

Section 3.5 Special Solution Sets

Equations with Special Solution Sets

1. Solve each equation.

a. $3x + 7 - x = 2x - 11$

$$\begin{array}{r} \cancel{2x} + 7 = \cancel{2x} - 11 \\ -\cancel{2x} \quad -\cancel{2x} \\ \hline 7 = -11 \end{array}$$

No solution ← webwork

 \emptyset Null set $\{ \}$

b. $2x + 5 + x = 3(x - 1) + 8$

$3x + 5 = 3x - 3 + 8$

$$\begin{array}{r} 3x + 5 = 3x + 5 \\ -\cancel{3x} \quad -\cancel{3x} \\ \hline \end{array}$$

$$\begin{array}{r} 5 = 5 \\ -5 \quad -5 \\ \hline 0 = 0 \end{array}$$

Simplify each side first

All real numbers ← webwork

 \mathbb{R} $\{x \mid x \text{ is a real number}\}$ $(-\infty, \infty)$

c. $5x + 1 = 6x + 1$

$$\begin{array}{r} + 1 = + 1 \\ -1 \quad -1 \\ \hline 5x = 6x \\ -5x \quad -5x \\ \hline 0 = x \end{array}$$

 $\{0\}$

check

$5(0) + 1 \stackrel{?}{=} 6(0) + 1$

$0 + 1 = 0 + 1 \checkmark$

An equation or inequality is inconsistent if it is not true for any real number. In this case we write

- No solution
- the empty set: \emptyset , or
- empty brackets: $\{ \}$

An equation or inequality is an identity if it is true for all real numbers. In this case we write

- All real numbers
- \mathbb{R}
- $\{x \mid x \text{ is a real number}\}$

2. Write the solution for each equation or inequality.

a. $x = x$

\mathbb{R}
all real numbers

b. $x > x$

\emptyset
no solution

c. $0 = 4$

no solution

d. $2 \leq 4$

all real numbers

e. $2x = 2x + 1$

$$\begin{array}{r} -2x \quad -2x \\ \hline 0 = 1 \end{array}$$

no solution

f. $x \geq x$

all real numbers

g. $2x - 7 > 2(x - 5) + 3$

$$2x - 7 > 2x - 10 + 3$$

$$2x - 7 > 2x - 7$$

$$\begin{array}{r} -2x \quad -2x \\ \hline -7 > -7 \end{array}$$

$$-7 > -7$$

no solution

h. $-2(x + 3) = 4x - 6(x + 1)$

$$-2x - 6 = 4x - 6x - 6$$

$$-2x - 6 = -2x - 6$$

all real numbers

3. Emil and Julia are saving money in their accounts. Emil starts with \$100 and saves \$60 per month. Julia starts with \$200 and saves \$60 per month. To determine when they will have the same amount in savings, let x be the number of months they save and solve the equation:

$$100 + 60x = 200 + 60x$$

$$\begin{array}{r} 100 + 60x = 200 + 60x \\ -60x \quad -60x \\ \hline 100 = 200 \end{array} \quad \text{No solution}$$

Section 3.4 Ratios and Proportions

A rate or ratio compares two quantities using division.

A proportion states that two rates or ratios are equal.

4. Solve the following proportions. Check each solution and write the solution set.

a. $\frac{x+2}{5} = \frac{3}{4}$

LCD = 20

$$4 \cdot \frac{x+2}{5} = 5 \cdot \frac{3}{4}$$

$$4x + 8 = 15$$

$$\frac{4x}{4} = \frac{7}{4} \quad x = \frac{7}{4}$$

LCD = 12 c. $\frac{x+3}{4} = \frac{7}{12}$

$$12 \cdot \frac{x+3}{4} = 12 \cdot \frac{7}{12}$$

$$3x + 9 = 7$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$\left\{ -\frac{2}{3} \right\}$$

