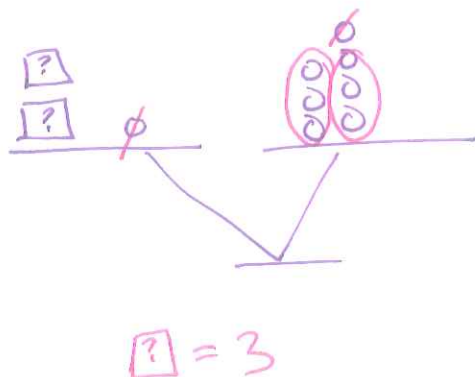


Section 3.1 Multistep Linear Equations and Inequalities

Multistep Equations with the Variable on One Side

1. First we will draw a scale to represent a two-step equation. Then solve and check the equation.

$$2x + 1 = 7$$



$$\begin{array}{r} 2x + 1 = 7 \\ -1 \quad -1 \\ \hline 2x = 6 \\ \frac{2x}{2} = \frac{6}{2} \\ x = 3 \end{array}$$

check: $2(3) + 1 \stackrel{?}{=} 7$
 $6 + 1 = 7$
 $7 = 7 \checkmark$

$\{3\}$
 $\{x | x = 3\}$

Undoing Operations in the Reverse Order

2. Draw a box around the variable. Then draw boxes outward following the order of operations. Solve and check each equation.

a. $7x - 5 = 37$

$$\begin{array}{r} +5 \quad +5 \\ \hline 7x = 42 \\ \frac{7x}{7} = \frac{42}{7} \\ x = 6 \end{array}$$

check: $7(6) - 5 \stackrel{?}{=} 37$
 $42 - 5 = 37 \checkmark$

$\{6\}$

b. $27 = -2x + 5$

$$\begin{array}{r} -5 \quad -5 \\ \hline 22 = -2x \\ \frac{22}{-2} = \frac{-2x}{-2} \\ -11 = x \end{array}$$

check: $27 \stackrel{?}{=} -2(-11) + 5$
 $27 = 22 + 5$
 $27 = 27 \checkmark$

$\{-11\}$

3. Solve the following equations, showing your operations. State the solution set and check your answer.

a. $3y + 10 = 40$
 $-10 \quad -10$

$$\frac{3y}{3} = \frac{30}{3}$$

$$y = 10 \quad \{10\}$$

$$3(10) + 10 \stackrel{?}{=} 40$$

$$30 + 10 = 40 \checkmark$$

b. $5 = 6z - 2$
 $+2 \quad +2$

$$\frac{7}{6} = \frac{6z}{6}$$

$$\frac{7}{6} = z$$

or

$$z = \frac{7}{6} \quad \left\{ \frac{7}{6} \right\}$$

$$5 \stackrel{?}{=} \frac{6}{1} \left(\frac{7}{6} \right) - 2$$

$$5 = 7 - 2 \checkmark$$

c. $-2t + 4 = -14$
 $-4 \quad -4$

$$\frac{-2t}{-2} = \frac{-18}{-2}$$

$$t = 9 \quad \{9\}$$

$$-2(9) + 4 \stackrel{?}{=} -14$$

$$-18 + 4 = -14 \checkmark$$

d. $\frac{1}{2}x + 7 = 15$
 $-7 \quad -7$

$$\frac{1}{1} \cdot \frac{1}{2}x = 8 \cdot \frac{2}{1}$$

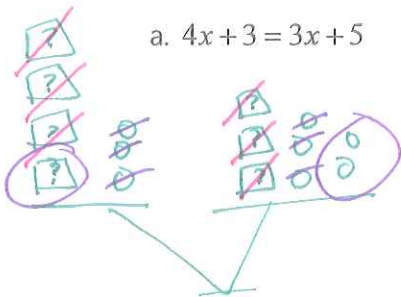
$$x = 16 \quad \{16\}$$

$$\frac{1}{2}(16) + 7 \stackrel{?}{=} 15$$

$$8 + 7 = 15 \checkmark$$

Equations with a Variable on Both Sides

4. Solve the following equations, showing your operations. State the solution set and check your answer.



a. $4x + 3 = 3x + 5$

$$4x + 3 = 3x + 5$$

$$\frac{-3x \quad -3x}{x + 3 = 5}$$

$$\frac{-3 \quad -3}{x = 2 \quad \{2\}}$$

$$4(2) + 3 \stackrel{?}{=} 3(2) + 5$$

$$8 + 3 \stackrel{?}{=} 6 + 5 \checkmark$$

b. $-4(k + 5) = -6k + 2$

$$\frac{-4k - 20 = -6k + 2}{+6k \quad +6k}$$

$$\frac{2k - 20 = 2}{+20 \quad +20}$$

$$\frac{2k = 22}{2 \quad 2}$$

$$k = 11 \quad \{11\}$$

$$-4(11 + 5) \stackrel{?}{=} -6(11) + 2$$

$$-4(16) \stackrel{?}{=} -66 + 2$$

$$-64 \stackrel{?}{=} -64 \checkmark$$

5. Solve the following equations, showing your operations. State the solution set and check your answer.

a. $11k + 8 = 10k - 7$
 $-10k \quad -10k$
 $k + 8 = -7$
 $-8 \quad -8$
 $k = -15$
 $\{-15\}$

b. $9x - 3 = 8x + 10$
 $-8x \quad -8x$
 $x - 3 = 10$
 $+3 \quad +3$
 $x = 13 \quad \{13\}$

