Math 60, Sections 4.5 and 4.6 Notes and Practice Name: Solutions

Section 4.5 Slope-Intercept Form

Slope and y-intercept in context

1. At the Mad Genius Escape Room on Hawthorne, the cost is $30 per person. If you want the room to be private for your party the cost is $27 plus $24 per person.
Source: https://www.madgeniusescapes.com/faq

Write an equation for the total cost, y, in terms of the number of people in your party, x.

a. For a private room.

$y = 27 + 24x$

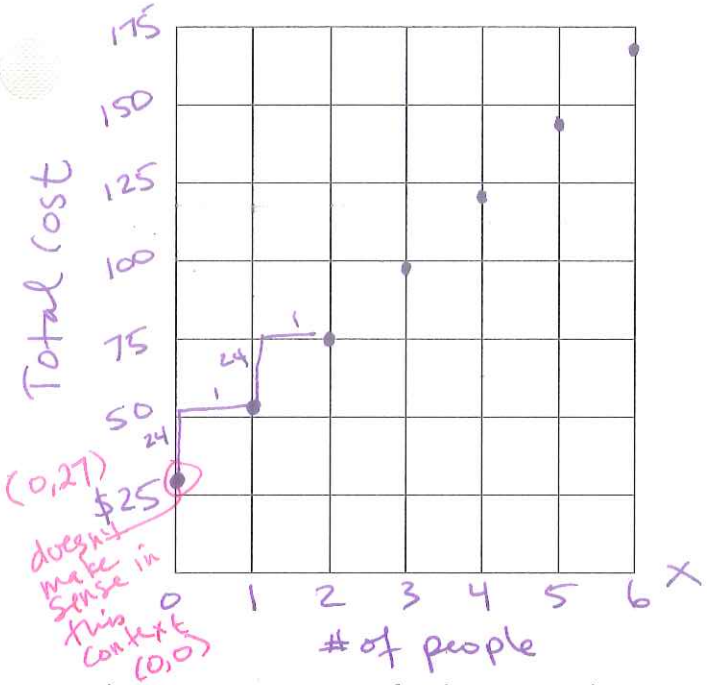
What is the rate of change or slope?

$\frac{\$24}{1 \text{ person}}$ or \$24 per person

What is the starting value or y-intercept?

\$27 base fee

Graph this linear equation:



b. For a non-private room.

$y = 30x$

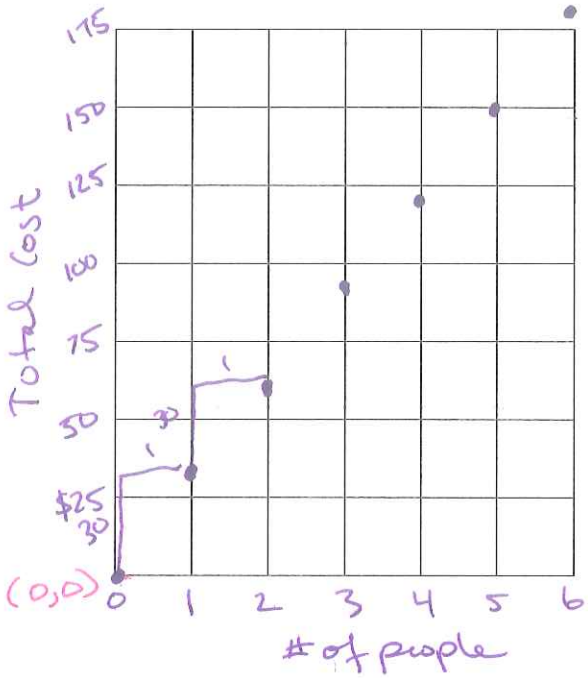
What is the rate of change or slope?

$\frac{\$30}{1 \text{ person}}$ or \$30/person

What is the starting value or y-intercept?

