

Domain of $f^{-1}: \{x \mid x \neq -2\}$

Range of $f: \{x \mid x \neq -2\}$

Range of $f^{-1}: \{x \mid x \neq 1\}$

The domain of $f$ is the range of $f^{-1}$. Similarly, the range of $f$ is the domain of $f^{-1}$.

Check:

$$f^{-1}(f(x)) = f^{-1}\left(-\frac{2x}{x-1}\right)$$

$$= \frac{\frac{-2x}{\cancel{x-1}} \cdot \left(\frac{\cancel{x-1}}{1}\right)}{\left(2 + \frac{-2x}{x-1}\right)\cdot\left(\frac{x-1}{1}\right)}$$

$$= \frac{-2x}{2\left(\frac{x-1}{1}\right) + \left(\frac{-2x}{\cancel{x-1}}\right)\left(\frac{\cancel{x-1}}{1}\right)}$$

$$= \frac{-2x}{\cancel{2x} - 2 - \cancel{2x}}$$

$$= \frac{\cancel{-2}x}{\cancel{-2}}$$

$$= x$$

The **horizontal line test** is a way of determining if a function is one-to-one. It states that if every horizontal line passes through a graph at most once, then the function is one-to-one.
In the same way that the vertical line test verifies if a graph represents a function, the horizontal line test verifies if the graph of a function is one-to-one (and thus invertible).

**Example 6.** Graph $y = f(x)$ for $\boxed{f(x) = x^2}$ in Figure 2. Then graph $y = g(x)$ for $\boxed{g(x) = x^2, x \geq 0}$ in Figure 3. Is either function invertible? Why or why not?

$y=x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| -1 | 1 |
| -2 | 4 |

**Figure 2.** Graph of $y = f(x)$

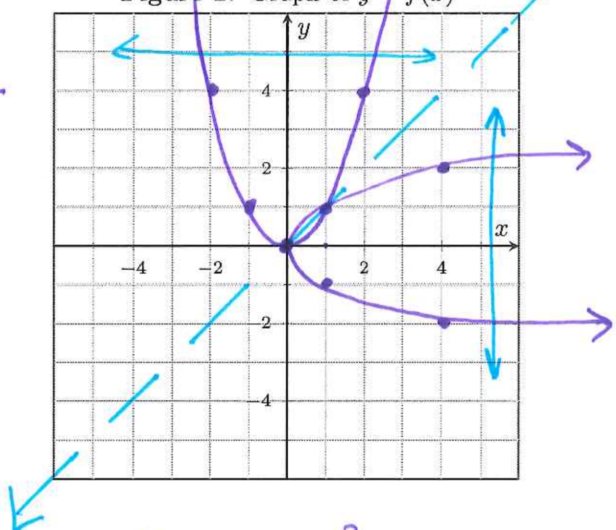
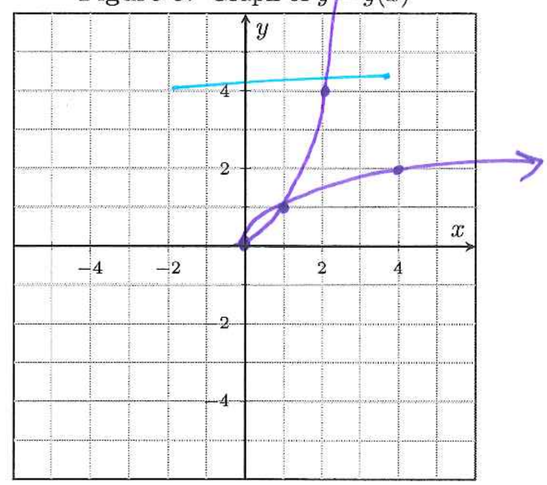



**Figure 3.** Graph of $y = g(x)$




$f(x)=x^2$
The inverse is not a function so $f(x)=x^2$ is not invertible
It doesn't pass the horizontal line test

$f(x)=x^2, x \geq 0$
is one-to-one.
It passes the horizontal line test so it is invertible

$f(x)=x^2, x \geq 0$
$f^{-1}(x)=\sqrt{x}, x \geq 0$

**Example 7.** The function $g$ defined by $g(x) = \sqrt[3]{x+8}$ is one-to-one. Find the inverse function and confirm that it is the inverse by showing $g\left(g^{-1}(x)\right) = x$ and $g^{-1}\left(g(x)\right) = x$. In Figure 4, use transformations to sketch $y = g(x)$, $y = g^{-1}(x)$ and $y = x$.

