

Section 4.3 Rates of Change

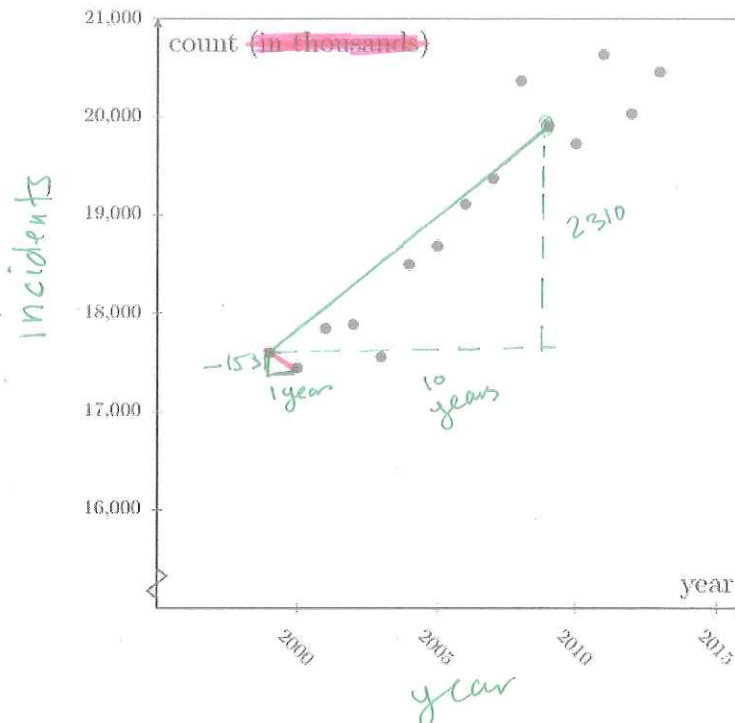
Modeling Data with Two Variables

1. Use the data in Table 1 and Figure 2 that shows incidents of invasive cancer reported in Oregon from 1999 to 2013 to answer each question.

TABLE 1. Raw Data

Year	Incidents
1999	17599
2000	17446
2001	17847
2002	17887
2003	17559
2004	18499
2005	18682
2006	19112
2007	19376
2008	20370
2009	19909
2010	19727
2011	20636
2012	20035
2013	20458

FIGURE 2. Invasive Cancer Incidents from 1999 through 2013



a. State the data values for the years 1999 and 2000. What was the rate of change in that year?

$$\begin{aligned}
 & (1999, 17599) \quad (2000, 17446) \\
 & 17599 - 17446 = \frac{-153 \text{ cancer incidents}}{1 \text{ year}} \\
 & = \boxed{-153 \text{ incidents/year}}
 \end{aligned}$$

b. State the data values for the years 1999 and 2009. What was the rate of change during that time?

$$\begin{aligned}
 & (1999, 17599) \quad (2009, 19909) \\
 & 19909 - 17599 = \frac{2310 \text{ incidents}}{10 \text{ years}} \\
 & = \boxed{231 \text{ incidents/year}}
 \end{aligned}$$

Patterns in Tables

2. Identify the pattern in each table below. In other words how could y be calculated given x ? Write an equation in the form $y = \dots$. Then find the rate of change if it is constant.

a.

x	y
-2	14
-1	7
0	0
1	-7
2	-14
3	-21

Handwritten annotations: $+1$ in the x column and -7 in the y column for each row transition.

Equation: $y = -7x$

Rate of Change: $\frac{-7}{1} = -7$

b.

x	y
-2	-1
-1	1
0	3
1	5
2	7
3	9

Handwritten annotations: $+1$ in the x column and $+2$ in the y column for each row transition.

Equation: $y = 2x + 3$

Rate of Change: $\frac{2}{1} = 2$

c.

x	y
-2	-8
-1	-1
0	0
1	1
2	8
3	27

Handwritten annotations: $+1$ in the x column and $+7, +1, +1, +7, +19$ in the y column for each row transition.

Equation: $y = x^3$

Rate of Change: not linear

d.

x	y
-20	-18
-10	-8
0	-2
10	8
20	18
30	28

Handwritten annotations: $+10$ in the x column and $+10, +6, 10, 10, 10$ in the y column for each row transition. A note says "not the same rate of change".

