

Section 4.5 Slope-Intercept Form

Slope and y-intercept in context

1. The cost to join LA Fitness is \$25 per month (rounded from \$24.99), plus a \$99 initiation fee.
 Source: <https://www.lafitness.com>

a. Write an equation for the total cost, y , in terms of the number of months, x , after joining the gym.

$y = 25x + 99$ 124 99 or 124 is ok
 slope

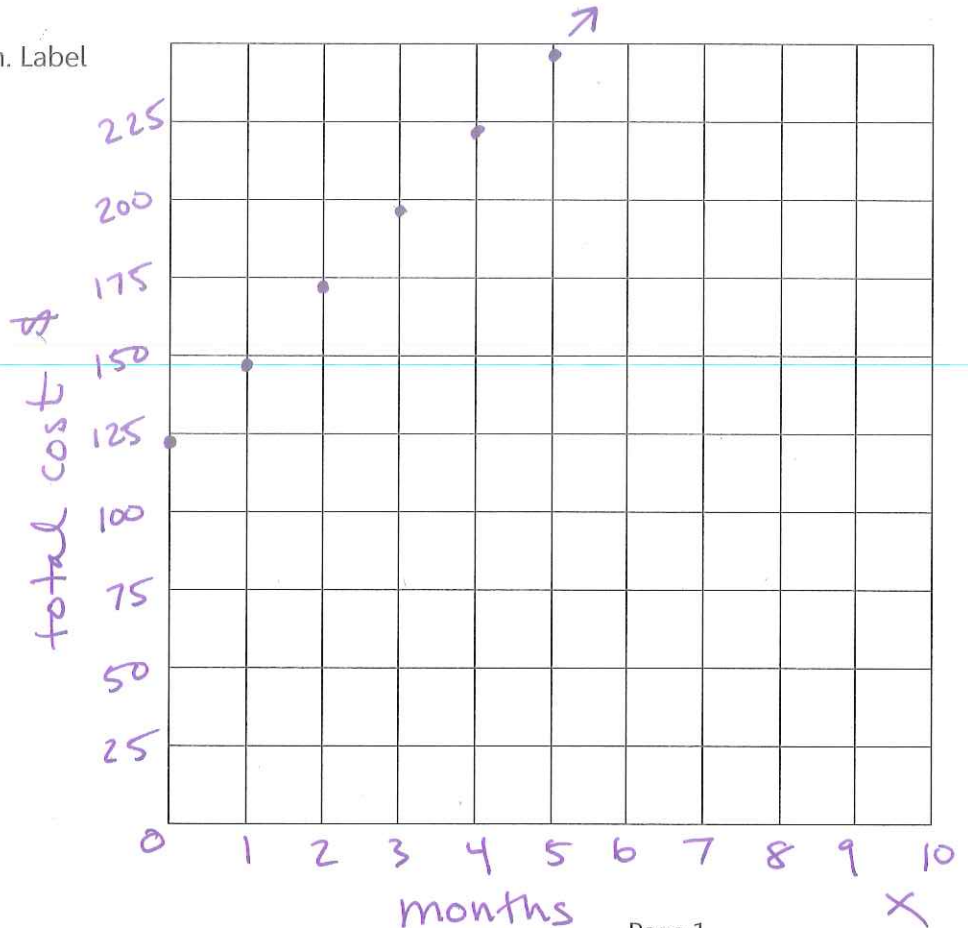
b. What is the rate of change or slope in this context?

$\frac{\$25}{1 \text{ month}}$ The monthly fee is \$25 per month
 \$25/month.

c. What is the starting value or y-intercept in this context?

The y-intercept is ~~(0, 99)~~ (0, 124)
 Its the \$99 initiation fee + \$25 = \$124
 you have to pay the 1st month also

d. Graph this linear equation. Label your axes and scale.



