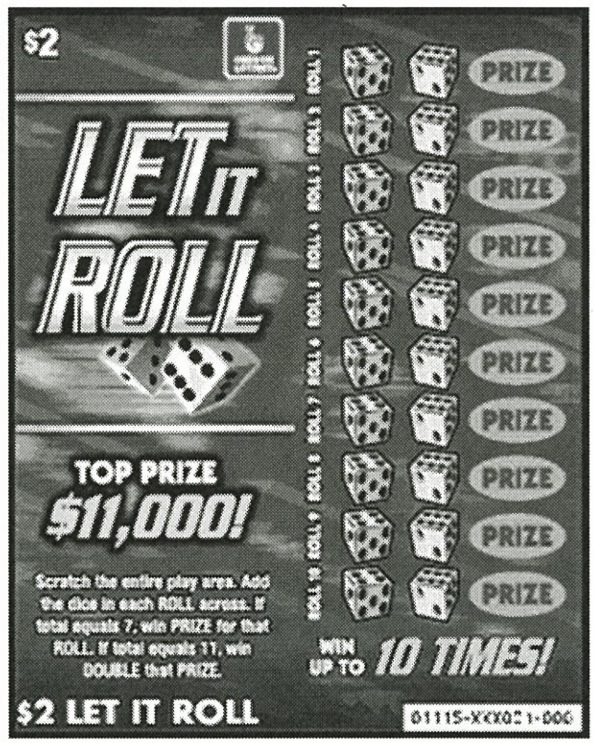
Overview

- Discrete vs. Continuous Random Variables
- Probability Distribution
- Calculating Expected Value
- Calculating Variance and Standard Deviation



The lottery should be played for entertainment only and not investment purposes!

On the back of this ticket it says:

Overall odds of winning: 1 in 3.84

Prize Payout: 64.00%

Random Variables:

A **random variable** is a variable whose value is determined by the outcome of a random event.

Example: Winnings from a lottery ticket

Discrete random variable:

Only certain values are possible (finite number)

Continuous random variable:

Can be any value in a range (0, $11,000)

Expected Value: "average"

long-run amount we would expect to win per ticket. (expected winnings)

expected profit/loss

Probability distribution: The collection of all possible values and the probabilities that they will occur. For a discrete random variable these can be listed in a table.

Image Source: http://www.oregonlottery.org/

Discrete Random Variables

Example 1. Making a Probability Model

$\frac{1}{280,000} = \boxed{\frac{3}{840,000}}$

Here is the prize chart for the same lottery ticket. http://www.oregonlottery.org/games/scratch-its/details/1115

Win	No. of Wins	Prize	Odds	Total No. of Prizes	Total of Prizes
$11,000	1	$ 11,000	1: 280,000.00	3	$ 33,000
$500	1	$ 500	1: 420,000.00	2	$ 1,000
$500 ($50 x 10)	10	$ 500	1: 280,000.00	3 10	$ 1,500
$500 ($250"D")	1	$ 500	1: 168,000.00	5	$ 2,500
$250	1	$ 250	1: 42,000.00	20	$ 5,000
$100	1	$ 100	1: 84,000.00	10	$ 1,000
$100 ($10 + $20x2 + $50)	4	$ 100	1: 56,000.00	15	$ 1,500
$100 ($10x10)	10	$ 100	1: 56,000.00	15	$ 1,500
100 ($50"D")	1	$ 100	1: 28,000.00	30	$ 3,000
$50	1	$ 50	1: 4,200.00	200	$ 10,000
$50 ($5x10)	10	$ 50	1: 2,333.33	360	$ 18,000
$50 ($5x2 + $10x4)	6	$ 50	1: 2,736.16	307	$ 15,350
$30	1	$ 30	1: 1,680.00	500	$ 15,000
$30 ($2x5 + $5x4)	9	$ 30	1: 982.46	855	$ 25,650
$30 ($5 + $10 + $15)	3	$ 30	1: 982.46	855	$ 25,650
$30 ($15"D")	1	$ 30	1: 600.00	1,400	$ 42,000
$15	1	$ 15	1: 400.00	2,100	$ 31,500
$15 ($5 x 3)	3	$ 15	1: 200.00	4,200	$ 63,000
$10	1	$ 10	1: 400.00	2,100	$ 21,000
$10 ($5"D")	1	$ 10	1: 133.33	6,300	$ 63,000
$10 ($2 x 5)	5	$ 10	1: 100.00	8,400	$ 84,000
$5	1	$ 5	1: 26.67	31,500	$ 157,500
$4 ($2"D")	1	$ 4	1: 12.50	67,200	$ 268,800
$2	1	$ 2	1: 9.09	92,400	$ 184,800
TOTAL			1: 3.84	218,780	$ 1,075,250

Let's make a probability distribution model for the winnings. $840,000 - 218,780 = 621,220$

Winnings X	$11,000	$500	$250	$100	$50	$30	$15	$10	$5	$4	$2	$0
Probability $P(X = x)$	$\frac{3}{840,000}$	$\frac{10}{840,000}$	$\frac{20}{840,000}$	$\frac{70}{840,000}$	$\frac{867}{840,000}$	$\frac{3610}{840,000}$	$\frac{6300}{840,000}$	$\frac{16,800}{840,000}$	$\frac{31,500}{840,000}$	$\frac{67,200}{840,000}$	$\frac{92,400}{840,000}$	$\frac{621,220}{840,000}$

What are the **expected** winnings?

$$E(X) = \$11,000\left(\frac{3}{840,000}\right) + \$500\left(\frac{10}{840,000}\right) + \ldots \$0\left(\frac{621,220}{840,000}\right)$$

$$= \frac{\$1,075,250}{840,000} = \$1.28 \text{ expected winnings}$$

Expected profit : $\$1.28 - 2 = -\$.72$

add value probability

Expected Value: $\mu = E(X) = \sum x \cdot P(x)$

Example 2. Find the Expected Value and Standard Deviation of a Random Variable

A coffee shop has tracked their morning sales of cups of coffee and observed the following distribution.

