

# Math 111 Lecture Notes

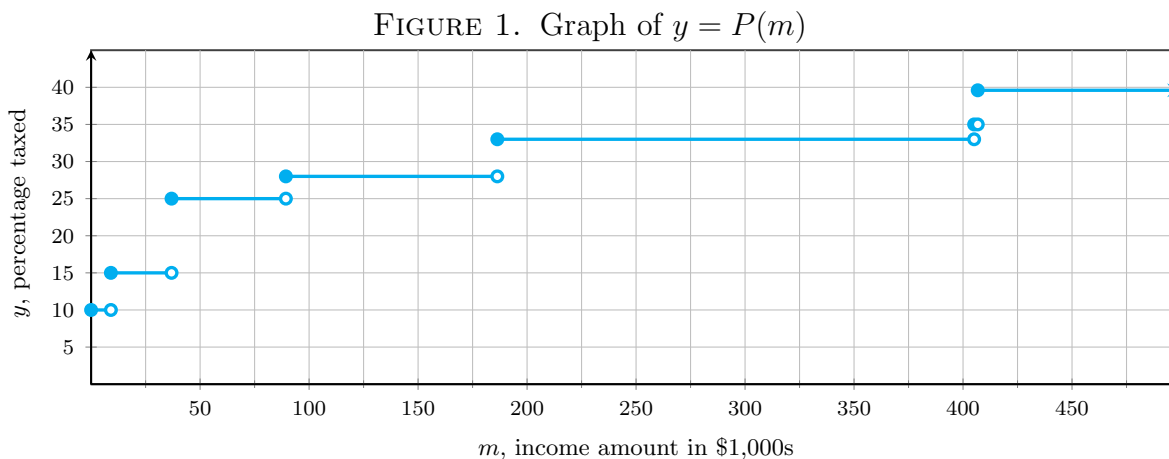
## SECTION 1.4: PIECEWISE-DEFINED FUNCTIONS

In Table 1, the 2014 federal income tax rates<sup>1</sup> for 2014 are shown.

TABLE 1. Federal Income Tax Percentage Rates for 2014 (Single Filing Status)

Income Amount ( $m$ )	Percentage of Income Taxed ( $P(m)$ , in %)
$0 \leq m < 9075$	10
$9075 \leq m < 36900$	15
$36900 \leq m < 89350$	25
$89350 \leq m < 186350$	28
$186350 \leq m < 405100$	33
$405100 \leq m < 406750$	35
$m \geq 406750$	39.6

Notice that for each interval, the percentage of income taxed as a function of income is *constant*. If we graph each *piece* over its respective interval, we obtain the following:



<sup>1</sup><http://taxfoundation.org/article/2014-tax-brackets>

A function that is defined by different formulas on different parts of its domain is a **piecewise-defined function**.

**Example 1.** Use the piecewise-defined function  $f$  defined below to answer the following.

$$f(x) = \begin{cases} \frac{3}{x-4} & \text{if } x \leq -2 \\ 7x - 8 & \text{if } -2 < x \leq 5 \\ -11 & \text{if } x > 5 \end{cases}$$

(a)  $f(0)$

(c)  $f(-6)$

(e)  $f(-2)$

(b)  $f(2)$

(d)  $f(8)$

(f)  $f(5)$

**Example 2.** As a prelude to graphing piecewise functions, let's graph just a few of the "pieces."

- Graph the linear function defined by  $f(x) = -x - 2$  for values of  $x$  where  $-4 < x \leq -1$ .
- Graph the constant function defined by  $f(x) = 3$  for values of  $x$  where  $2 < x < 4$ .
- Graph the linear function defined by  $f(x) = x^2$  for values of  $x$  where  $x \geq -1$ .