Slope-Intercept Form of a Linear Equation: $y = mx + b$

\swarrow slope
 \nwarrow y-intercept

2. Identify the slope and the y -intercept of each line below.

a. $y = 3x - 1$

$m = 3$

y -intercept = -1
 or
 $(0, -1)$

b. $y = -9x - \frac{1}{2}$

$m = -9$

$b = (0, -\frac{1}{2})$

c. $y = -\frac{1}{8}x + 15$

$m = -\frac{1}{8}$

$b = (0, 15)$

3. The weight y (in pounds) of a plastic tank holding x gallons of water can be modeled by the equation $y = 8.3454x + 67$. Suppose that a truck will be hauling this plastic tank.

a. What is the slope of this linear equation, with units? Explain what the slope represents in this context.

$m = \frac{8.3454 \text{ pounds}}{1 \text{ gallon}} \frac{\Delta y}{\Delta x}$

The slope is the weight of the water, which is 8.3454 pounds/gallon.

b. What is the y -intercept of this linear equation? Explain what it represents in this context.

The y -intercept is 67 pounds. With 0 gallons of water, the tank weighs 67 pounds.

$(0, 67)$

c. If the weight of the tank is 1,318.81 pounds, how many gallons of water are in the tank?

$y = 8.3454x + 67$

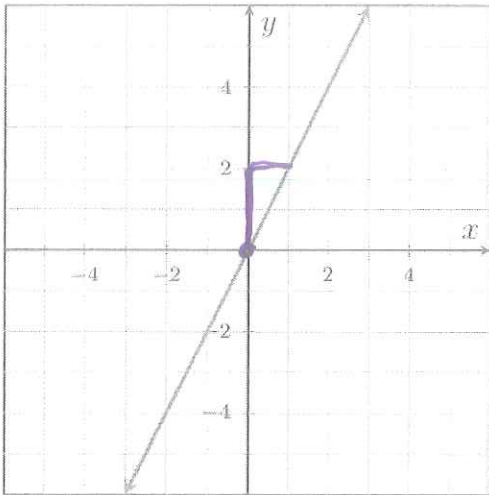
$1318.81 = 8.3454x + 67$
 $-67 \quad \quad \quad -67$

$\frac{1251.81}{8.3454} = \frac{8.3454x}{8.3454}$

$150 \text{ gallons } \times$

4. Write the equation of each line by finding the slope and y -intercept from the graph.

a.

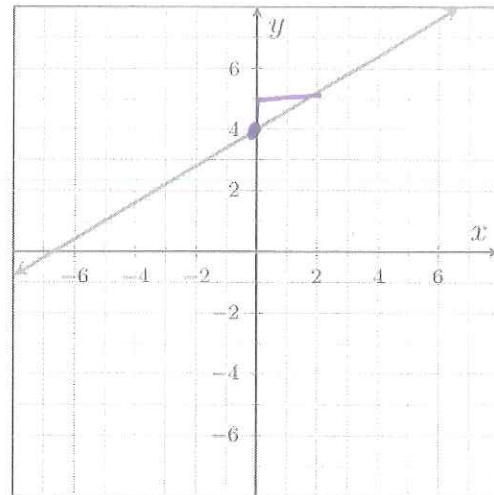


Slope: $\frac{2}{1} = 2$

y -intercept: $(0,0)$

Equation: $y = mx + b$
 $y = 2x + 0$ $y = 2x$

b.



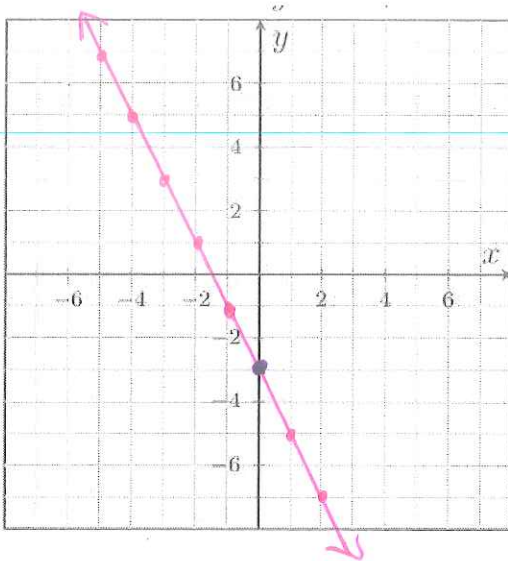
Slope:

y -intercept:

