Math 243 **Sections 3.1-3.2** **Introduction to Probability Lab**

Overview

- Outcomes, Events, Sample Space, Trials
- Probabilities and Complements (not)
- Theoretical vs. Empirical Probability
- The Law of Large Numbers (LLN)
- Disjoint "OR" Events and the Addition Rule
- Independent "AND" Events and the Multiplication Rule
- Conditional Probability

There are three activities to guide you through the language and concepts of probability. Read each section carefully and help each other.

Flipping a Coin

1. If you flip a coin, there are two possible **outcomes**.

Let event H = the outcome of heads. An **event** is a combination of outcomes.

Let event T = the outcome of tails.

2. The **sample space** is the set of all possible outcomes. List the sample space for flipping one coin.

S = { ______ , ______ }

3. For each event we can state the probability or chance that it will occur. The **theoretical probability** is based on the idea of a **fair** coin where both outcomes are equally likely. A probability can be written as a fraction or as a decimal between 0 and 1.

P(H) means the probability of getting heads. P(H) =

P(T) means the probability of getting tails. P(T) =

4. Get pennies for each group member and flip them 20 times. Record your results below:

Trial	1	2	3	4	5	6	7	8	9	10
Outcome										

Trial	11	12	13	14	15	16	17	18	19	20
Outcome										

5. “Heads” or event H, was the result what percentage of the time? This is an **empirical probability** because it is the result of observations or an experiment. Find 3 other people and write down their empirical probabilities.

Yours ____________ 3 Classmates ________________ ________________ ________________

6. How do your **empirical** and **theoretical** probabilities compare?

7. Is each flip of the coin **independent**? If knowing the result of one flip does not change the probability of the next flip they are **independent** events.

8. What would happen to the percentage of “heads” if you flipped the coin 100 times? 1,000 times? 1,000,000 times?

If you said your percentage of “heads” would get closer and closer to 50% you are right!

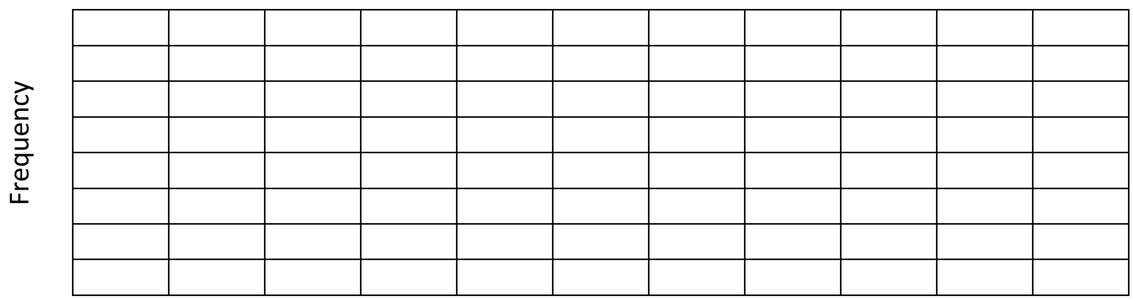
The Law of Large Numbers (LLN) states that the long-run relative frequency of repeated independent events gets closer and closer to the *true* relative frequency as the number of trials increases.

Rolling 2 Dice

1. If you roll two fair dice and add all the dots, what are the possible outcomes? List the sample space.

S =

2. Label the horizontal axis on the graph with the possible outcomes, one under each column. Roll two dice and add them together. Color in a space for that outcome, starting at the bottom. Repeat for a total of 36 rolls and make a histogram.



The sum of two dice

3. Look at the shape of your histogram. Did you expect that shape? Why do you think it has that shape?

4. You may have noticed that the outcomes from 2-12 are not equally likely. To find the theoretical probabilities we need to list all of the possible rolls of the dice. The rows in the table below represent the outcome of the first die and the columns represent the outcome of the second die. Write the sum for each roll in the corresponding space.

Second Die

First Die

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

5. Now it is easier to see why the histogram is not uniform. How many ways can you get a sum of two versus a sum of six? To find the theoretical probabilities, take the number of ways the event can occur and divide it by the total number of possibilities.

$$P(2) = \frac{\text{The number of ways to roll a sum of 2}}{\text{The total number of possible rolls}} = \underline{\hspace{3cm}}$$

Theoretical Probabilities

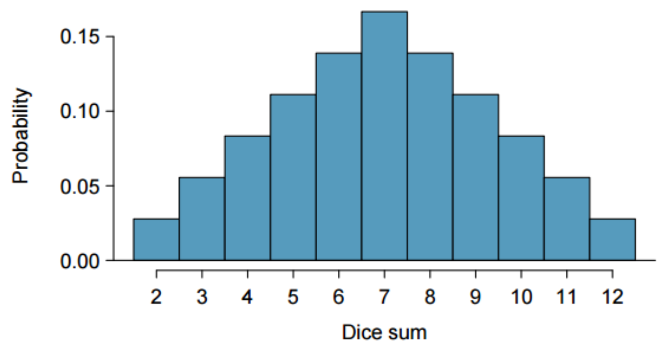
P(2) = P(3) = P(4) =

P(5) = P(6) = P(7) =

P(8) = P(9) = P(10) =

P(11) = P(12) =

This is a histogram of the theoretical probabilities. It is unimodal and symmetric.