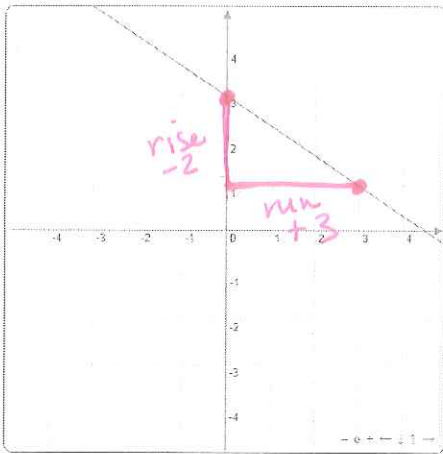
Equation: not linear

Rate of Change:

Section 4.4 Slope

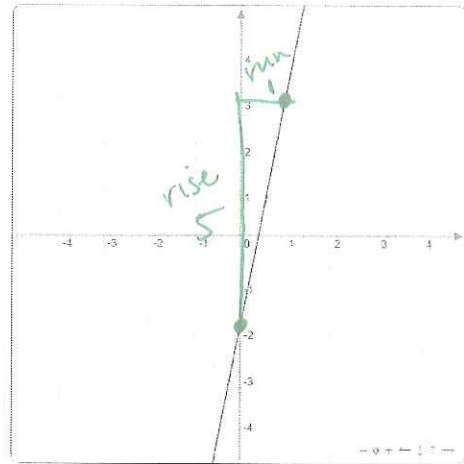
3. Find the slope of each line using its graph.

a.



$$m = \frac{\text{rise}}{\text{run}} = -\frac{2}{3}$$

b.



$$m = \frac{\text{rise}}{\text{run}} = \frac{5}{1} = 5$$

4. Find the slope of each line again, using the formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

a. x_1, y_1 and x_2, y_2
 $(0, 3)$ and $(3, 1)$

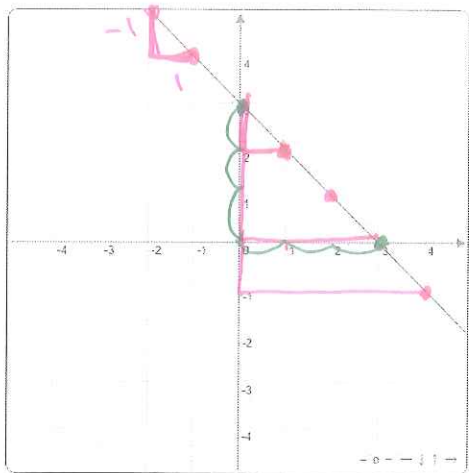
$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{1 - 3}{3 - 0} \\ &= -\frac{2}{3} \\ &= -\frac{2}{3} \end{aligned}$$

x_1, y_1 and x_2, y_2
 $(0, -2)$ and $(1, 3)$

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{3 - (-2)}{1 - 0} = \frac{5}{1} \\ &= 5 \end{aligned}$$

5. Find the slope of each line using its graph.

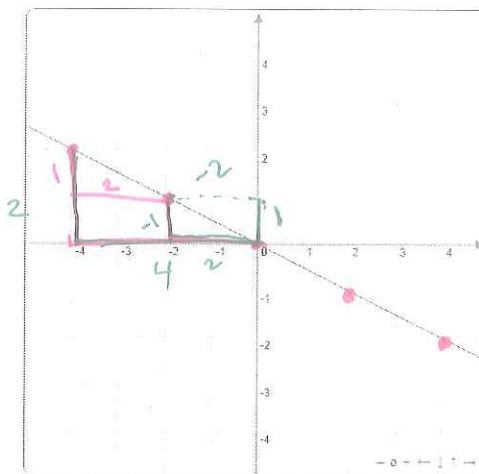
a.



$$\frac{\text{rise}}{\text{run}} = -\frac{3}{3} = -1$$

$$\frac{-4}{4} = -1$$

b.



$$\frac{\text{rise}}{\text{run}} = -\frac{2}{4} = -\frac{1}{2}$$

$$\frac{1}{-2} = -\frac{1}{2}$$

6. Find the slope of each line again. Write down the coordinates and use the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{0 - 1}{0 - (-2)} = \frac{-1}{2}$$

a. $(0, 3)$ $(1, 2)$

b. x_1, y_1 x_2, y_2
 $(-2, 1)$ $(0, 0)$

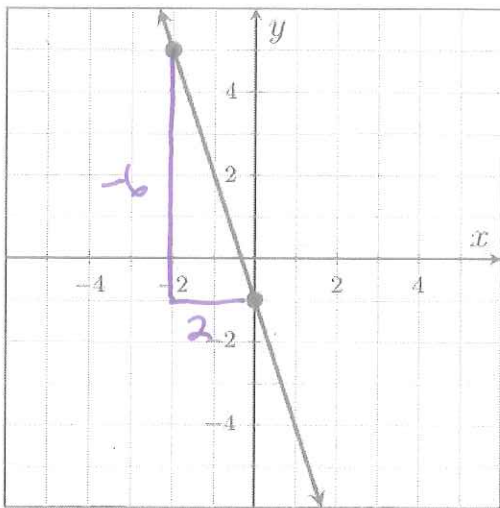
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{1 - 0} = \frac{-1}{1} = -1$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{0 - (-2)} = \frac{-1}{-2} = \frac{1}{2} = -\frac{1}{2}$$