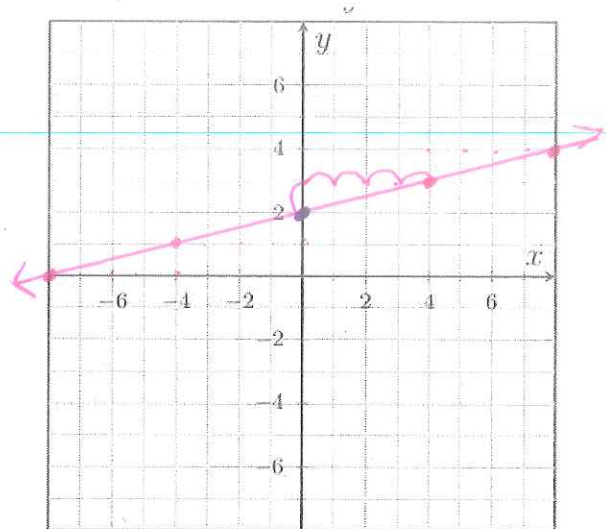
Equation: $y = \frac{1}{2}x + 4$

5. Graph each equation by plotting the y -intercept and then using the slope to plot additional points.

a. $y = -2x - 3$ $m = -\frac{2}{1}$ $\leftarrow y$ -intercept $(0, -3)$



b. $y = \frac{1}{4}x + 2$ $\leftarrow y$ -int



Section 4.6 Point-Slope Form

The Point-Slope Form of a line with slope m that passes through the point (x_1, y_1) is $y = m(x - x_1) + y_1$

$$y = -\frac{3}{4}(x - (-5)) - 6$$

6. Identify the slope and a point that is on each line below.

a. $y = -2(x - 3) + 1$
 m x_1 y_1

$$m = -2$$

$$(3, 1)$$

b. $y = -\frac{3}{4}(x + 5) - 6$

$$m = -\frac{3}{4}$$

$$(-5, -6)$$

opposite sign for x
 same sign for y

c. $y = -7(x + 4) + 8$

$$m = -7$$

$$(-4, 8)$$

7. Find the equation of the line with a slope of $\frac{3}{1}$ that passes through the point $(1, 8)$. Then simplify the equation to slope-intercept form.

$$y = m(x - x_1) + y_1$$

$$y = 3(x - 1) + 8$$
 point-slope form

$$y = 3x - 3 + 8$$

$$y = 3x + 5$$

slope-intercept form

8. Find the equation of the line with a slope of $\frac{1}{2}$ that passes through the point $(-2, 6)$. Then simplify the equation to slope-intercept form.

$$y = \frac{1}{2}(x - (-2)) + 6$$

$$y = \frac{1}{2}(x + 2) + 6$$
 point-slope

$$y = \frac{1}{2}x + \frac{1}{2} \cdot \frac{2}{1} + 6$$

$$y = \frac{1}{2}x + 1 + 6$$

$$y = \frac{1}{2}x + 7$$

9. Find the equation of the line that passes through the points $(-1, -2)$ and $(5, -4)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-2)}{5 - (-1)} = \frac{-2}{6} = -\frac{1}{3}$$

$$y = m(x - x_1) + y_1$$

$$y = -\frac{1}{3}(x - (-1)) + (-2)$$

$$y = -\frac{1}{3}(x + 1) - 2$$

$$y = -\frac{1}{3}(x + 1) - 2$$

$$y = -\frac{1}{3}x - \frac{1}{3} - 2 \cdot \frac{3}{1} \cdot \frac{3}{3}$$

$$y = -\frac{1}{3}x - \frac{1}{3} - \frac{6}{3}$$

$$y = -\frac{1}{3}x - \frac{7}{3}$$

More Practice

14. Metromile auto insurance charges \$30 per month and 3.2 cents per mile (rates may vary).

a. Write a linear equation representing the monthly cost, M , if you drive x miles per month.

$$M = 30 + .032x \quad \text{or} \quad M = .032x + 30$$

b. What is the slope in context?

