Math 60, Sections 4.3 and 4.4 Notes and Practice Name: Solutions

Section 4.3 Rates of Change

Modeling Data with Two Variables

1. Use Figure 1 to answer each question.

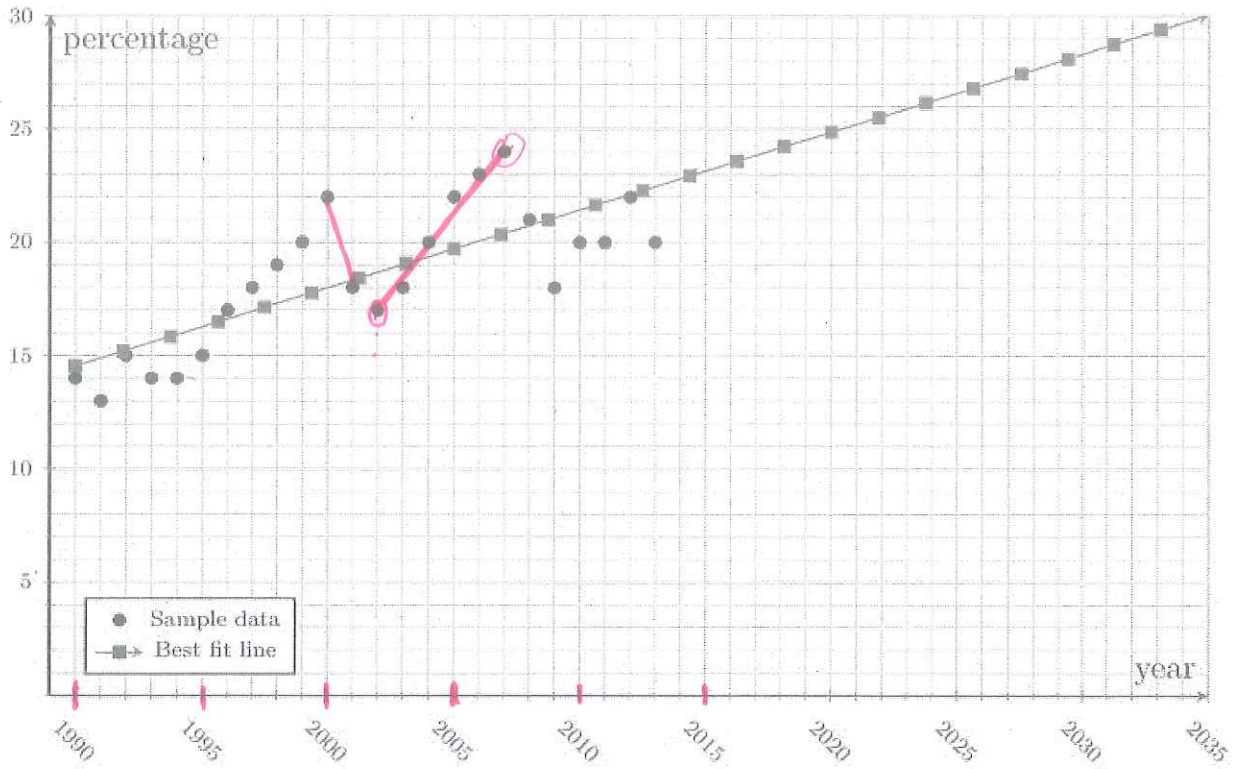


FIGURE 1. Share of all income held by the top 1% in the United States, 1990-2013
Source: http://epi.org/blog/new-data-show-top-1-percent-really-are-different-from-you-and-me/

a. State the data values for the years 2000 and 2001. How did the percentage change in this year?

(2000, 22%) and (2001, 18%)

-4% change

It went down 4%. rate: $\frac{-4\%}{1 \text{ year}}$

b. State the data values for the years 2002 and 2007. How did the percentage change over these 5 years?

(2002, 17%) (2007, 24%)

+7% change

$\frac{7\%}{5 \text{ years}} = 1.4\% \text{ per year}$

c. State the answers from parts a and b as rates of change.