\$0

Graph this linear equation:



Slope-Intercept Form of a Linear Equation: $y = mx + b$

m b $(0, b)$

2. Identify the slope and the y-intercept of each line below.

a. $y = -9x + 4$

$m = -9$

$b = 4$ $(0, 4)$

b. $y = \frac{2}{3}x - 6$

$m = \frac{2}{3}$

$b = (0, -6)$

c. $y = -x + 3$

$m = -1$

$b = (0, 3)$

3. The weight y (in pounds) of a plastic tank holding x gallons of water can be modeled by the equation $y = 8.3454x + 67$. Suppose that a truck will be hauling this plastic tank.

x gal

a. What is the slope of this linear equation, with units? Explain what the slope represents in this context.

$m = 8.3454 \frac{lbs}{gal}$ This is the weight per gallon of water.

$y = 8.3454 \frac{lbs}{gal} \cdot x \text{ gallons} + 67 \text{ lbs}$

lbs

b. What is the y-intercept of this linear equation? Explain what it represents in this context.

$y = 67 \text{ lbs}$ $(0, 67)$

0 gal water

The y-intercept represents the weight of the tank by itself.

y-value

c. If the weight of the tank is 1,318.81 pounds, how many gallons of water are in the tank?

$y = 8.3454x + 67$

$1{,}318.81 = 8.3454x + 67$

$-67 \quad -67$

$1251.81 = 8.3454x$

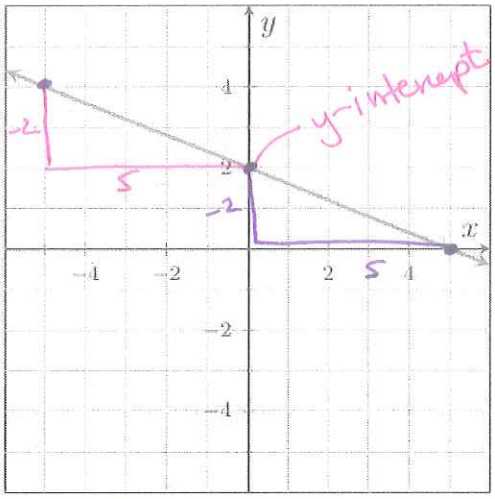
$\frac{1251.81}{8.3454} = \frac{8.3454x}{8.3454}$

$150 = x$ gallons

There are 150 gallons in the tank

4. Write the equation of each line by finding the slope and y-intercept from the graph.

a.



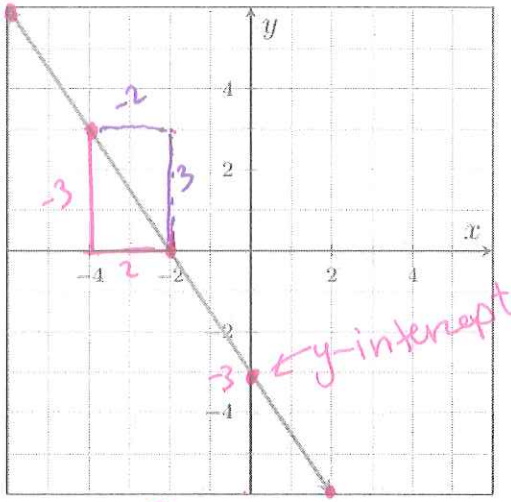
Slope: $\frac{rise}{run} = -\frac{2}{5}$

y-intercept $(0, 2)$

Equation: $y = mx + b$

$y = -\frac{2}{5}x + 2$

b.



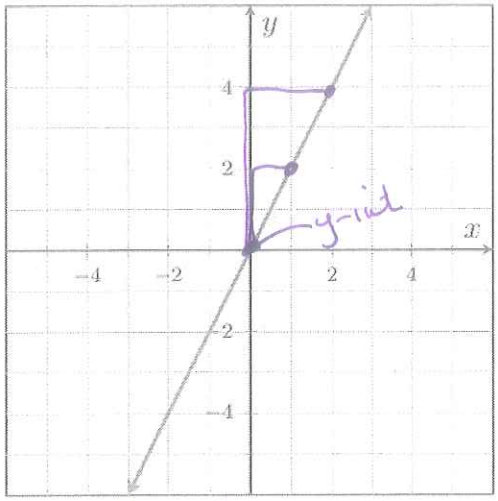
Slope: $-\frac{3}{2}$

y-intercept: $(0, -3)$

Equation: $y = -\frac{3}{2}x - 3$

5. Write the equation of each line by finding the slope and y-intercept from the graph.

a.