More Practice

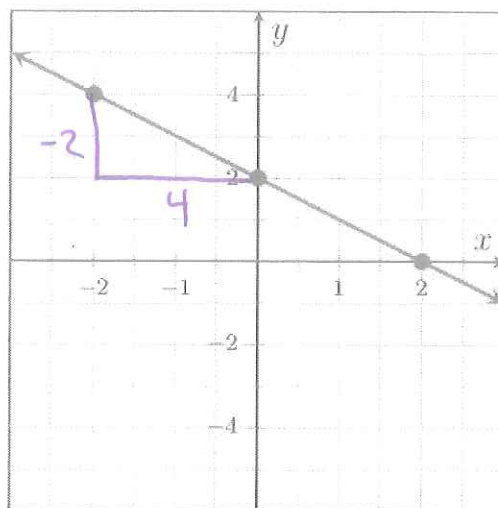
7. Find the slope of each line using its graph.

a.



$$m = \frac{-6}{2} = -3$$

b.



$$m = \frac{-2}{4} = -\frac{1}{2}$$

8. Without graphing, find the slope of the line between each pair of points.

a. (1, -3) and (-1, -5)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned} m &= \frac{-5 - (-3)}{-1 - 1} \\ &= \frac{-2}{-2} \\ &= 1 \end{aligned}$$

b. (1, -9) and (7, 11)

$$\begin{aligned} m &= \frac{11 - (-9)}{7 - 1} \\ &= \frac{20}{6} \\ &= \frac{10}{3} \end{aligned}$$

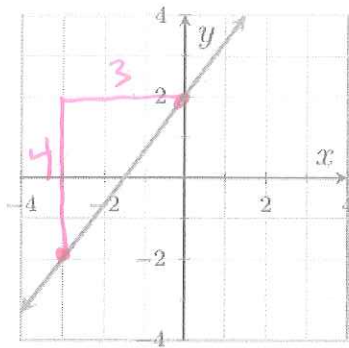
c. (-6, 1) and (-6, -1)

$$\begin{aligned} m &= \frac{-1 - 1}{-6 - (-6)} \\ &= \frac{-2}{0} \\ &= \text{undefined} \\ &= \text{vertical line} \end{aligned}$$

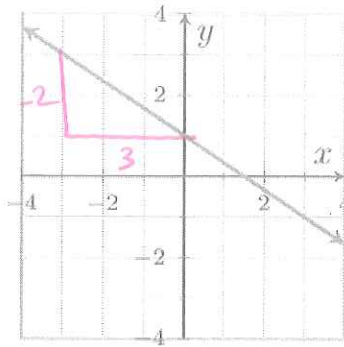
d. (-3, -2) and (4, -2)

$$\begin{aligned} m &= \frac{-2 - (-2)}{4 - (-3)} \\ &= \frac{0}{7} \\ &= 0 \quad \text{(horizontal line)} \end{aligned}$$

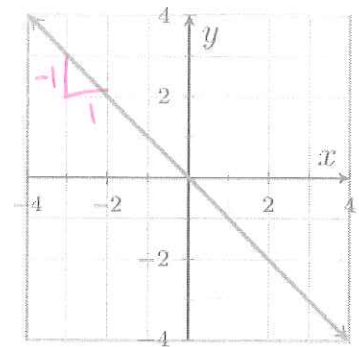
9. Identify two points on each line and find the slope.



$$m = \frac{4}{3}$$



$$m = -\frac{2}{3}$$



$$m = -\frac{1}{1} = -1$$

10. Without graphing, find the slope of the line between each pair of points.

a. (2,1) and (3,4)

$$\begin{aligned} m &= \frac{4-1}{3-2} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

b. (3,1) and (-1,1)

$$\begin{aligned} m &= \frac{1-1}{-1-3} \\ &= \frac{0}{-4} \\ &= 0 \end{aligned}$$

(horizontal line)

c. (6,-4) and (4,-2)

$$\begin{aligned} m &= \frac{-2 - (-4)}{4-6} \\ &= \frac{2}{-2} \\ &= -1 \end{aligned}$$

d. (-4,5) and (-4,3)

$$\begin{aligned} m &= \frac{3-5}{-4 - (-4)} \\ &= \frac{-2}{0} \\ &= \text{undefined} \end{aligned}$$

(vertical line)