$g(x) = \sqrt[3]{x+8}$

$y = \sqrt[3]{x+8}$ switch $x$ + $y$

$(x)^3 = \left(\sqrt[3]{y+8}\right)^3$ solve for $y$

$x^3 = y + 8$

$x^3 - 8 = y$

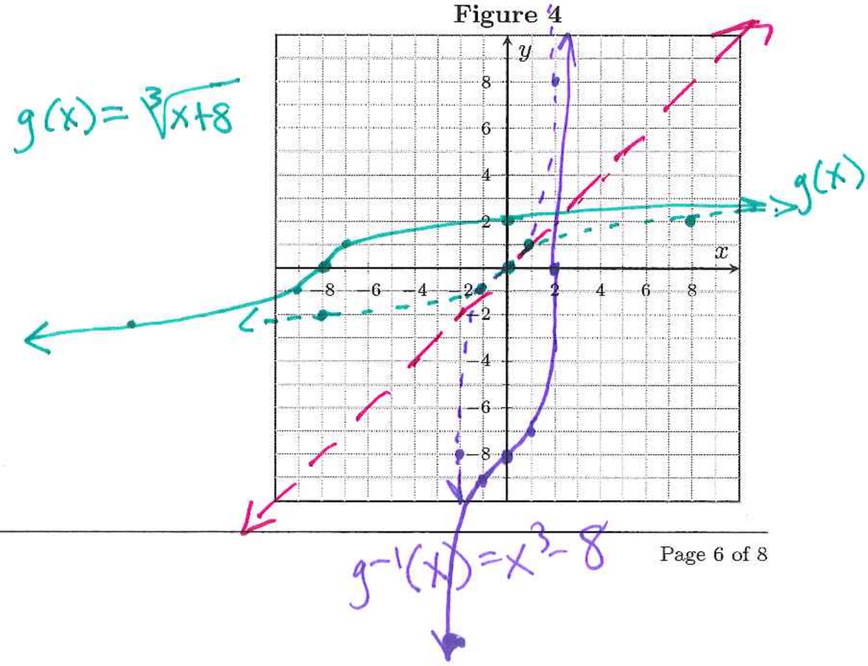
$y = x^3 - 8$

$g^{-1}(x) = x^3 - 8$

$$g(g^{-1}(x)) = g(x^3 - 8)$$
$$= \sqrt[3]{(x^3 \cancel{-8}) \cancel{+8}}$$
$$= \sqrt[\cancel{3}]{x^{\cancel{3}}}$$
$$= x$$

$$g^{-1}(g(x)) = g^{-1}\left(\sqrt[3]{x+8}\right)$$
$$= \left(\left(\sqrt[\cancel{3}]{x+8}\right)^{\cancel{3}} - 8\right)$$
$$= x + 8 - 8$$
$$= x$$

**Figure 4**



**Example 8.** Use the functions $f$ and $g$ given in Table 1 to determine the following.

| $x$ | 5 | 4 | 2 | -1 | 1 |
|---|---|---|---|---|---|
| $f^{-1}(x)$ | -2 | -1 | 0 | 1 | 2 |

**Table 1**

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 5 | 4 | 2 | -1 | 1 |
| $g(x)$ | 7 | 2 | 0 | -2 | 9 |

| $x$ | 7 | 2 | 0 | -2 | 9 |
|---|---|---|---|---|---|
| $g^{-1}(x)$ | -2 | -1 | 0 | 1 | 2 |

(a) $g^{-1}(-2) = 1$
$\uparrow$
$x$

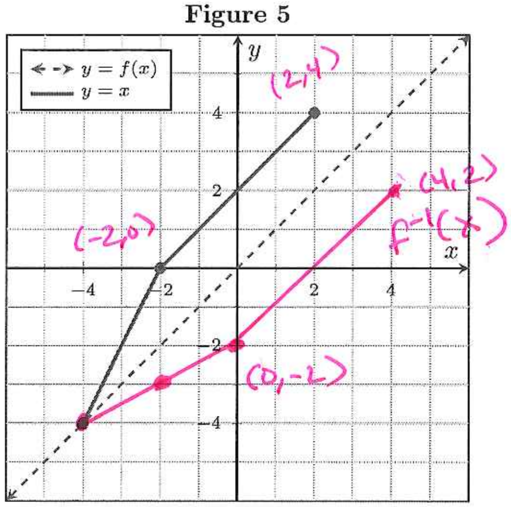
(b) $f^{-1}(2) = 0$

(c) $f^{-1}(0) =$ undefined

(d) $f(g^{-1}(0))$
$= f(0)$
$= 2$

**Example 9.** Graph the inverse function of $f$ in Figure 5. Then use your sketch to find the values of $f^{-1}$ below.

**Figure 5**




(a) $f^{-1}(-4) = -4$

(b) $f^{-1}(-2) = -3$

(c) $f^{-1}(0) = -2$

(d) $f^{-1}(2) = 0$

(e) $f^{-1}(4) = 2$

**Example 10.** The diameter of a Window-Pane oyster, $d$ (in mm), as a function of its weight, $w$ (in grams) can be modeled by

$$d = f(w) = 25 + 20w^{1/3}$$

Find the inverse function by solving $d = 25 + 20w^{1/3}$ for $w$. Write this inverse function as $g(d)$.