Simplify first

c. $m - 3 - 3m = -6 - 6m + 31$
 $-2m - 3 = -6m + 25$
 $+6m \quad +6m$
 $4m - 3 = 25$
 $+3 \quad +3$
 $4m = 28$
 $\frac{4m}{4} = \frac{28}{4} \quad m = 7 \quad \{7\}$

d. $3(x - 2) + 4 = -6x + 7$
 $3x - 6 + 4 = -6x + 7$
 $3x - 2 = -6x + 7$
 $+6x \quad +6x$
 $9x - 2 = 7$
 $+2 \quad +2$
 $\frac{9x}{9} = \frac{9}{9} \quad x = 1 \quad \{1\}$

6. In our Section 2.6 notes we wrote an equation for the Mad Genius Escape Room on Hawthorne. The cost is \$30 per person if you don't mind being with other guests. If you want the room to be private for your party the cost is \$27 plus \$24 per person. How many people would you need to get a private room at no extra cost?

Now we can solve this equation algebraically. State your answer in a complete sentence.

$$30p = 27 + 24p$$

$$-24p \quad -24p$$

$$\frac{6p}{6} = \frac{27}{6}$$

$$p = 4.5 \uparrow 5$$

round up to a whole person

You would need 5 people to have the private room but 4 might be close enough.

7. In our Section 2.6 notes we also wrote an inequality for MetroMile insurance to represent the number of miles we would need to drive to spend less than a plan that is \$800 per year.

Now we can solve this inequality algebraically. State your answer in a complete sentence.

$$360 + 0.032m \leq 800$$

$$-360 \quad -360$$

$$\frac{.032m}{.032} \leq \frac{440}{.032}$$

$$m \leq 13,750 \text{ miles/year}$$

If you drive less than or equal to 13,750 miles per year, MetroMile would be cheaper

Solving Multistep Inequalities:

8. Solve each inequality and graph each solution on a number line. Write the solution set in interval and set-builder notation.

** when you multiply or divide by a negative, flip the symbol*

Solve the Inequality	Number Line Graph	Interval	Set-builder Notation
<p>a. $-3x + 5 > 11$</p> $\begin{array}{r} -3x + 5 > 11 \\ -5 \quad -5 \\ \hline -3x > 6 \\ \frac{-3x}{-3} > \frac{6}{-3} \\ x < -2 \end{array}$ <p><i>* put variable on the left</i></p>		$(-\infty, -2)$	$\{x x < -2\}$
<p>b. $8t - 7 \leq 3t - 2$</p> $\begin{array}{r} 8t - 7 \leq 3t - 2 \\ -3t \quad -3t \\ \hline 5t - 7 \leq -2 \\ +7 \quad +7 \\ \hline 5t \leq 5 \\ \frac{5t}{5} \leq \frac{5}{5} \\ t \leq 1 \end{array}$		$(-\infty, 1]$	$\{t t \leq 1\}$
<p>c. $-5x - 9 > 2(x - 3)$</p> $\begin{array}{r} -5x - 9 > 2x - 6 \\ -2x \quad -2x \\ \hline -7x - 9 > -6 \\ +9 \quad +9 \\ \hline -7x > +3 \\ \frac{-7x}{-7} > \frac{+3}{-7} \quad x < -\frac{3}{7} \end{array}$		$(-\infty, -3/7)$	$\{x x < -\frac{3}{7}\}$
<p>d. $-4(8 - y) \geq 7(y - 2) - y$</p> $\begin{array}{r} -32 + 4y \geq 7y - 14 - y \\ -32 + 4y \geq 6y - 14 \\ -6y \quad -6y \\ \hline -32 - 2y \geq -14 \\ +32 \quad +32 \\ \hline -2y \geq 18 \\ \frac{-2y}{-2} \geq \frac{18}{-2} \\ y \leq -9 \end{array}$		$(-\infty, -9]$	$\{y y \leq -9\}$

Writing and Solving Equations and Inequalities

9. Write and solve an equation in each scenario given. Give your answer as a complete sentence, including units.

a. A school purchased boxes of pens from an office supply company. Each box was \$2 and they gave the school a \$16 rebate. ^{one-time coupon} If the school spent \$380, how many boxes of pens did they buy?

Let $x = \# \text{ of boxes}$

$$\begin{array}{r} 2x - 16 = 380 \\ + 16 \quad + 16 \end{array}$$

$$\begin{array}{r} 2x = 396 \\ \underline{\quad} \quad \underline{\quad} \\ x = 198 \end{array}$$

The school purchased 198 boxes.

b. Vien and Terry were reading the Everybody Reads Book for the year. Vien read 20 fewer pages than Terry did. Together they read 300 pages. How many pages did Terry read?