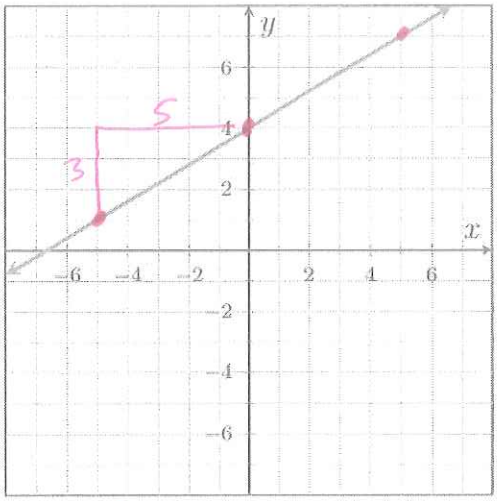
Slope: $\frac{4}{2} = \frac{2}{1}$

y-intercept $(0,0)$

Equation: $y = \frac{2}{1}x + 0$

$y = 2x$

b.



Slope: $\frac{3}{5}$

y-intercept: $(0,4)$

Equation: $y = \frac{3}{5}x + 4$

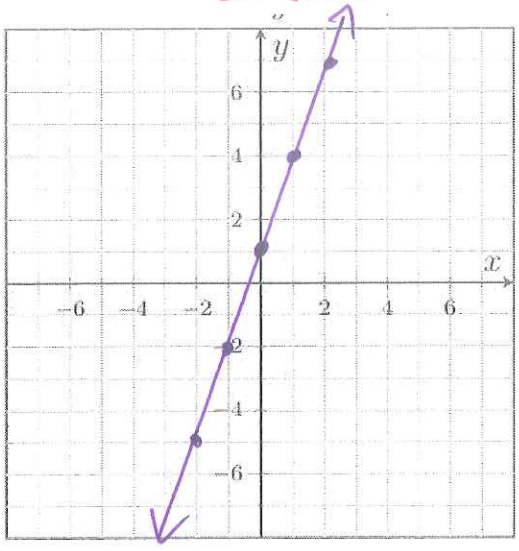
6. Graph each equation by plotting the y-intercept and then using the slope to plot additional points.

