Math 111 — Tues, 5/10

Please turn in Bonus

Questions on 4.4

Finish page 6 of the 4.4 packet + 4.4 supplement

Section 4.5

Checkpoint 6 on Thurs (4.3, 4.4)

Project due on Thursday

Mission 3 (Individual) Handed out

multicultural Night Tonight 4:30-7:30 in the Great Hall

math 3D Printing - Need 4 helpers to work with high school students on a Saturday (May 14th or June 4th) 9am - noon [Gift card + bonus points]

4.4 29, 31

29. $\log_{1/2} 16$

$\left(\frac{1}{2}\right)^{?} = 16$

$2^{?} = 16$

$2^{4} = 16$

$\left(\frac{1}{2}\right)^{-4} = 16$

$\log_{1/2} 16 = -4$

31. $\log_{10} \sqrt{10}$

$10^{\frac{1}{2}} = \sqrt{10}$

$\log_{10} \sqrt{10} = \frac{1}{2}$

$\sqrt{9} = 9^{1/2}$

$\sqrt[3]{27} = 27^{1/3}$

roots are fractional powers

35. $\ln_e \sqrt{e}$

$\ln = \log_e$

$\ln_e \sqrt{e} = \frac{1}{2}$

log → common log
base 10

36. $\ln e^3 = 3$

$e^{?} = e^{3}$

Math 111 Lecture Notes

Section 4.5: Properties of Logarithms

Example 1. Calculate the following:

(a) $\log_5(1) = 0$

because

$5^0 = 1$

(c) $\log_2(1) = 0$

because

$2^0 = 1$

(e) $\log(1) = 0$

$10^0 = 1$

(g) $\ln(1) = 0$

$e^0 = 1$

(b) $\log_5(5) = 1$

because

$5^1 = 5$

(d) $\log_2(2) = 1$

$2^1 = 2$

(f) $\log(10) = 1$

$10^1 = 10$

(h) $\ln(e) = 1$

$e^1 = e$

For any positive real number a, $a \neq 1$, it holds that

- $\log_a(1) = 0$
- $\log_a(a) = 1$

Example 2. We have said that the functions defined by $\boxed{g(x) = \log_2(x)}$ and $\boxed{f(x) = 2^x}$ are inverse functions. Find $f(g(x))$ and $g(f(x))$. Since f and g are inverses, what should these be equivalent to?

$$f(g(x)) = f(\log_2 x) = 2^{\log_2 x} = x$$

$$g(f(x)) = g(2^x) = \log_2 2^x = x$$

For any positive real numbers x and a, $a \neq 1$, it holds that

- $\log_a(a^x) = x$
- $a^{\log_a(x)} = x$

Example 3. Compare the following expressions:

$$\log_2(8) + \log_2(4) \qquad \text{vs.} \qquad \log_2(32)$$

$3 + 2$ $\qquad$ 5

5

Example 4. Compare the following expressions:

$$\log_3(81) - \log_3(3) \qquad \text{vs.} \qquad \log_3(27)$$

$4 - 1$ $\qquad$ 3

3

Example 5. Compare the following expressions:

$$4\log(10) \qquad \text{vs.} \qquad \log_{10}(10{,}000)$$

$4 \cdot 1$ $\qquad$ 4 $\qquad$ $10^4 = 10{,}000$

4 $\qquad$ $\log_{10} 10^4$

$4 \log_{10} 10$

For any positive real numbers M, N, and a, $a \neq 1$, it holds that

- $\log_a(MN) = \log_a(M) + \log_a(N)$
- $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$
- $\log_a(M^r) = r\log_a(M)$

Example 6. Use the properties of logarithms to find the exact value of the following expressions. Do not use a calculator.

(a) $\log_4(4^{-5})$ $= -5$

property 3

(b) $\log_6(9) + \log_6(4)$

$= \log_6(36)$

$= 2$ property 5

(c) $e^{\ln(7)}$ property 4

$= 7$

(d) $2^{\log_2(15)}$ $= 15$

property 4

(e) $\log(250) - \log(25)$

$= \log\left(\frac{250}{25}\right)$ prop 6

$= \log(10)$

$= 1$ prop 2

(f) $5^{\log_5(6)+\log_5(7)}$

$= 5^{\log_5(42)}$ prop 5

$= 42$ prop 4

Example 7. Write each expression as a sum and/or difference of logarithms. Express powers as factors.

(a) $\log\left(\frac{1}{x-3}\right)$, $x > 3$

property 6

$= \log 1 - \log(x-3)$

$= 0 - \log(x-3)$

$= -\log(x-3)$

domain

$x - 3 > 0$

$+3 \quad +3$

$x > 3$

(b) $\ln\left(x^4\sqrt{1+x^2}\right)$

$= \ln x^4 + \ln\sqrt{1+x^2}$ prop 5

$= 4\ln x + \ln(1+x^2)^{\frac{1}{2}}$ prop 7

$= 4\ln x + \frac{1}{2}\ln(1+x^2)$

(c) $\log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right)$

$= \log_5\sqrt[3]{x^2+1} - \log_5(x^2-1)$ prop 6

$= \log_5(x^2+1)^{1/3} - \log_5((x+1)(x-1))$

$= \frac{1}{3}\log_5(x^2+1) - [\log_5(x+1) + \log_5(x-1)]$ prop 5

$= \frac{1}{3}\log_5(x^2+1) - \log_5(x+1) - \log_5(x-1)$

Example 8. Write each expression as a single logarithm.