Let $T = \# \text{ pages that Terry read}$
 $V = \# \text{ pages that Vien read}$

$$V = T - 20$$

$$T + V = 300$$

$$T + (T - 20) = 300$$

$$\begin{array}{r} 2T - 20 = 300 \\ + 20 \quad + 20 \end{array}$$

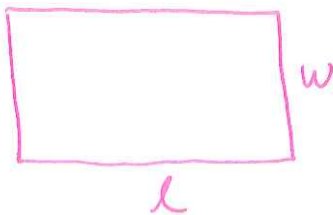
$$2T = 320$$

$$\begin{array}{r} 2T = 320 \\ \underline{\quad} \quad \underline{\quad} \\ T = 160 \end{array}$$

$$T = 160$$

Terry read 160 pages

c. A rectangle's perimeter is 252 feet. Its length is 2 feet shorter than 3 times the width. Draw a picture and use an equation to find the rectangle's length and width.



$$l = 3w - 2$$

$$P = 2l + 2w$$

$$252 = 2(3w - 2) + 2w$$

$$252 = 6w - 4 + 2w$$

$$\begin{array}{r} 252 = 8w - 4 \\ + 4 \quad \quad + 4 \end{array}$$

$$\begin{array}{r} 256 = 8w \\ \underline{\quad} \quad \underline{\quad} \\ 32 = w \end{array}$$

The rectangle is 32 by 94 feet.

$$\begin{array}{l} w = 32 \text{ feet} \\ l = 3(32) - 2 \\ = 96 - 2 \\ = 94 \text{ feet} \end{array}$$

More Practice

10. Solve and check each equation and write the solution set.

a. $-10 - 5x = x + 26$
 $-x - x$
 $-10 - 6x = 26$
 $-10 - 5(-6) = -6 + 26$
 $-10 + 30 = 20$
 $20 = 20 \checkmark$

$-10 - 6x = 26$
 $+10 \quad +10$
 $-6x = 36$
 $\frac{-6x}{-6} = \frac{36}{-6}$
 $x = -6$
 $\{-6\}$

b. $\frac{3}{4}x + 2 = 20$
 $-2 -2$

$\frac{3}{4}x = 18$
 $\frac{3}{4}x = 18 \cdot \frac{4}{3}$

$x = 24$
 $\{24\}$

$\frac{3}{4}(24) + 2 = 20$
 $18 + 2 = 20 \checkmark$

c. $5(x+2) - 8x = 6(x-1) + 5$
check on separate page

$5x + 10 - 8x = 6x - 6 + 5$
 $-3x + 10 = 6x - 1$
 $-6x \quad -6x$
 $-9x + 10 = -1$
 $-10 \quad -10$
 $-9x = -11$
 $\frac{-9x}{-9} = \frac{-11}{-9} \quad x = \frac{11}{9}$
 $\{\frac{11}{9}\}$

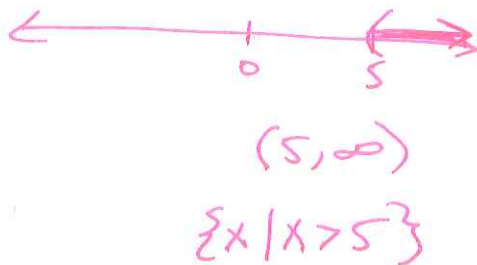
d. $(t-4) - (t-9) = 5t$
 $(1-4) - (1-9) = 5(1)$
 $-3 - (-8) = 5$
 $5 = 5 \checkmark$

$t - 4 - t + 9 = 5t$
 $5 = 5t$
 $\frac{5}{5} = \frac{5t}{5}$
 $1 = t$
 $\{1\}$

11. Solve each inequality and draw the solution set on a number line. Write the solution set in interval and set-builder notation.

a. $-6x + 18 < -12$
 $-18 \quad -18$

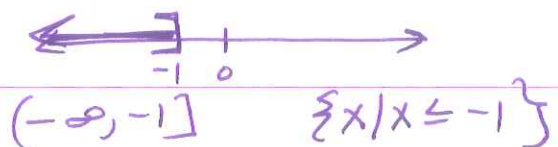
$-6x < -30$
 $\frac{-6x}{-6} < \frac{-30}{-6}$
 $x > 5$



b. $46 \leq 1 - 5(z-8)$

$46 \leq 1 - 5z + 40$
 $46 \leq -5z + 41$
 $-41 \quad -41$
 $\frac{5}{-5} \leq \frac{-5z}{-5}$

$-1 \geq z$
 $z \leq -1$
Put the variable on the left



10c. check:

$$5\left(\frac{11}{9}+2\right)-8\left(\frac{11}{9}\right) \stackrel{?}{=} 6\left(\frac{11}{9}-1\right)+5$$

$$\frac{55}{9}+10-\frac{88}{9} \stackrel{?}{=} \frac{66}{9}-6+5$$

$$-\frac{33}{9}+10\frac{1}{9} \stackrel{?}{=} \frac{66}{9}-1\frac{1}{9}$$

$$-\frac{33}{9}+\frac{90}{9} \stackrel{?}{=} \frac{66}{9}-\frac{9}{9}$$

$$\frac{57}{9} = \frac{57}{9} \checkmark$$