$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x} \quad \frac{\%}{\text{years}}$$

a. $\frac{-4\%}{1 \text{ year}} = -4\% \text{ per year}$

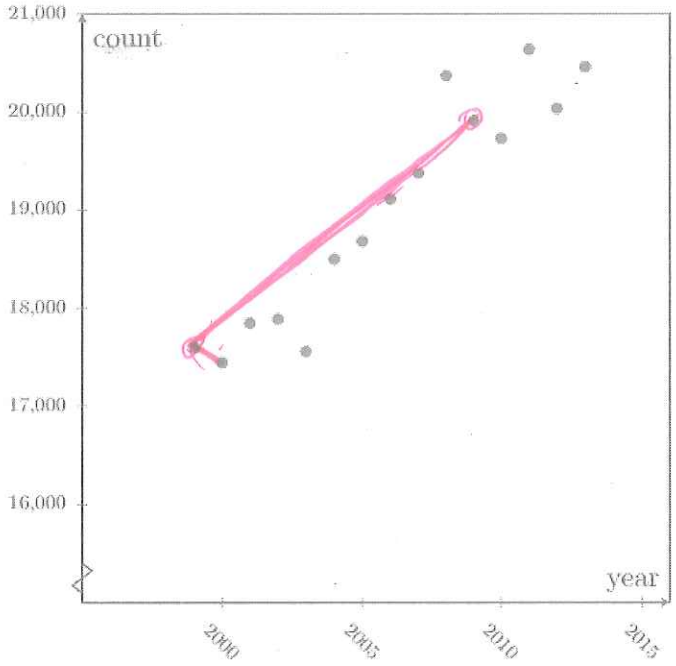
b. $\frac{7\%}{5 \text{ years}} = 1.4\% \text{ per year}$

2. Use the data in Table 1 and Figure 2 that shows incidents of invasive cancer reported in Oregon from 1999 to 2013 to answer each question.

TABLE 1. Raw Data

Year	Incidents
1999	17599
2000	17446
2001	17847
2002	17887
2003	17559
2004	18499
2005	18682
2006	19112
2007	19376
2008	20370
2009	19909
2010	19727
2011	20636
2012	20035
2013	20458

FIGURE 2. Invasive Cancer Incidents from 1999 through 2013



a. State the data values for the years 1999 and 2000. What was the rate of change in that year?

(1999, 17599) (2000, 17446)

Rate of change = $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{17599 - 17446}{1 \text{ year}}$

$\frac{2000 - 1999}{1999 - 2000}$

$= \frac{153 \text{ people}}{-1 \text{ year}}$

-153 people/year ←

b. State the data values for the years 1999 and 2009. What was the rate of change during this time?

(1999, 17599) (2009, 19909)

$\frac{\text{change in } y}{\text{change in } x} = \frac{19909 - 17599}{10 \text{ years}} = \frac{2310 \text{ people}}{10 \text{ years}}$

$= 231 \text{ people/year}$

Patterns in Tables

3. Identify the pattern in each table below. In other words how could y be calculated given x? Write an equation in the form $y = \ldots$. Then find the rate of change if it is constant.

a.

x	y
-2	-8
-1	-4
0	0
1	4
2	8
3	12

$\frac{\Delta y}{\Delta x}$

x4 x4 x4 x4 x4 x4

+1 +1 +1 +1 +1

+4 +4 +4 +4 +4

webwork

Equation: $y = 4x$

Rate of Change: $\frac{\Delta y}{\Delta x} = \frac{4}{1} = 4$

b.

x	y
-2	-7
-1	-3
0	1
1	5
2	9
3	13

+1 +1 +1 +1 +1

+4 +4 +4 +4 +1

Equation: $y = 4x + 1$

Rate of Change: $\frac{4}{1} = 4$

c.

x	y
-2	4
-1	1
0	0
1	1
2	4
3	9

Equation: $y = x^2$

Rate of Change: not linear

d.