a. $y = 3x + 1$

$m = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$

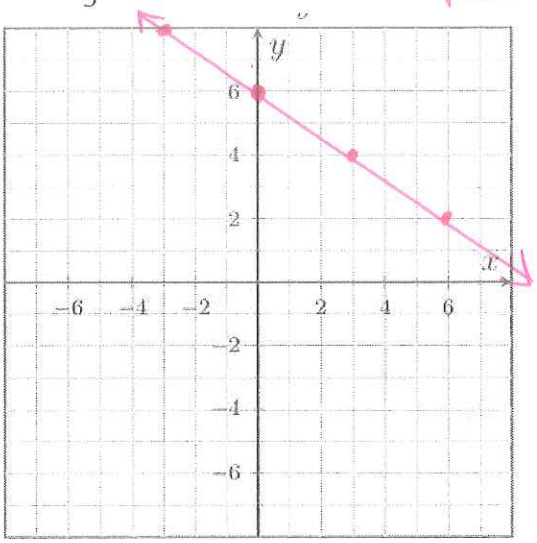
$b = (0,1)$

Start with y-int



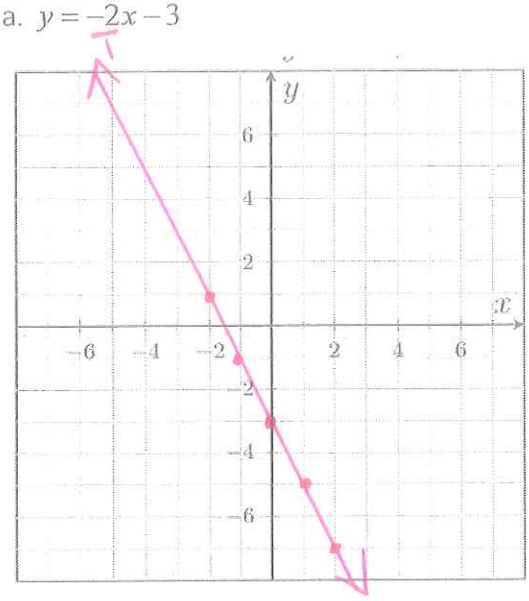
b. $y = -\frac{2}{3}x + 6$ ← $(0,6)$

$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$ or $\frac{2}{-3}$

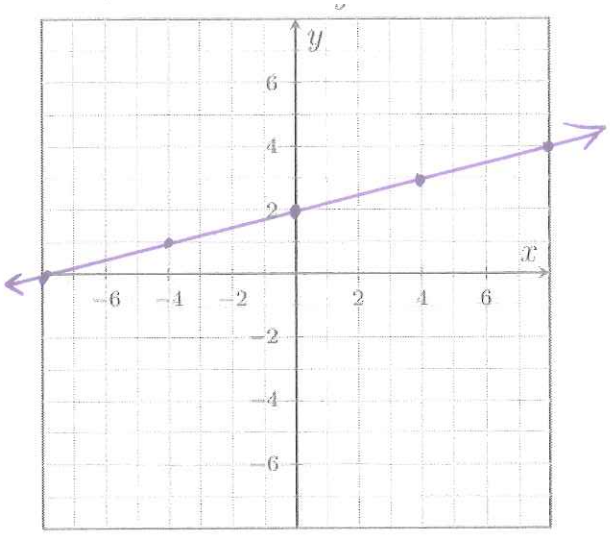


7. Graph each equation by plotting the y-intercept and then using the slope to plot additional points.

a. $y=-2x-3$



b. $y=\frac{1}{4}x+2$



Section 4.6 Point-Slope Form

The Point-Slope Form of a line with slope m that passes through the point (x_1, y_1) is $y=m(x-x_1)+y_1$

(x_1,y_1) m

$y=m(x-x_1)+y_1$

8. Identify the slope and a point that is on each line below.

a. $y=4(x-1)+5$

$m=4$

point $(1,5)$

b. $y=-\frac{1}{3}(x+2)-4$ (same sign; opposite sign)

$m=-\frac{1}{3}$

$(-2,-4)$

c. $y=-9(x+1)-10$

$m=-9$

$(-1,-10)$

9. Find the equation of the line with a slope of -2 (m) that passes through the point $(3,-5)$. Then simplify the equation to slope-intercept form.

$y=m(x-x_1)+y_1$

$y=-2(x-3)-5$

$y=-2x+6-5$

$y=-2x+1$

10. Find the equation of the line with a slope of $\frac{1}{2}$ that passes through the point $(-2,6)$. Then simplify the equation to slope-intercept form.

$m = \frac{1}{2}$ $(-2,6)$

$y = m(x-x_1)+y_1$

$y = \frac{1}{2}(x-(-2))+6$

$y = \frac{1}{2}(x+2)+6$

$y = \frac{1}{2}x+1+6$

$y = \frac{1}{2}x+7$

11. Find the equation of the line that passes through the points $(2,0)$ and $(4,2)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$x_1\ y_1$ $x_2\ y_2$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-0}{4-2} = \frac{2}{2} = 1$

use $(2,0)$

$y = m(x-x_1)+y_1$

$y = 1(x-2)+0 \rightarrow y = x-2$

use $(4,2)$

$y = 1(x-4)+2$

$y = x-4+2$

$y = x-2$

12. Find the equation of the line that passes through the points $(-1,-2)$ and $(5,-4)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-(-2)}{5-(-1)} = \frac{-4+2}{5+1} = \frac{-2}{6} = -\frac{1}{3}$

$y = m(x-x_1)+y_1$

$y = -\frac{1}{3}(x-(-1))+(-2)$

$y = -\frac{1}{3}x-\frac{1}{3}-\frac{2}{1}\cdot\frac{3}{3}$

$y = -\frac{1}{3}x-\frac{1}{3}-\frac{6}{3}$

$y = -\frac{1}{3}x-\frac{7}{3}$

13. A bakery owner has learned that by pricing pastries at \$2.50 each, sales reach 100 pastries per day. If they price them at \$3.50 each, they only sell 60 per day. Let y be the number of pastries the bakery sells per day at x dollars each. Write a linear equation that models the number of pastries sold per day when the price is x dollars each.

