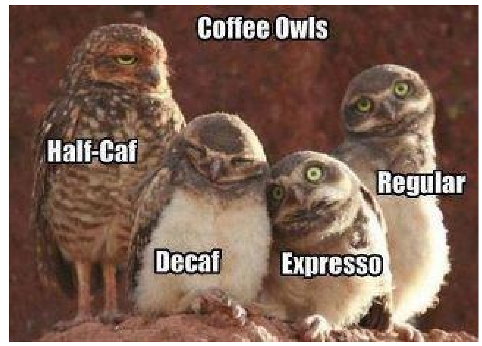
Math 111 Lecture Notes

Section 4.1: Function Composition



When a person consumes caffeine, it is absorbed into their blood. Over time, the amount of caffeine in the bloodstream decreases (assuming they stop consuming caffeine). The result of caffeine being in the bloodstream is that the person's heart rate is elevated. This "chain reaction" is a simple example of a *composite function*. The person's heart rate depends on the amount of caffeine in their bloodstream, which depends on the amount of time since it was consumed. It makes sense then that we should be able to combine these two functions and determine person's heart rate at a given time.

Example 1. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

$$g(x) = -10x + 90, \qquad h(y) = 3y - 90$$

(a) Find and interpret $g(3)$.

(b) Find and interpret $h(60)$.

(c) Find and interpret $h\big(g(3)\big)$.

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read "f composed with g" or "f of g") is defined by

$$(f \circ g)(x) = f(g(x))$$

The function g is referred to as the *inside function* and the function f is referred to as the *outside function*.
In determining the domain for the composite function, the domain of the inside function and the domain for the resultant composite function must be accounted for.

Example 2. Use the functions f and g given in Table 1 to determine the following.

TABLE 1

x	-2	-1	0	1	2
$f(x)$	5	4	-3	2	0
$g(x)$	0	-2	6	9	-1

(a) $(g \circ f)(2)$

(c) $(g \circ g)(-1)$

(b) $(f \circ g)(2)$

(d) $(f \circ f)(-2)$

Example 3. Use Figure 1 to complete the following, if they exist.

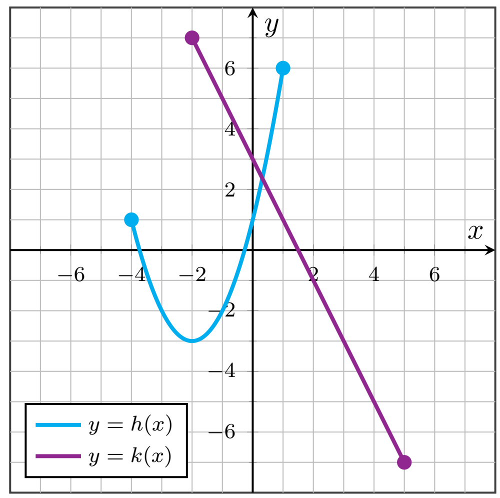
(a) $(h \circ k)(2)$

(b) $(k \circ h)(-3)$

(c) $(h \circ h)(1)$

(d) $(k \circ k)(-1)$

Figure 1



Example 4. Let $f(x) = \dfrac{\sqrt{x+4}}{3x-6}$ and $g(x) = |2x-8|$. Compute the following:

(a) $(f \circ g)(-2)$

(b) $(f \circ f)(-4)$

Example 5. Let $j(x) = 5x^2 + 3x - 1$ and $k(x) = 2x + 7$. Find and fully simplify each of the following:

(a) $(k \circ j)(x)$

(b) $(k \circ k)(x)$

(c) $(j \circ k)(x)$

What is the domain of $k \circ j$?

Example 6. Find $(g \circ f)(x)$ if $f(x) = \dfrac{7}{x+4}$ and $g(x) = \dfrac{3x}{2x-5}$. State the domain of $g \circ f$.

Example 7. Let $g(x)$ be the amount of caffeine (in mg) in your bloodstream after x hours. Let $h(y)$ be your heart rate when there are y mg of caffeine in your bloodstream. These two functions will be modeled by:

$$g(x) = -10x + 90, \qquad h(y) = 3y - 90$$

Write the composite function $(h \circ g)(x)$. What does this function represent?

Group Work 1. Let $f(x) = 5x - 7$, $g(x) = \dfrac{2x}{x-3}$, and $h(x) = \sqrt{4x+8}$. Find and fully simplify each of the following. Also state the domain of $g \circ f$ and $f \circ h$.

(a) $(g \circ h)(2)$

(b) $(g \circ f)(x)$

(c) $(f \circ f)(-4)$

(d) $(f \circ h)(x)$

Group Work 2. Use Table 2 and Figure 2 to complete the following, if they exist.

(a) $(a \circ m)(3)$

(b) $(m \circ m)(4)$

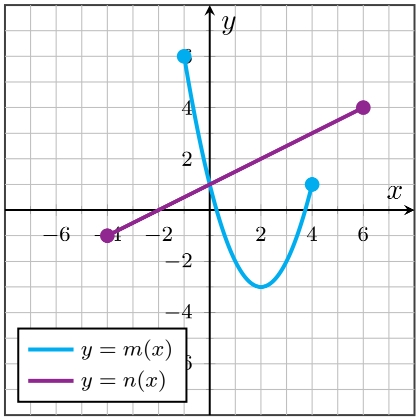
(c) $(b \circ a)(0)$

(d) $(n \circ b)(1)$

TABLE 2

x	-2	-1	0	1	2
$a(x)$	5	4	2	-1	1
$b(x)$	7	2	0	-9	-4

FIGURE 2



Example 8. Let $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}(x - 5)$. Show that both $(f \circ g)(x) = x$ and that $(g \circ f)(x) = x$ for every x in the respective domains of $f \circ g$ and $g \circ f$.

Example 9. For the following examples, find the functions f and g such that $H = f \circ g$. Do not choose $f(x) = x$ or $g(x) = x$.

(a) $H(x) = \sqrt{3x+1}$

(b) $H(x) = (5x-3)^2$

(c) $H(x) = \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}+1}$

(d) $H(x) = (x^2-1)^3$

(e) $H(x) = \dfrac{2}{x-3}$