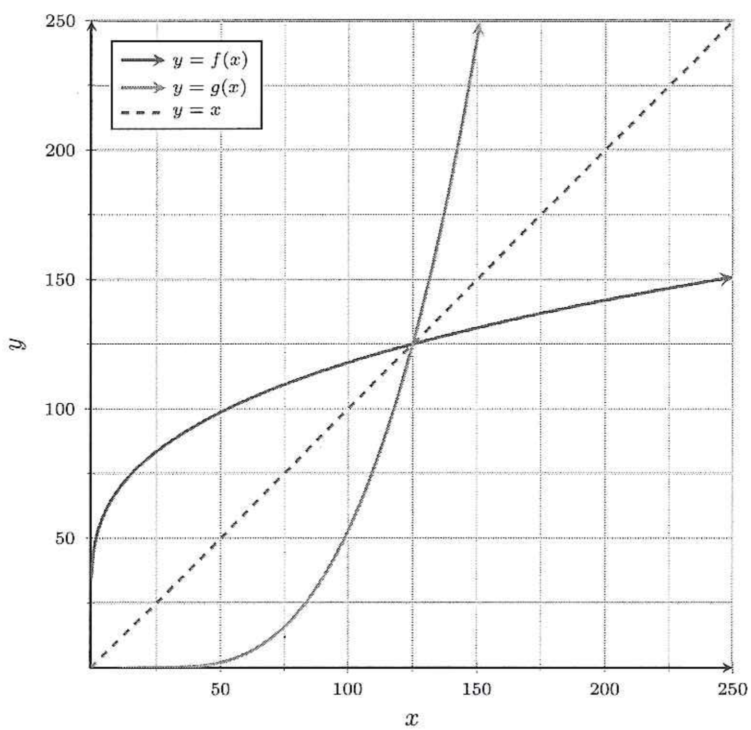
$d = 25 + 20\sqrt[3]{w}$ solve for $w$

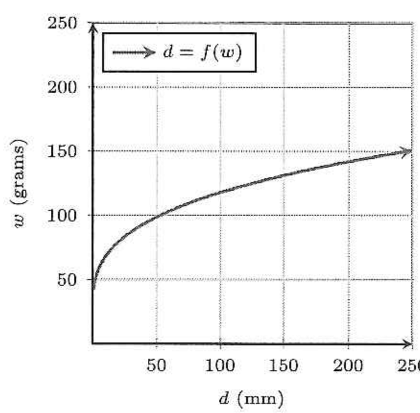
$$\frac{d-25}{20} = \frac{20\sqrt[3]{w}}{20}$$

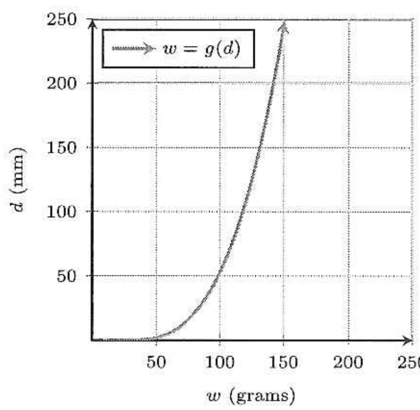
$$\left(\frac{d-25}{20}\right)^3 = \left(\sqrt[3]{w}\right)^3$$

$$\left(\frac{d-25}{20}\right)^3 = w$$

$$w = g(d) = \left(\frac{d-25}{20}\right)^3$$



Math 111 Inverse Matching Activity
Section 4.2

1. Find the person that has the inverse to your function. Complete this sheet together.

2. Write down your functions.

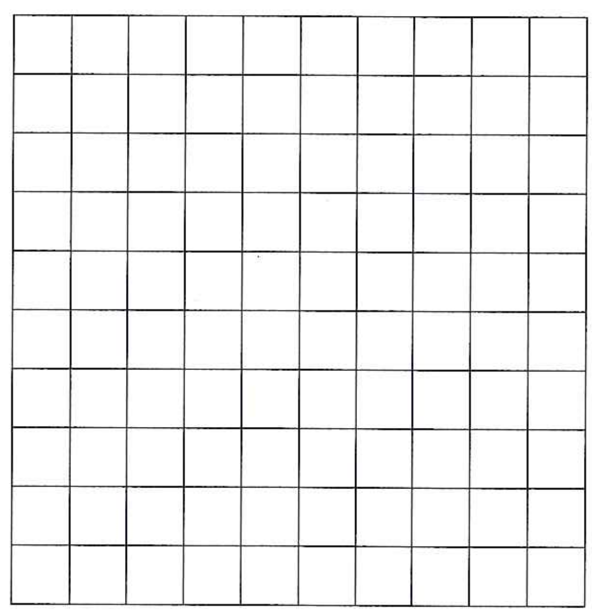
| Your Function | The Inverse Function |
| --- | --- |
| $f(x) = 2x^3 + 3$ | $g(x) = \sqrt[3]{\frac{x-3}{2}}$ |

3. Verify that your functions are inverses of each other.

$$f(g(x)) = f\left(\sqrt[3]{\frac{x-3}{2}}\right)$$
$$= 2\left(\cancel{\sqrt[3]{\frac{x-3}{2}}}\right)^{\cancel{3}} + 3$$
$$= \cancel{2}\left(\frac{x-3}{\cancel{2}}\right) + 3$$
$$= x - 3 + 3$$
$$= x$$

$$g(f(x)) = g(2x^3 + 3)$$

4. Use transformations to graph both functions and the line y=x on the same grid.



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