Zombie Tag!

A Zombie is loose in our classroom!

How long until we are all infected?

Example 1. Fill in the table for each scenario.

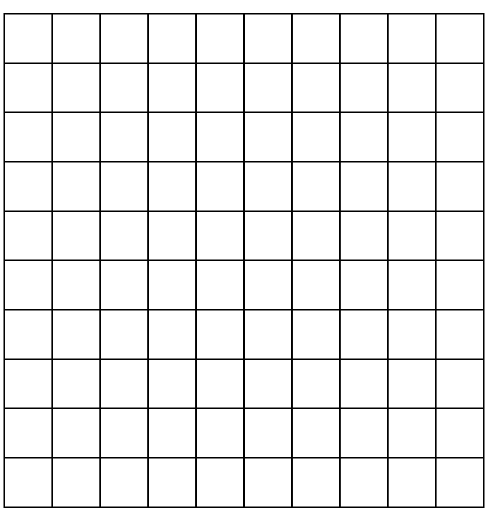
Scenario 1: The initial zombie infects one new person in our class per day. Newly infected zombies cannot infect others.

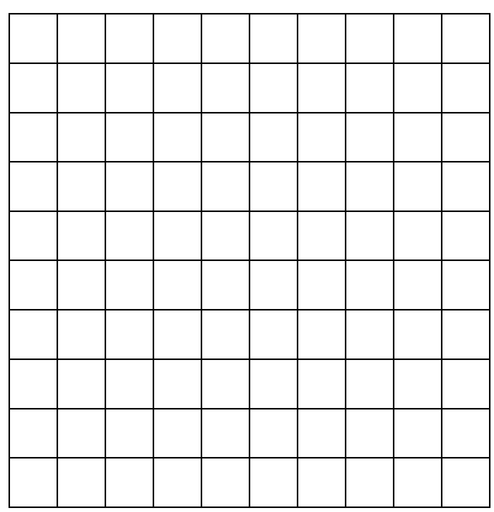
Days	# of People Infected
Day 0	1
Day 1	
Day 2	
Day 3	
Day 4	
Day 5	
Day 6	
Day 7	
Day 8	

Scenario 2: The initial zombie and each infected person infect one new person per day.

Days	# of People Infected
Day 0	1
Day 1	
Day 2	
Day 3	
Day 4	
Day 5	
Day 6	
Day 7	
Day 8	

a. Graph each scenario. How many days will it take to infect our whole class in each scenario?





b. Write an equation for each scenario:

Scenario 1: Scenario 2:

c. How many people would be infected on day 30?

Scenario 1: Scenario 2:

d. On which day would the zombie outbreak infect one million people?

Scenario 1: Scenario 2:

An **exponential function** is of the form

$$f(x) = C\,a^x$$

where

- C is the initial value
- a is the growth factor and $a > 0$

Consequently, an exponential function is a function that increases or decreases at a constant percent rate. Let's review percent increase and decrease as we work through these examples.

Example 2. You start a new job with an initial salary of $36,000 per year. Each year thereafter, you receive a 3% raise. Let $S(t)$ be your salary t years after you start your new job.

(a) Write the formula for $S(t)$.

(b) What will your salary be after 10 years?

(c) When will your salary reach $50,000? (Use your graphing calculator to solve this).

Example 3. A compost pile has 27 cubic feet of waste and decays at a rate of 10% per month. Let $Q(t)$ be the volume of compost (in cubic feet) t months since decay began. Write the formula for this decreasing exponential function.

Example 4. Graph of $y = 2^x$ in Figure 3. Use this to graph the various transformations listed.