$(2.50, 100)$ $(3.50, 60)$

\$ pastries

x y

$m = \frac{60-100}{3.50-2.50}$

$= \frac{-40}{1.00}$

$= -40$

$y = -40(x-2.50)+100$

$y = -40x+100+100$

$y = -40x+200$

More Practice

14. Metromile auto insurance charges $30 per month and 3.2 cents per mile (rates may vary). ← convert to dollars $.032

a. Write a linear equation representing the monthly cost, M, if you drive x miles per month.

$M = 30 + .032x$ or $M = .032x + 30$

b. What is the slope and what does it mean in this context?

The slope is 3.2 ¢ per mile. It is the rate per mile driven.

c. What is the M-intercept and what does it mean in this context?

The M-intercept is the fixed monthly fee of $30.

d. If you drive 100 miles in a month, how much will you be billed?

$M = 30 + .032(100)$
$= 30 + 3.2$
$= \$33.2$

I would be billed $33.20 for 100 miles.

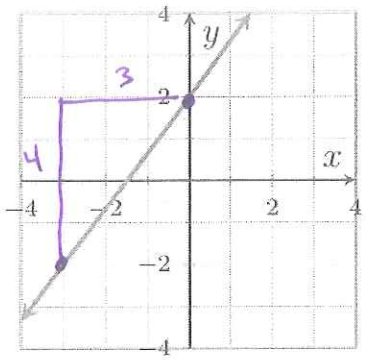
e. If your insurance bill for the month was $43.60, how many miles did you drive?

$43.60 = 30 + .032x$
$-30 \quad -30$
$\frac{13.60}{.032} = \frac{.032x}{.032}$
$425 = x$

I would have driven 425 miles.

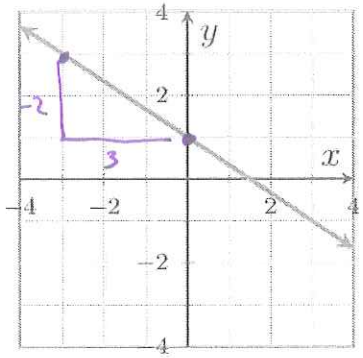
15. Write the equation for each line in slope-intercept form.

a.



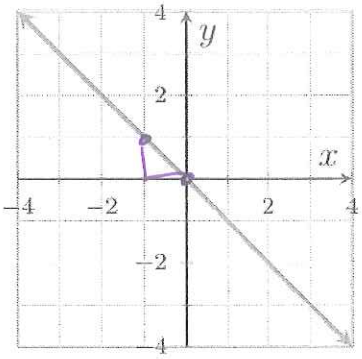
$y = \frac{4}{3}x + 2$

b.



$y = -\frac{2}{3}x + 1$

c.