x	y
-200	-20
-100	-10
0	0
100	10
200	20
300	30

+100 +100

+10 +10

$y = \frac{1}{10}x$

Equation: $y = \frac{x}{10}$ or $y = x \div 10$

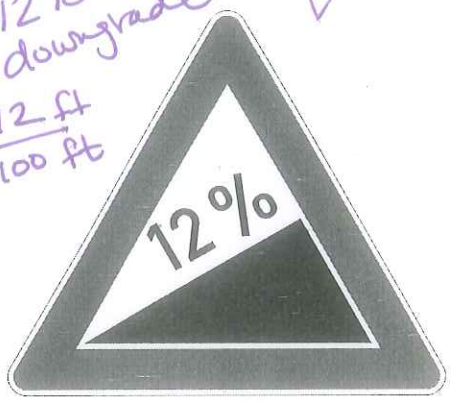
Rate of Change: linear

$\frac{\Delta y}{\Delta x} = \frac{10}{100} = \frac{1}{10}$

4. Which of these signs involve rate of change?

12% downgrade

$\frac{12 \text{ ft}}{100 \text{ ft}}$



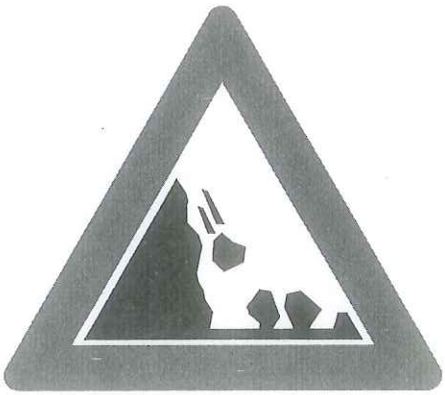
✓

X

$\frac{8 \text{ ft}}{100 \text{ ft}}$

✓

X



?

✓

requirements for ramps $\frac{1 \text{ inch}}{1 \text{ ft}}$

✓

✓

X

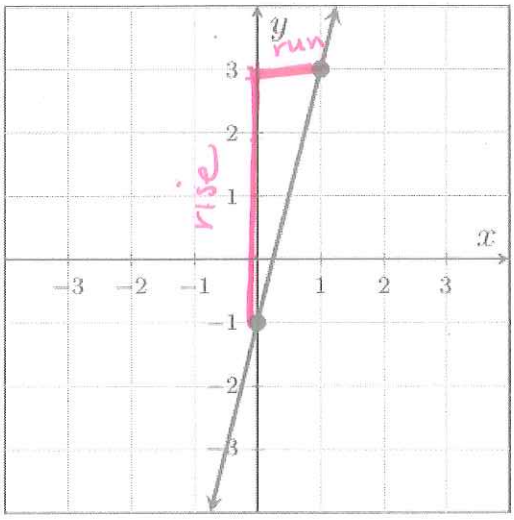
$\frac{1 \text{ ft}}{5 \text{ ft}} \quad \frac{1 \text{ m}}{5 \text{ m}} = 20\%$

Slope is a rate of change

$$\text{Slope} = \text{Rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

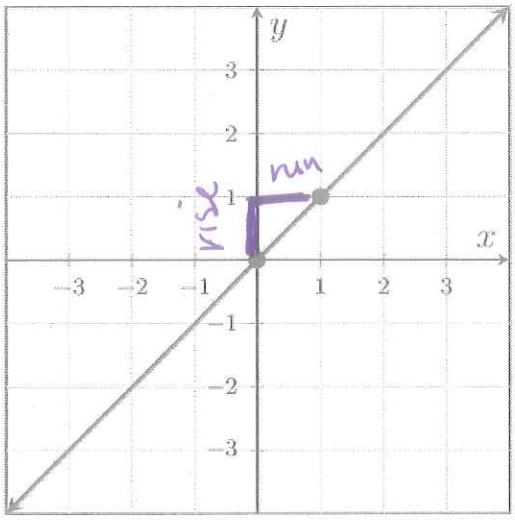
5. Find the slope of each line using its graph.

a.



$m = \frac{\text{rise}}{\text{run}} = \frac{4}{1} = 4$

b.