$$\frac{-\frac{2}{3} + 3 \cdot \frac{2}{3}}{4} = \frac{7}{12}$$

$$\frac{-\frac{2}{3} + 9}{4} = \frac{7}{12}$$

$$\frac{7}{3} \div \frac{4}{1} = \frac{7}{12}$$

$$\frac{7}{3} \cdot \frac{1}{4} = \frac{7}{12} \checkmark$$

b. $\frac{10}{y-1} = \frac{16}{y+2}$

LCD = $(y-1)(y+2)$

$$(y-1)(y+2) \cdot \frac{10}{y-1} = (y-1)(y+2) \cdot \frac{16}{y+2}$$

$$10(y+2) = 16(y-1)$$

$$10y + 20 = 16y - 16$$

$$-6y + 20 = -16$$

$$-6y = -36$$

$$y = 6 \quad \{6\}$$

d. $\frac{z}{9} = \frac{z-1}{6}$

LCD = 18

$$18 \cdot \frac{z}{9} = 18 \cdot \frac{z-1}{6}$$

$$2z = 3z - 3$$

$$-3z = -3$$

$$z = 1$$

$$z = 3$$

$$\{3\}$$

$$\frac{3}{9} = \frac{3-1}{6}$$

$$\frac{1}{3} = \frac{2}{6} \checkmark$$

Beyond math 60

4a. ~~$\frac{x+2}{5} = \frac{3}{4}$~~

~~$\frac{1}{2} = \frac{3}{4}$~~

$4(x+2) = 3 \cdot 5$

$4x + 8 = 15$

check:

$\frac{\frac{7}{4} + 2 \cdot \frac{1}{4}}{5} \stackrel{?}{=} \frac{3}{4}$

$\frac{\frac{7}{4} + \frac{8}{4}}{5} \stackrel{?}{=} \frac{3}{4}$

$\frac{\frac{15}{4}}{5} \div \frac{5}{1} \stackrel{?}{=} \frac{3}{4}$

$\frac{\frac{3}{4}}{1} \cdot \frac{1}{5} \stackrel{?}{=} \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4} \checkmark$

b. check:

$\frac{10}{y-1} \stackrel{?}{=} \frac{16}{y+2}$

$\frac{10}{6-1} \stackrel{?}{=} \frac{16}{6+2}$

$\frac{10}{5} = \frac{16}{8}$
 $2 = 2 \checkmark$

5. A car is driving 60 mph. Set up and solve a proportion to find out how long will it take the car to travel 132 miles? Remember to define your variable.

Let $t =$ time in hours

$$\frac{60 \text{ miles}}{1 \text{ hour}} = \frac{132 \text{ miles}}{t}$$

$$\frac{60t}{60} = \frac{132(1)}{60}$$

$$t = 2.2 \text{ hours}$$

It would take 2.2 hours
or 2 hours and 12 minutes.

Similar triangles have the same angles and their sides are proportional.

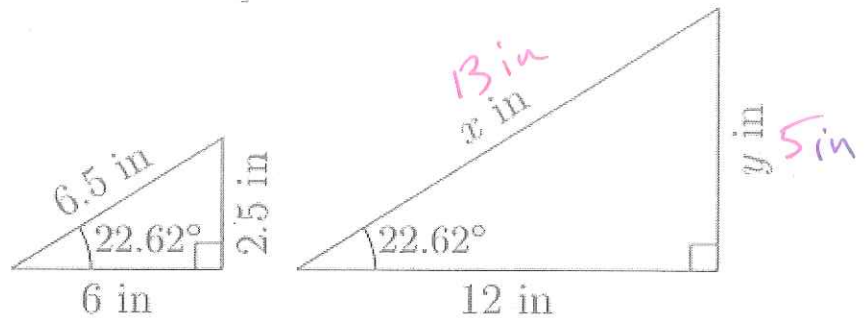
6. Write and solve proportions for the similar triangles to find x and y .