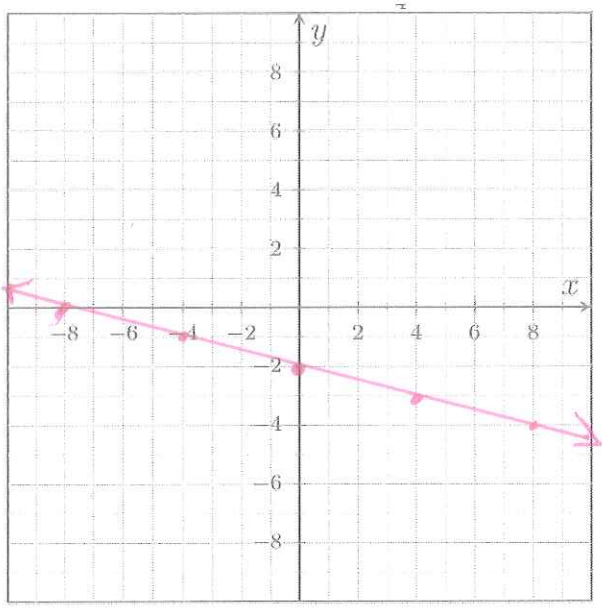


$y = -x + 0$ or $y = -x$

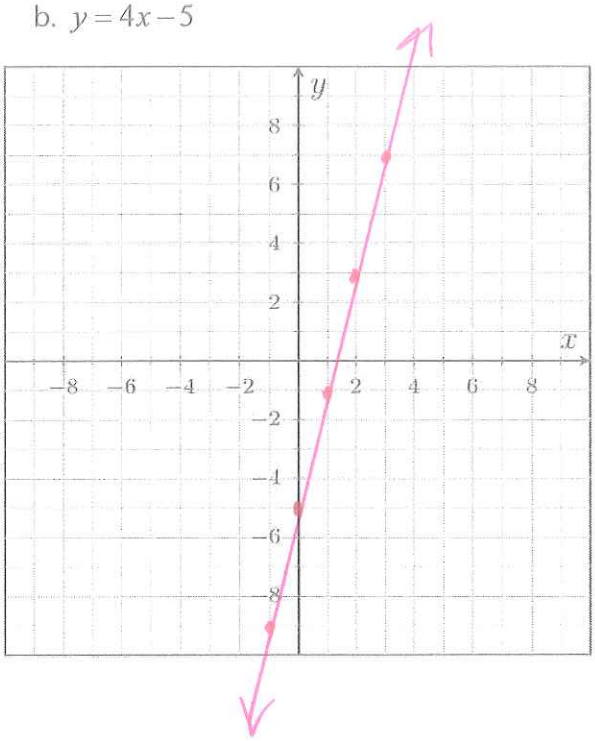
or $y = -\frac{1}{1}x + 0$

16. Graph each line using the slope and y-intercept.

a. $y = -\frac{1}{4}x - 2$



b. $y = 4x - 5$



17. Find the equation of a line with a slope of 7 that passes through the point $(10,5)$. Then simplify the equation to point-slope form.

$y = m(x - x_1) + y_1$

$y = 7(x - 10) + 5$

$y = 7x - 70 + 5$

$y = 7x - 65$

18. Find the equation of the line that passes through the points $(-2,-10)$ and $(1,8)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$m = \frac{8-(-10)}{1-(-2)} = \frac{8+10}{1+3} = \frac{18}{4} = \frac{9}{2}$

$y = \frac{9}{2}(x-1) + 8$

$y = \frac{9}{2}x - \frac{9}{2} + \frac{8}{1}\cdot\frac{2}{2}$

$y = \frac{9}{2}x - \frac{9}{2} - \frac{16}{2}$

$y = \frac{9}{2} - \frac{25}{2}$

19. Find the equation of the line that passes through the points $(2,3)$ and $(7,9)$. First write the equation in point-slope form, then simplify the equation to slope-intercept form.

$m = \frac{9-3}{7-2} = \frac{6}{5}$

$y = \frac{6}{5}(x-2)+3$

$y = \frac{6}{5}x - \frac{12}{5} + \frac{3}{1}\cdot\frac{5}{5}$

$y = \frac{6}{5}x - \frac{12}{5} + \frac{15}{5}$

$y = \frac{6}{5}x + \frac{3}{5}$

20. A company set aside a certain amount of money in the year 2000. The company spent the same amount from that fund each year on perks for its employees. In 2003, there was still $490,000 in the fund. In 2007 there was $318,000 left in the fund.

(2003, 490,000)
(2007, 318,000)
or
(3, 490)
(7, 318)
to make it easier

a. Write an equation for the amount of money in the fund, y, in year x.

$m = \frac{318-490}{7-3} = \frac{-172}{4} = -43$ thousand/year

$y = -43(x-3)+490$
$= -43x + 129 + 490$
$y = -43x + 619$

b. How much is the company spending each year from this fund?

The company is spending $43,000 per year (slope)

c. How much money did they start with in the fund?

The company started with $619,000 in the fund. (y-intercept)

d. In the year 2011, how much was left in the fund?

$y = -43(11)+619$
$y = -473 + 619$
$= 146$

In 2011 there was $146,000 left in the fund

e. If they continue the same trend, in which year will the fund run out?

$y = -43x + 619$
$0 = -43x + 619$
$-619 \quad -619$
$\frac{-619}{-43} = \frac{-43x}{-43}$
$x \approx 14.4$

After 2014 the fund would run out.