$m = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$

slope is a measure of steepness

6. Find the slope of each line again, using the formula.

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

a. $(3,1)$ and $(0,-1)$ ($x_1\ y_1$ $x_2\ y_2$)

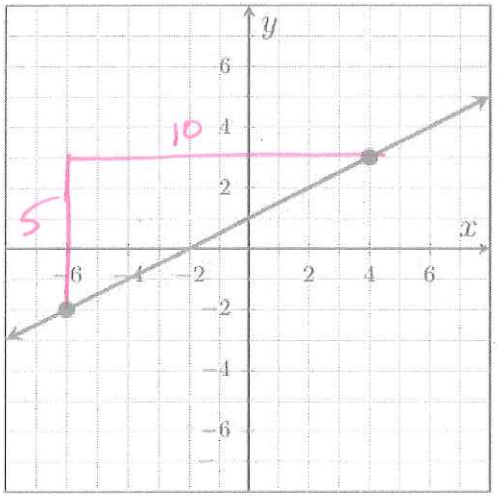
$$m = \frac{-1-1}{0-3} = \frac{-2}{-3} = \frac{2}{3}$$

b. $(1,1)$ and $(0,0)$ ($x_1\ y_1$ $x_2\ y_2$)

$$m = \frac{0-1}{0-1} = \frac{-1}{-1} = 1$$

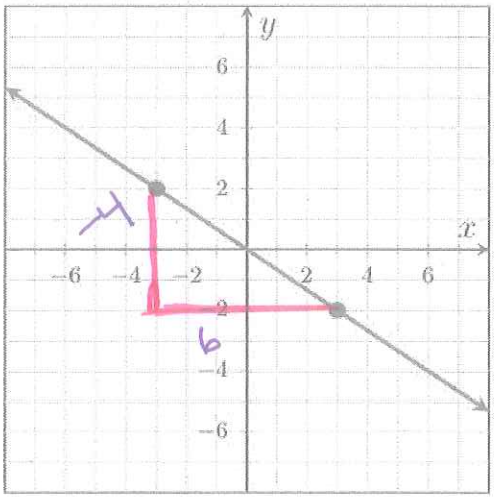
7. Find the slope of each line using its graph.

a.



$m = \frac{5}{10} = \frac{1}{2}$

b.



$m = \frac{-4}{6} = -\frac{2}{3}$

8. Find the slope of each line again. Write the down the coordinates and use the slope formula.

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

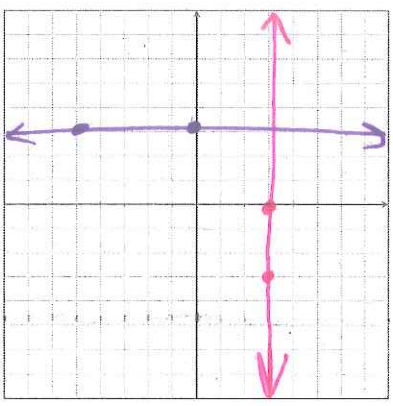
a. $(4,3)\ (-6,-2)$ ($x_1\ y_1$ $x_2\ y_2$)

$m = \frac{-2-3}{-6-4} = \frac{-5}{-10} = \frac{1}{2}$

b. $(-3,2)\ (3,-2)$

$m = \frac{-2-2}{3-(-3)} = \frac{-4}{6} = -\frac{2}{3}$

9. What would be the slope be of a horizontal line? A vertical line?



horizontal line

$m = \frac{rise}{run} = \frac{0}{run} = 0$

vertical line

$m = \frac{rise}{run} = \frac{rise}{0} =$ undefined

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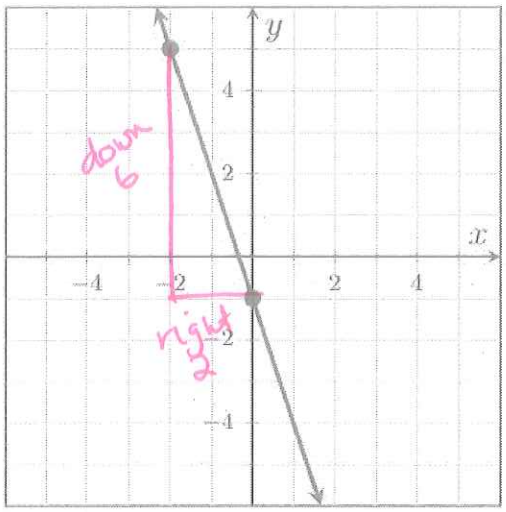
More Practice