~~$$\frac{x}{6.5} = \frac{12}{6}$$~~

$$6x = 12(6.5)$$

$$\frac{6x}{6} = \frac{78}{6}$$

$$x = 13 \text{ in}$$



~~$$\frac{y}{2.5} = \frac{12}{6}$$~~

$$6y = 12(2.5)$$

$$\frac{6y}{6} = \frac{30}{6}$$

$$y = 5 \text{ in}$$

7. An 8-ounce can of soda contains 36 milligrams of caffeine. How much caffeine is contained in a 20-ounce bottle?

~~$$\frac{8 \text{ oz}}{36 \text{ mg}} = \frac{20 \text{ oz}}{c \text{ mg}}$$~~

$$8c = 20(36)$$

$$\frac{8c}{8} = \frac{720}{8}$$

$$c = 90 \text{ mg}$$

There are
90 mg of
caffeine.

More Practice

8. The annual property taxes for a house assessed at \$250,000 were \$2,463. Find the property taxes for a house assessed at \$325,000.

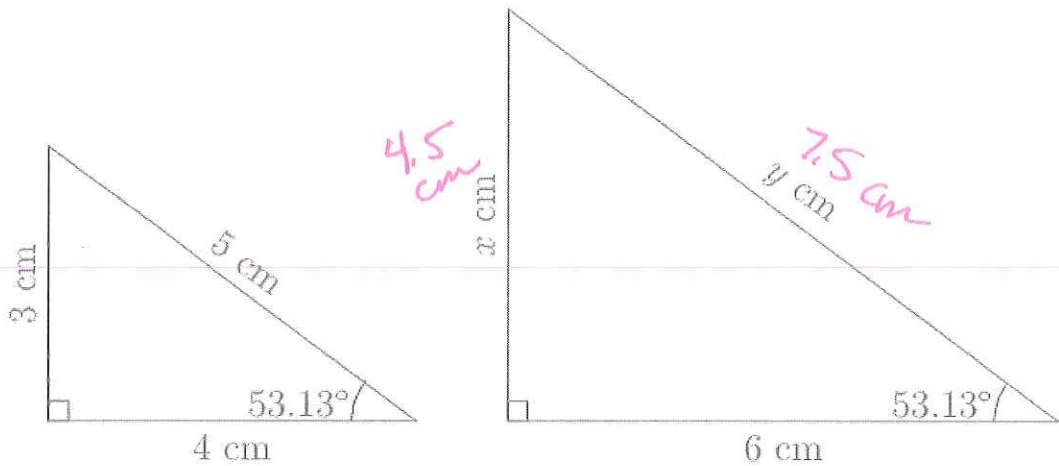
$$\frac{\text{value } \$250,000}{\$325,000} = \frac{\text{tax } \$2,463}{x}$$

$$\frac{250,000x}{250,000} = \frac{325,000(2,463)}{250,000}$$

$$x = \$3,201.90$$

The property taxes are \$3,201.90

9. Write and solve proportions to find the value of x and y.



$$\frac{x}{3} = \frac{6}{4} \quad \text{LCD} = 12$$

$$4 \cdot \frac{x}{3} = 4 \cdot \frac{6}{4}$$

$$\frac{4x}{4} = \frac{18}{4}$$

$$x = \frac{9}{2} \text{ or } 4.5 \text{ cm}$$

$$\frac{y}{5} = \frac{6}{4} \quad \text{LCD} = 20$$

$$4 \cdot \frac{y}{5} = 4 \cdot \frac{6}{4}$$

$$\frac{4y}{4} = \frac{30}{4}$$

$$y = \frac{15}{2} \text{ or } 7.5 \text{ cm}$$

10. Solve each equation below and state the solution set.

a. $5(t+3) \geq 2(t-3) + 3(t-7)$

$$5t + 15 \geq \underline{2t - 6} + \underline{3t - 21}$$

$$\begin{array}{r} 5t + 15 \geq 5t - 27 \\ -5t \quad -5t \\ \hline \end{array}$$

$$15 \geq -27$$

All real numbers

b. $5(2y-1) - 5 = 12y - 2(y+3)$

$$10y - \underline{5} - \underline{5} = \underline{12y} - \underline{2y} - 3$$

$$\begin{array}{r} 10y \quad = 10y - 3 \\ -10y \quad -10y \\ \hline 0 = -3 \end{array}$$

No solution