FIGURE 3. $y = 2^x$

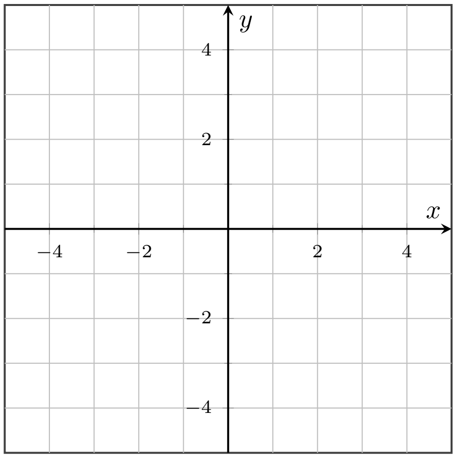


FIGURE 4. $y = 2^x + 1$

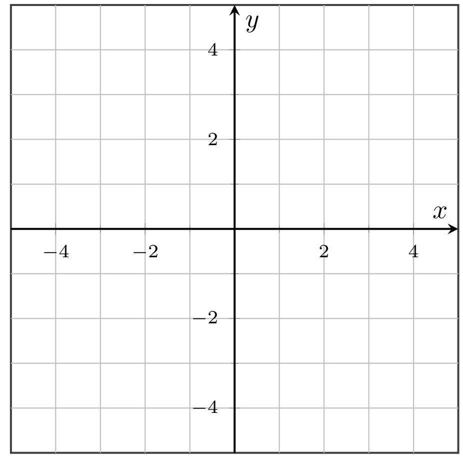


FIGURE 5. $y = 3 \cdot 2^x$

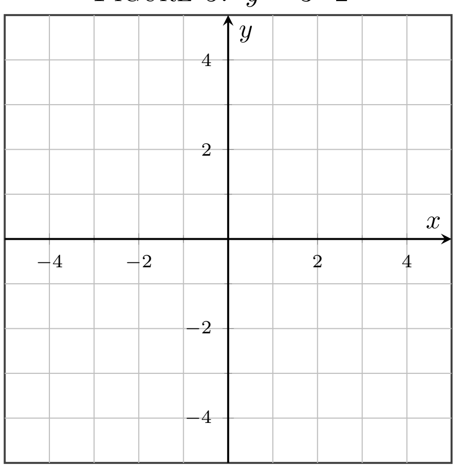


FIGURE 6. $y = -2^x$

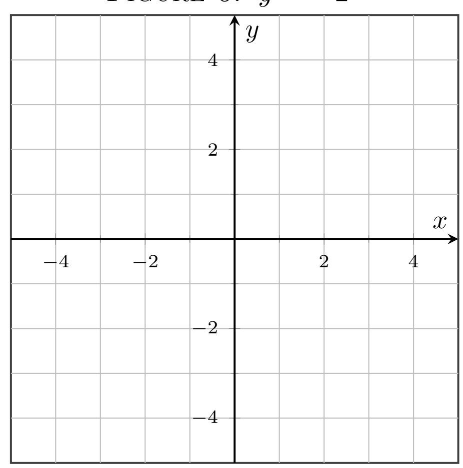


FIGURE 7. $y = 2^{-x}$

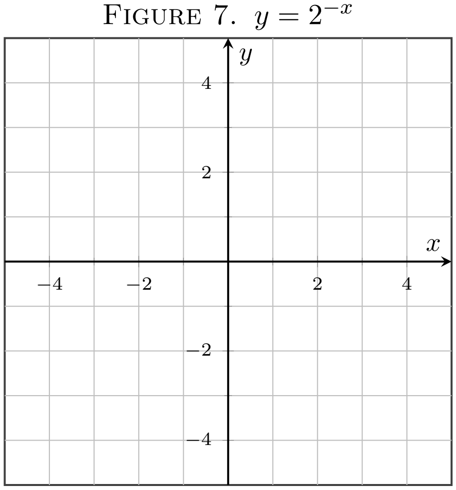
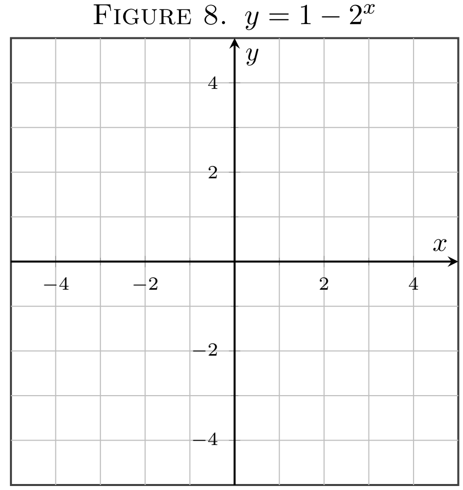


FIGURE 8. $y = 1 - 2^x$



Example 5. Solve the following equations. List your solution set.