(a) $\log_2\left(\frac{x-3}{x+5}\right) + \log_2\left(\frac{3x+15}{x-4}\right)$ property 5

$= \log_2\left(\frac{x-3}{x+5} \cdot \frac{3x+15}{x-4}\right)$

$= \log_2\left(\frac{x-3}{\cancel{x+5}} \cdot \frac{3\cancel{(x+5)}}{x-4}\right)$

$= \log_2\left(\frac{3(x-3)}{x-4}\right)$

(b) $\log_4\left(\frac{5}{x}\right) - \log_4\left(\frac{x+2}{x^3}\right)$ property 6

$= \log_4\left(\frac{5}{\cancel{x}} \cdot \frac{x^{\cancel{3}\,2}}{x+2}\right)$ mult by the reciprocal

$\log_4\left(\frac{\frac{5}{x}}{\frac{x+2}{x^3}}\right)$

$= \log_4\left(\frac{5x^2}{x+2}\right)$

(c) $\log(x^2+3x+2) - 2\log(x+1)$ property 7

$= \log(x^2+3x+2) - \log(x+1)^2$ prop 6

$= \log\left(\frac{x^2+3x+2}{(x+1)^2}\right)$

$= \log\left(\frac{\cancel{(x+1)}(x+2)}{(x+1)^{\cancel{2}}}\right)$

$= \log\left(\frac{x+2}{x+1}\right)$

(d) $\ln\left(x^2-9\right)+\ln\left(\frac{x}{x-3}\right)-\ln\left(\frac{x+3}{x}\right)$

$\underbrace{\ln((x+3)(x-3)) + \ln\left(\frac{x}{x-3}\right)} - \ln\left(\frac{x+3}{x}\right)$ prop 5

$\ln\left(\frac{x(x+3)\cancel{(x-3)}}{\cancel{x-3}}\right) - \ln\left(\frac{x+3}{x}\right)$

prop 6

$\ln\left(x\cancel{(x+3)} \cdot \frac{x}{\cancel{x+3}}\right)$

$\ln(x^2)$

$2\ln x$ prop 7

Change-of-Base Formula If a, b, and M are positive real numbers, $a \neq 1$ and $b \neq 1$, then

$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

In practice, we primarily use one of the following forms of this formula:

$$\log_a(M) = \frac{\log(M)}{\log(a)} \quad \text{or} \quad \log_a(M) = \frac{\ln(M)}{\ln(a)}$$

Example 9. Use the Change-of-Base formula to write the following logarithmic expressions in terms of the natural logarithmic function or common logarithmic function. Then approximate each in your calculator.

(a) $\log_4(15) = \frac{\log 15}{\log 4} \approx 1.953$

7 log (

or $\frac{\ln 15}{\ln 4} \approx 1.953$

(b) $\log_5\left(\frac{1}{7}\right) = \frac{\log(1/7)}{\log 5} \approx -1.209$

or $\frac{\ln(1/7)}{\ln(5)} \approx -1.209$

3. A population decreases at a rate of 13.2% per 5 years. Find the approximate value for the following:

a. 1-year factor of decay and 1-year rate of decay.

b. 5-year factor of decay and 5-year rate of decay.

c. 10-year factor of decay and 10-year rate of decay.

SUPPLEMENTAL PROBLEMS FOR §4.4

EXAMPLE: The graph of $f(x) = \log_a(x)$ is given in Figure 17. Find a. (Note that the points $(1, 0)$ and $(9, 2)$ are on the graph of f.)

(1,0) and (9,2)

$y = \log_a x$

$0 = \log_a 1$

$a^0 = 1$ ← (0,1) is on every log graph

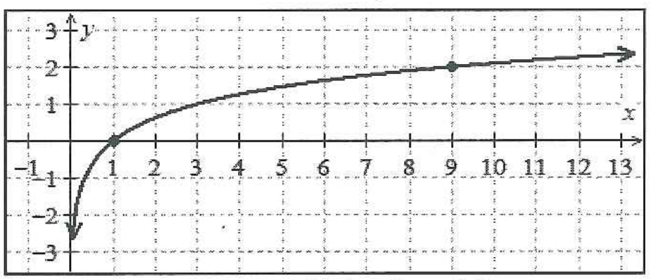

Figure 17: $f(x) = \log_a(x)$

$\log_3 x$

$2 = \log_a 9$

$a^2 = 9$ $a = 3$

Solution:

Since the function has form $f(x) = \log_a(x)$ and since the point $(9, 2)$ is on the graph, we know that $f(9) = 2$. Thus,

$$f(9) = 2$$

$$\Rightarrow \log_a(9) = 2 \quad \text{(since } f(9) = \log_a(9)\text{)}$$

$$\Rightarrow \quad a^2 = 9 \quad \text{(translate the logarithmic statement into an exponential one)}$$

$$\Rightarrow \quad a = 3 \quad \text{(take the positive square root of 9 because bases of logs are positive)}$$

Notice that we didn't attempt to use $(1, 0)$, the other obvious point on the graph of $f(x) = \log_a(x)$, to find a. Why not? (The point $(1, 0)$ is on the graph of *all* functions of the form $f(x) = \log_a(x)$ so it doesn't provide information that will help us find the particular function graphed here.)