Number of cups sold, X	145	150	155	160	170
Probability, $P(X = x)$	0.15	0.22	0.37	0.19	0.07

← probabilities must add up to 100% or 1.0

a. Compute the expected sales.

$$\mu = E(x) = 145(.15) + 150(.22) + 155(.37) + 160(.19) + 170(.07)$$
$$= 154.4 \text{ cups sold}$$

"mu"

add value mean probability

Variance: $\sigma^2 = Var(X) = \sum (x - \mu)^2 \cdot P(x)$

Standard Deviation: $\sigma = SD(X) = \sqrt{Var(X)}$

b. Compute the standard deviation of the sales.

$$Var(x) = (145 - 154.4)^2(.15) + (150 - 154.4)^2(.22) + (155 - 154.4)^2(.37$$
$$+ (160 - 154.4)^2(.19) + (170 - 154.4)^2(.07)$$
$$= 40.64 \text{ cups}^2$$
$$\sigma = SD(x) = \sqrt{40.64} = 6.37 \text{ cups}$$

"average deviation from the mean"

Example 3. Is it a Fair Game?

You roll a die. If it comes up a 6, you win \$100. If not, you get to roll again. If you get a 6 the second time, you win \$50. If not, you lose.

a. Create a probability distribution for the amount you win.

winnings, X	\$100	\$50	\$0
probability P(x)	1/6	$\frac{5}{6}\cdot\frac{1}{6} = \frac{5}{36}$	$\frac{5}{6}\cdot\frac{5}{6} = \frac{25}{36}$

b. Find the expected amount you'll win.

$$E(X) = \$100\left(\tfrac{1}{6}\right) + 50\left(\tfrac{5}{36}\right) + 0\left(\tfrac{25}{36}\right)$$
$$= \$23.61$$

c. Find the standard deviation.

$$Var(x) = (\$100 - 23.61)^2(1/6) + (50 - 23.61)^2(5/36) + (0 - 23.61)^2(25/36)$$
$$= \$1456.40 \text{ dollars}^2$$

$$SD(x) = \sqrt{1456.40} = \$38.16$$ "average deviation from the mean"

d. Should you play this game if it costs \$25? Why or why not?

No, I would not pay \$25 to play because the expected value (winnings) is \$23.61

Expected profit: $23.61 - 25 = -\$1.39$

A **fair game** is defined as a game that costs as much as its expected payout. The expected profit is 0.

Expected payout = cost

e. What price would make this game fair?

what's the expected value?

\$23.61

f. Are lottery and gambling games usually fair?

No, they are designed to have an expected loss for the consumer.

Practice

1. The probability model below describes the number of repair calls that an appliance repair shop may receive during an hour.

X = # of repair calls	0	1	2	3
$P(X = x)$	0.1	0.3	0.4	.2

a. Complete the table.

$1 - .1 - .3 - .4 = .2$

b. How many calls should the shop expect per hour? Include units on all means.

$E(x) = 0(.1) + 1(.3) + 2(.4) + 3(.2)$

$= 1.7$ repair calls per hour

c. What is the standard deviation? Include units on all standard deviations.

$Var(x) = (0-1.7)^2(.1) + (1-1.7)^2(.3) + (2-1.7)^2(.4) + (3-1.7)^2(.2)$

$= 0.81$ repair calls^2

$SD(x) = \sqrt{.81} = 0.9$ repair calls per hour

2. (Problem 3.46) A game of roulette, Part I. The game of roulette involves spinning a wheel with 38 slots. 18 red, 18 black, and 2 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. Gamblers can place bets on red or black. If the ball lands on their color, they double their money. If it lands on another color, they lose their money. Suppose you bet $1 on red. What's the expected value and standard deviation of your winnings.

a. Create a probability distribution for the amount you win.

	red	black	green
winnings, X	2	0	0
probability, P(x)	$\frac{18}{38}$	$\frac{18}{38}$	$\frac{2}{38}$

b. Find the expected winnings, including units.

$$E(x) = 2\left(\frac{18}{38}\right) + 0\left(\frac{18}{38}\right) + 0\left(\frac{2}{38}\right)$$
$$= \$0.95$$

c. Find the standard deviation of your winnings, including units.

$$var(x) = (2-.95)^2\left(\frac{18}{38}\right) + (0-.95)^2\left(\frac{18}{38}\right) + (0-.95)^2\left(\frac{2}{38}\right.$$
$$= .9972 \text{ dollars}^2$$
$$SD(x) = \sqrt{.9972} \approx \$1.00$$

d. Is this a fair game at $1?

No, it's not fair because you would lose 5¢ on average

e. What is your expected profit or loss?

expected profit: $.95 − 1.00 = −$.05