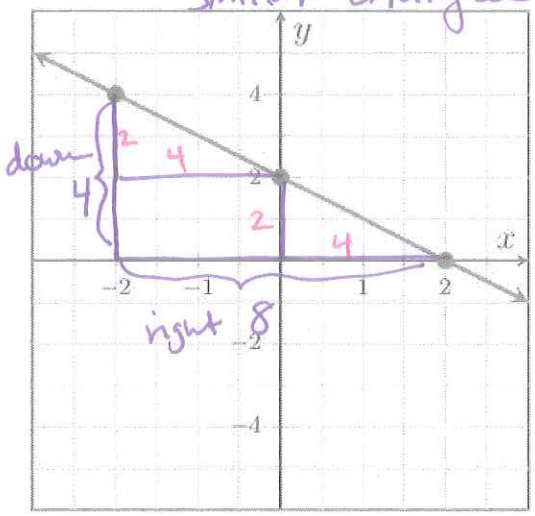
10. Find the slope of each line using its graph.

a.



$m = \frac{-6}{2} = -3$

b. use any 2 points similar triangles



$m = \frac{-4}{8}$ or $\frac{-2}{4} = -\frac{1}{2}$

11. Without graphing, find the slope of the line between each pair of points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

a. $(1,-3)$ and $(-1,-5)$ (x_1, y_1 and x_2, y_2)

$m = \frac{-5-(-3)}{-1-1}$

$= \frac{-2}{-2}$

$= 1$

b. $(1,-9)$ and $(7,11)$ (x_1, y_1 and x_2, y_2)

$m = \frac{11-(-9)}{7-1}$

$= \frac{20}{6}$

$= \frac{10}{3}$

c. $(-6,1)$ and $(-6,-1)$

$m = \frac{-1-1}{-6-(-6)}$

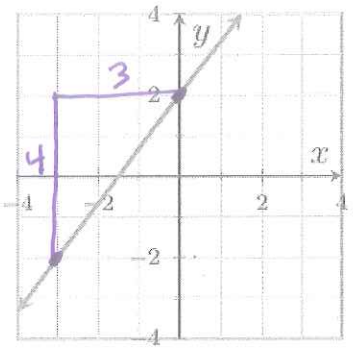
$= \frac{-2}{0}$ = undefined

d. $(-3,-2)$ and $(4,-2)$

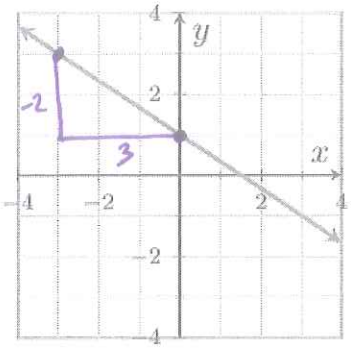
$m = \frac{-2-(-2)}{4-(-3)}$

$= \frac{0}{7} = 0$

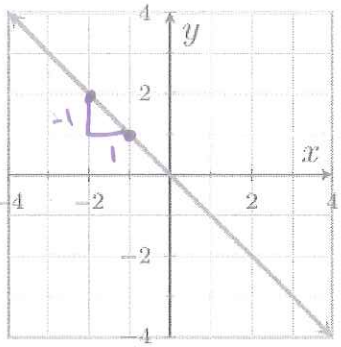
12. Identify two points on each line and find the slope.



$m = \frac{4}{3}$



$m = \frac{-2}{3}$



$m = -\frac{1}{1} = -1$

13. Without graphing, find the slope of the line between each pair of points.

a. $(2,1)$ and $(3,4)$

$m = \frac{4-1}{3-2}$

$= \frac{3}{1}$

$= 3$

b. $(3,1)$ and $(-1,1)$

$m = \frac{1-1}{-1-3}$

$= \frac{0}{-4}$

$= 0$

c. $(6,-4)$ and $(4,-2)$

$m = \frac{-2-(-4)}{4-6}$

$= \frac{2}{-2}$

$= -1$

d. $(-4,5)$ and $(-4,3)$

$m = \frac{3-5}{-4-(-4)}$

$= \frac{-2}{0}$

= undefined