FIGURE 2

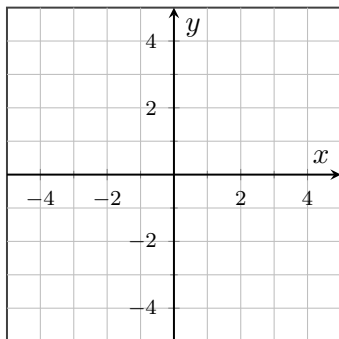


FIGURE 3

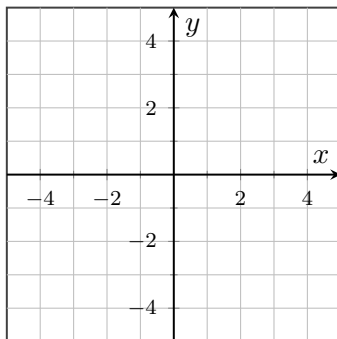
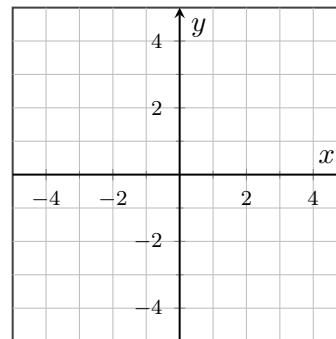
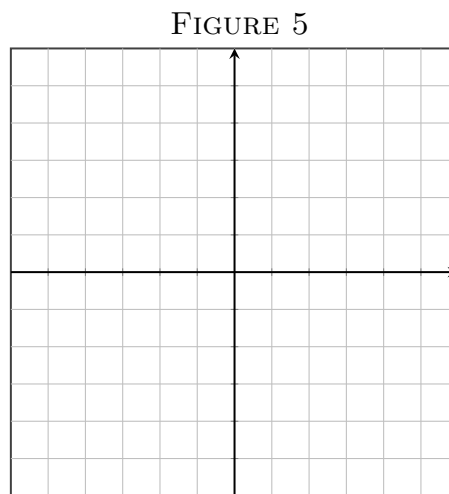


FIGURE 4



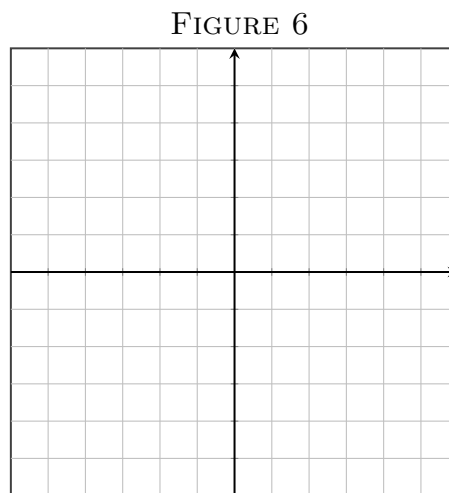
**Example 3.** Graph  $y = g(x)$  in Figure 5 for the piecewise-defined function  $g$  given below.

$$g(x) = \begin{cases} -3x - 2 & \text{if } x < -1 \\ 4 & \text{if } -1 \leq x < 2 \\ \frac{3}{2}x - 4 & \text{if } 2 < x \leq 4 \end{cases}$$

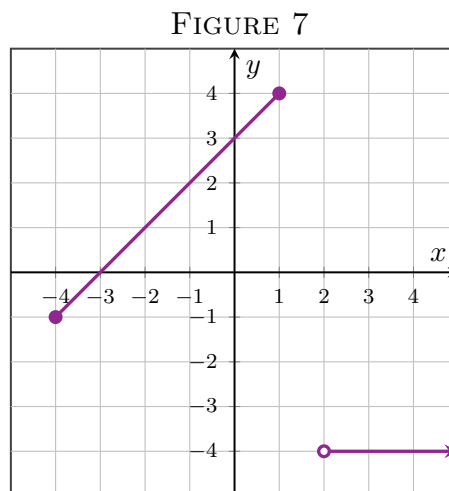


**Example 4.** Graph  $y = h(x)$  in Figure 6 for the piecewise-defined function  $h$  given below.

$$h(x) = \begin{cases} -x^2 + 4 & \text{if } -3 < x < 3 \\ -5 & \text{if } x \geq 4 \end{cases}$$



**Example 5.** Find the formula for the piecewise-defined function  $f$  graphed in Figure 7 below.



**Example 6.** The graph of a piecewise function  $g$  is graphed in Figure 8.

(a) State the domain and range of  $g$ .

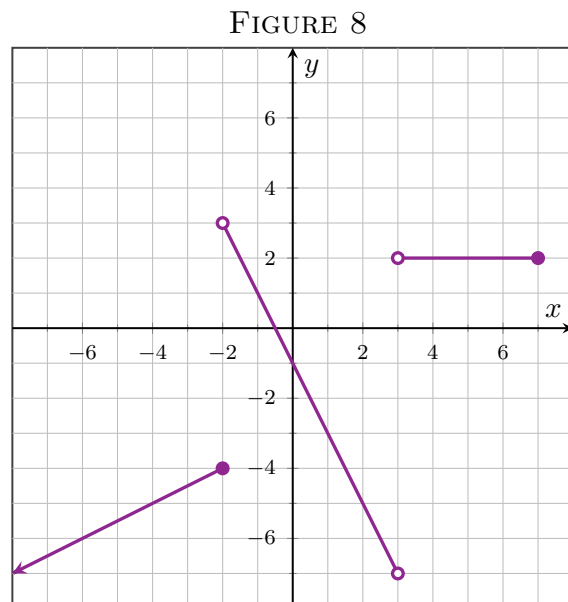
(b) Evaluate  $g(6)$ .

(c) Evaluate  $g(-2)$ .

(d) Solve  $g(x) = -3$ .

(e) Solve  $g(x) = -5$ .

(f) Write the formula for the function  $g$ .

