Section 3.5







Key for Rational function formula from graph worksheet

1.

- Factors in numerator are from *x* intercepts
- Factors in denominator are from vertical asymptotes
- Note that the horizontal asymptote is *y* = 3, which means the case is "match."
 - Since the numerator and denominator should have the same degree, one of the factors in the denominator must be repeated for formula to agree with the graph.
 - Since the graph points in the same direction on both sides of x = -2, the x + 2 factor must be squared.

$$r(x) = \frac{k(x+3)(x+1)(x-1)}{(x+2)^2(x-2)}$$

• Use the point to 5.76 = $\frac{k(3+3)(3+1)(3-1)}{(3+2)^2(3-2)}$ additional solve for k: $5.76 = \frac{k(6)(4)(2)}{(25)(1)}$ $5.76 = \frac{48}{25}k$ 3 = k • Conclusion:

$$r(x) = \frac{3(x+3)(x+1)(x-1)}{(x+2)^2(x-2)}$$

2.

- Factors in numerator are from *x*-intercepts
- Factors in denominator are from vertical asymptotes

• Initial set up:
$$y = \frac{k(x+2)(x-1)(x-3)}{(x+1)(x-4)}$$

- Note that there is no horizontal asymptote on the graph, which means the case is "top-heavy." The numerator should have a higher degree, and more specifically the degree will be exactly one higher (since there is an oblique asymptote on the graph). Our formula agrees.
- Since the graph points in opposite directions on both sides of each vertical asymptote, the factors in the denominator would not be squared.
- Use the additional point (7, -9) to solve for k:

$$-9 = \frac{k(7+2)(7-1)(7-3)}{(7+1)(7-4)}$$
$$-9 = \frac{k(9)(6)(4)}{(8)(3)}$$
$$-9 = 9k$$
$$-1 = k$$

• Conclusion:

$$R(x) = - \frac{(x+2)(x-1)(x-3)}{(x+1)(x-4)}$$