The slope is the cost of 3.2¢ per mile.

c. What is the M -intercept in context?

The M -intercept is the base fee of \$30.

d. If you drive 100 miles in a month, how much will you be billed?

$$\begin{aligned} M &= 30 + .032(100) \\ &= 30 + 3.2 \\ &= \$33.2 \end{aligned}$$

It would cost \$33.20.

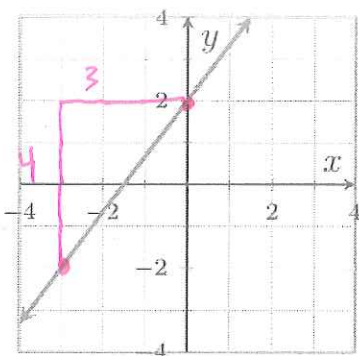
e. If your insurance bill for the month was \$43.60, how many miles did you drive?

$$\begin{aligned} M &= 30 + .032x \\ 43.60 &= 30 + .032x \\ -30 \quad -30 & \\ \hline 13.60 &= .032x \\ \frac{13.60}{.032} &= \frac{.032x}{.032} \quad x = 425 \end{aligned}$$

I drove 425 miles & I got charged \$43.60.

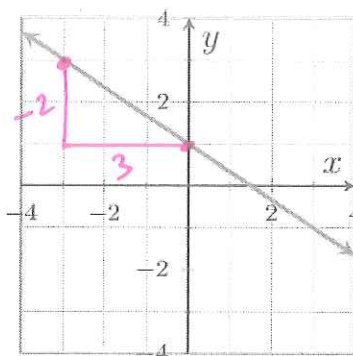
15. Write the equation for each line in slope-intercept form.

a.



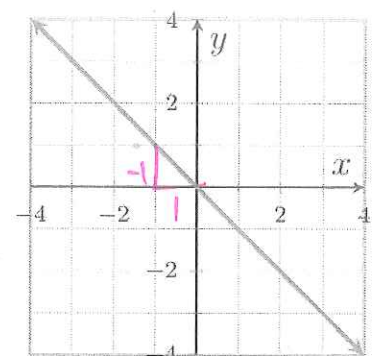
$$y = \frac{4}{3}x + 2$$

b.



$$y = -\frac{2}{3}x + 1$$

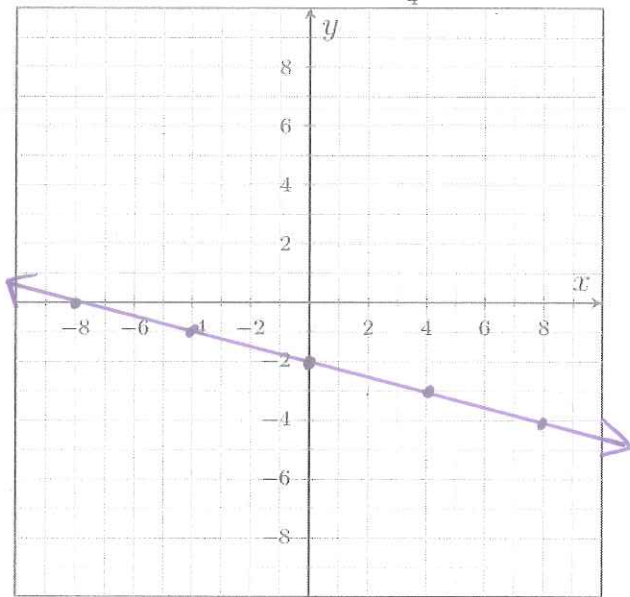
c.