(a) $5^x = 5^{-6}$

(b) $4^{2x-5} = \frac{1}{16}$

(c) $5^{x^2+8} = 125^{2x}$

(d) $2^{2x-1} = 4$

(e) $2^{3x-1} = 32$

(f) $9^{2x} \cdot 27^{x^2} = 3^{-1}$

What's "e"?

The number e is a number that occurs in nature, and is a frequent base for exponential and logarithmic expressions. It is defined by:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

It can also be expressed by the following:

$$e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots$$

This number is irrational and is approximated by 2.718281828. The graph of the function given by $y = e^x$ looks a lot like the graphs of the functions given by $y = 2^x$ and $y = 3^x$, as shown in Figure 9. In calculus, you will study that the special property of e is that the slope of the tangent line at zero is exactly 1, as shown in Figure 10.

Figure 9

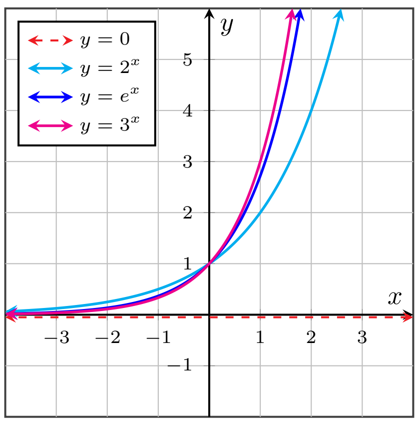
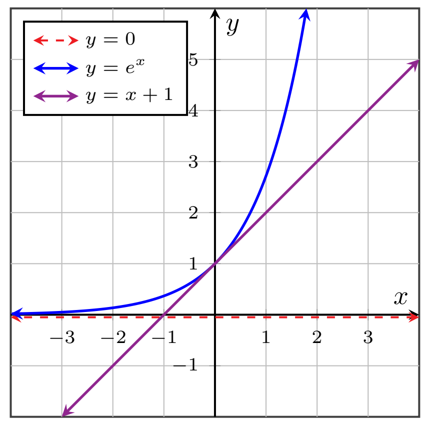


Figure 10



Example 6. Solve the following equation.

$$e^{3x} = e^{2-x}$$

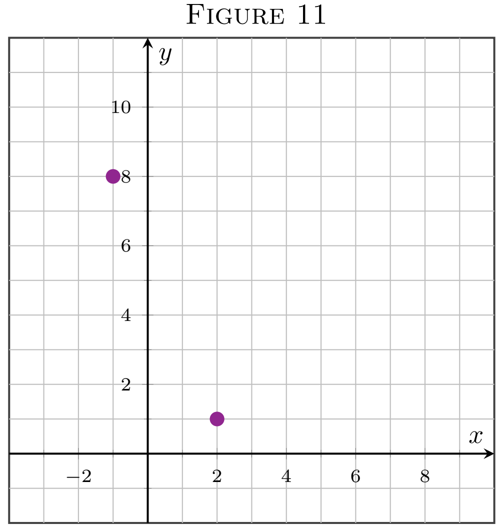
Example 7. In 1990, the population of Oregon was 2.84 million people. In 2010, the population of Oregon was 3.83 million people. Let $P(t)$ be the population of Oregon in millions, where t is the number of years after 2000. This can be modeled by $P(t) = 3.298e^{0.015t}$.

(a) According to this model, what will the population be in 2020?

(b) According to this model, when will the population reach 4 million people? Use your graphing calculator to solve this.

Example 8. Find an algebraic rule (or formula) for an exponential function f that passes through the points $(-1, 8)$ and $(2, 1)$. Also find the algebraic rule (or formula) for a linear function g that passes through the points $(-1, 8)$ and $(2, 1)$.

FIGURE 11



Example 9. Find an algebraic rule (or formula) for an exponential function f that passes through the points $\left(-2, \frac{3}{4}\right)$ and $(2, 12)$.

Example 10. Find an algebraic rule (or formula) for an exponential function f that passes through the points $(1, 8)$ and $(3, 128)$.

Example 11. After caffeine is consumed, it leaves the body at a fairly fixed rate. A person consumes 200 mg of caffeine at 8:00am. Four hours later, about 100 milligrams of caffeine are remaining in their bloodstream. Let $Q(t)$ be the number of milligrams of caffeine in the body t hours after consumption.

(a) Write the formula for the function modeling this exponential decay.

(b) How much caffeine will still be in the body at 8:00pm?