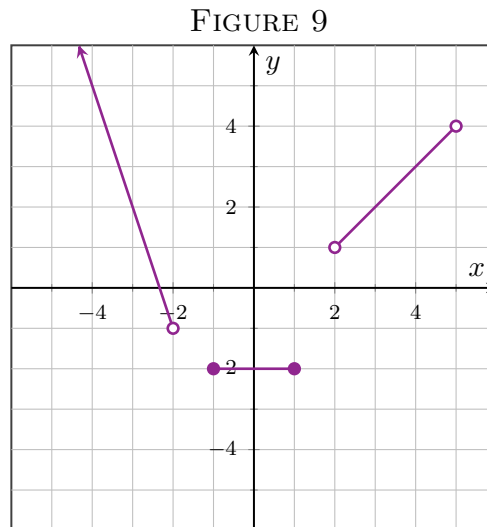


**Group Work 1.** The graph of the piecewise-defined function  $f$  is shown in Figure 9.

(a) Find the formula for this function.

(b) Find  $f(1)$ .

(c) Solve  $f(x) = 2$ .



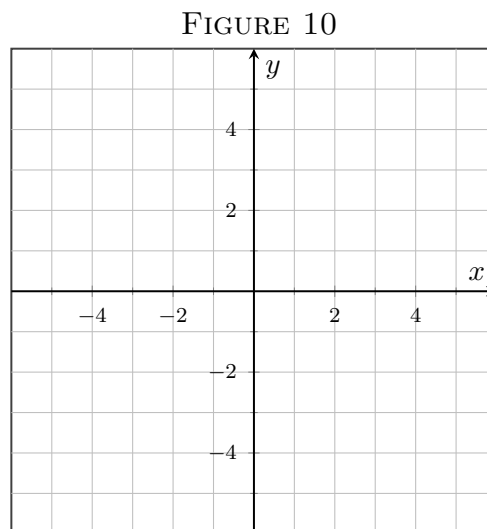
**Group Work 2.** Graph the function  $h$  defined below and then complete the following.

$$h(x) = \begin{cases} x^2 & \text{if } -2 \leq x < 1 \\ 3 & \text{if } 1 \leq x < 3 \\ -\frac{3}{2}(x - 5) & \text{if } 3 \leq x \leq 5 \end{cases}$$

(a) State the domain and range of  $h$ .

(b) State any horizontal and vertical intercepts.

(c) State the absolute maximum of  $h$  and where it occurs.



**Group Work 3.** When calculating your electricity bill, PGE uses the follows rates: It costs 5.124 cents per kWh for the first 250 kWh used in a month. After the first 250 kWh, it costs 6.899 cents for each additional kWh used. Let  $C(x)$  represent the monthly amount due (in dollars) for a PGE residential electricity bill where  $x$  kWh of energy were used that month.

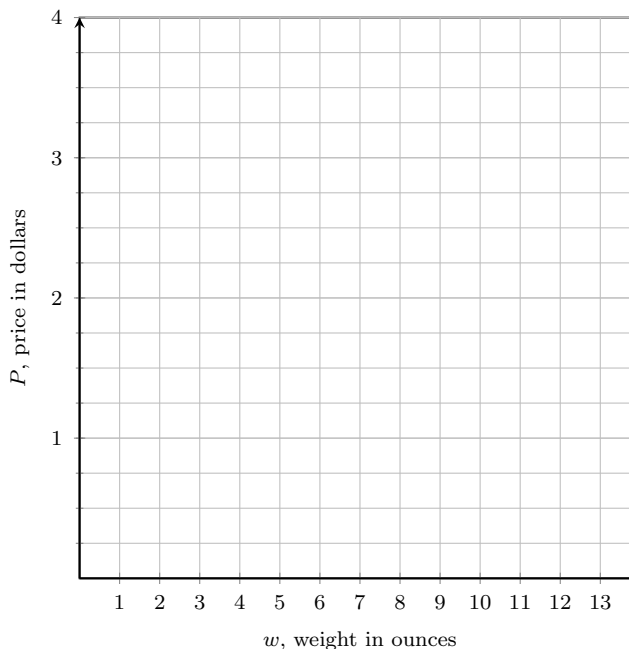
- (a) Write the formula for the piecewise-defined function  $C$ .
- (b) Use that formula to determine the amount due (before taxes and other fees) when you use 325 kWh of electricity in a month.

**Group Work 4.** The US Postal Service rates for large envelopes are given in Table 2, according to their weight.<sup>2</sup> Graph the cost  $P$  (in dollars) of mailing a large envelope as a function of the weight  $w$  (in ounces) in Figure 11.

TABLE 2. US Postal Service First-Class Mail Prices, Large Envelopes

Weight Not Over (in oz.)	Price (in \$)
1	0.90
2	1.10
3	1.30
4	1.50
5	1.70
6	1.90
7	2.10
8	2.30
9	2.50
10	2.70
11	2.90
12	3.10
13	3.30

FIGURE 11. US Postal Service First-Class Mail Prices for Large Envelopes



<sup>2</sup><http://pe.usps.com/cpim/ftp/manuals/dmm300/Notice123.pdf>