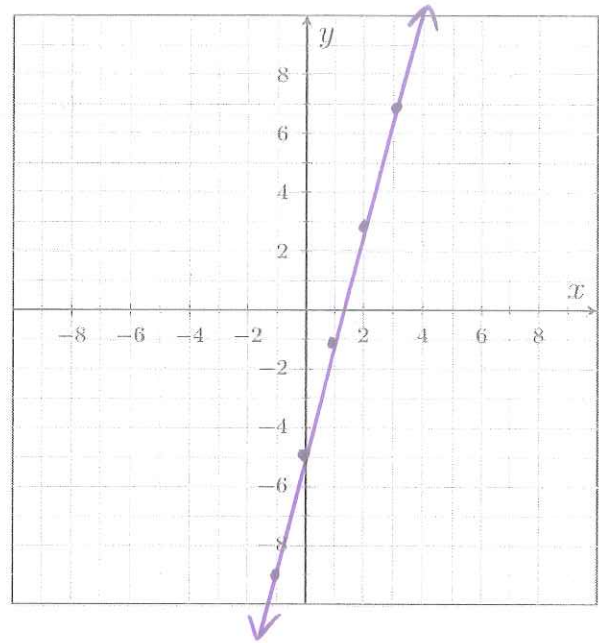
$$y = -x + 0 \quad \text{or} \quad y = -x$$

16. Graph each line using the slope and y-intercept.

a. $y = -\frac{1}{4}x - 2$



b. $y = 4x - 5$



17. Find the equation of a line with a slope of 7 that passes through the point $(10, 5)$. Then simplify the equation to point-slope form.

$$y = m(x - x_1) + y_1$$

$$y = 7(x - 10) + 5$$

$$y = 7x - 70 + 5$$

$$y = 7x - 65$$

18. Find the equation of the line that passes through the points $(-2, -10)$ and $(1, 8)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-10)}{1 - (-2)} = \frac{18}{3} = 6$$

$$y = m(x - x_1) + y_1$$

$$y = 6(x - 1) + 8$$

or $y = 6(x - (-2)) + (-10)$

$$y = 6x - 6 + 8$$

$$y = 6(x + 2) - 10$$

$$y = 6x + 2$$

$$y = 6x + 12 - 10 = y = 6x + 2$$

19. Find the equation of the line that passes through the points (2,3) and (7,9). First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$$m = \frac{9-3}{7-2} = \frac{6}{5}$$

$$y = \frac{6}{5}x - \frac{12}{5} + \frac{15}{5}$$

$$y = \frac{6}{5}(x-2) + 3$$

$$y = \frac{6}{5}x + \frac{3}{5}$$

$$y = \frac{6}{5}x - \frac{12}{5} + \frac{3 \cdot 5}{1 \cdot 5}$$

20. A company set aside a certain amount of money in the year 2000. The company spent the same amount from that fund each year on perks for its employees. In 2003, there was still \$490,000 in the fund. In 2007 there was \$318,000 left in the fund.

a. Write an equation for the amount of money in the fund, y , in year x .

$$m = \frac{318-490}{7-3} = \frac{-172}{4} = -43 \text{ thousand/year}$$

(2003, 490,000)

(2007, 318,000)

or

(3, 490)

(7, 318)

to make it easier

$$y = -43(x-3) + 490$$

$$= -43x + 129 + 490$$

$$y = -43x + 619$$

b. How much is the company spending each year from this fund?

The company is spending \$43,000 per year (slope)

c. How much money did they start with in the fund?

The company started with \$619,000 in the fund. (y-intercept)

d. In the year 2011, how much was left in the fund?

$$y = -43(11) + 619$$

$$y = -473 + 619$$

$$= 146$$

In 2011 there was \$146,000 left in the fund

e. If they continue the same trend, in which year will the fund run out?

$$y = -43x + 619$$

$$0 = -43x + 619$$

$$-619 \quad -619$$

After 2014 the fund would run out.

$$\frac{-619}{-43} = \frac{-43x}{-43} \quad x \approx 14.4$$