"OR" Events

If A and B are disjoint events, **P(A or B) = P(A) + P(B)**

6. Using your theoretical probabilities, what is the probability of rolling a sum of 2 or 12? These are **disjoint** or **mutually exclusive** events because they cannot happen at the same time.

To calculate an "OR" probability for disjoint events, add the probabilities together.

P(2 or 12) = P(2) + P(12)

7. What is the probability that you would roll a sum of 6, 7 or 8?

8. What is the probability that you would roll an even sum?

Complements

9. What is the probability that you would **<u>not</u>** roll a sum of 3? (This is the **complement** of rolling a 3, denoted by 3-prime or 3′.)

P(3′) or P(not 3) =

10. A probability and its complement add up to what number?

P(3) + P(3′) =

Another way to say this is P(3′) = 1 – P(3)

"AND" Events

Two events are called **independent** if knowing that one occurs does not change the probability that the other occurs. Are successive rolls of the dice independent? If they are then we can multiply the probabilities together.

For independent events, P(A and B) = P(A)·P(B)

11. For successive rolls, what is the probability of rolling a sum of 3 and then a sum of 6?

P(3 and 6) = P(3)·P(6)

12. What is the probability of rolling a sum of 10 and then a sum of 12?

13. What is the probability of rolling snake-eyes 5 times in a row?

Drawing Chips

1. Let's say you have the following chips in a bag: 3 black, 4 red, 2 white, 1 green.

   Let B be the event of drawing a black chip
   Let R be the event of drawing a red chip
   Let W be the event of drawing a white chip
   Let G be the event of drawing a green chip

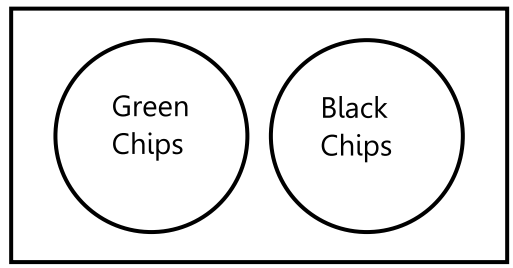
2. Write the theoretical probability of each event.

   P(B) =   P(R) =   P(W) =   P(G) =

Disjoint Events cannot occur at the same time or share no common outcomes (a chip cannot be green and black at the same time). They are **mutually exclusive**.

If A and B are disjoint events, add the probabilities.

P(A or B) = P(A) + P(B)



3. What is the probability of drawing a black or green chip?

   P(B or G) = P(B) + P(G) =

4. What is the probability of not drawing a green chip?

   P(G') =

Independent Events

Drawing with replacement. You are going to draw one chip and put it back each time before you draw another. Each draw is an independent event because you will be starting with the same chips each time.

Two events are called **independent** if knowing that one occurs does not change the probability that the other occurs. When you put the chips back each time the draws are independent.

For independent events, P(A and B) = P(A)·P(B)

5. For a series of 3 draws with replacement, what is the probability of getting

a. Three red chips?

P(3 red chips) = P(R)·P(R)·P(R) =

b. Three black chips?

Conditional Probabilities

A probability is **conditional** if it depends on knowing what has already happened. If you draw the chips without putting them back you are drawing **without replacement.** Now successive draws are not independent because the chips in the bag are different each time.

6. For a series of 3 draws without replacement, what is the probability of getting

a. Three red chips

b. Three black chips

These are conditional probabilities because when you keep the chip out it changes the distribution of chips in the bag. We write a conditional probability with a vertical line that means "**given**".

7. P(B|R) in this case means, "The probability of drawing a black chip **given** that you already drew a red chip"

P(B|R) =

We will look at conditional probabilities, independence, and events that are not disjoint in the next packet.

Concepts

The **complement** of any event A is the event that A does not occur. It's denoted by A′.

Two events are called **disjoint** if they have no outcomes in common and can never happen together.

Two events are called **independent** if knowing that one occurs does not change the probability that the other occurs.

A **conditional probability** is used when events are not independent. P(A|B) means the probability of A given B.

The "Law of Averages" is false. If you get a string of tails the next flip is no more likely to land on heads.

Probability Rules

1. $0 \leq P(A) \leq 1$, where P(sample space) = 1

2. $P(\text{not } A) = P(A') = 1 - P(A)$

3. $P(A \text{ or } B) = P(A) + P(B)$ if A and B are disjoint

4. $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

Practice Problem

Suppose that 40% of cars in your area are manufactured in the United States, 30% in Japan, 10% in Germany, and 20% in other countries. If cars are selected at random, find the probability that:

a. A car is not U.S.-made.

b. A car is made in Japan or Germany.

c. You select two cars in a row from Japan.

d. None of three cars came from Germany.

Answers: a. 0.6, b. 0.4, c. 0.09, d. 0.729