

Math 111 Lecture Notes

SECTION 4.6: LOGARITHMIC AND EXPONENTIAL EQUATIONS

Example 1. Suppose the formula $D = 5e^{-0.4h}$ can be used to find the number of milligrams D of a certain drug in a patient's bloodstream h hours after the drug was administered. When the number of milligrams reaches 1.5, the drug is to be administered again. What is the time between injections? In this problem, we solve the equation $1.5 = 5e^{-0.4h}$. We isolate the exponential expression and then convert to logarithmic form:

What if the base was not 10 or e ? What if we had the following instead: $1.5 = 5 \cdot 2^{-0.4h}$?

Example 2. A population of fruit flies initially has 2 fruit flies and grows exponentially. The population P of fruit flies after t days can be modeled by $P = 2(1.2)^t$. When will the population reach 100 fruit flies?

Example 3. Julie and Mia each made investments in 1999.¹ Let $J(t)$ be Julie's investment t years after 1999 and let $M(t)$ be Mia's investment t years after 1999. Their respective investments can be modeled by the functions

$$J(t) = 50000(1.092)^t \quad \text{and} \quad M(t) = 45000e^{0.09t}$$

(a) When will Julie's investment have doubled in value?

(b) When will the value of Mia's investment reach \$100,000? Solve this equation algebraically and then using your graphing calculator.

¹This was the year when Mia Hamm, Julie Foudy, and Brandi Chastain won the Women's World Cup and netted \$50,000 bonuses, but this problem is otherwise fictional.

Example 4. Solve the following exponential equations for x . Express irrational solutions in exact form.

(a) $3^{2x} = 15$

(c) $2^{-x} = 14$

(b) $4(2^{3x}) = 5$

(d) $3^x = 15^{x+1}$

Example 5. Solve the following exponential equations for x . Express irrational solutions in exact form and rounded to three decimal places.

$$2^{x+3} = 5^{2x+1}$$

Example 6. Solve the following logarithmic equations for x . Clearly state the solution set.

(a) $\log_4(x + 1) = \log_4(25)$

(b) $20 \log_4(x) = 10$

Example 7. The loudness $L(x)$ (in decibels) of a sound of intensity x (measured in watts per square meter) is defined by $L(x) = 10 \log\left(\frac{x}{I_0}\right)$, where $I = 10^{-12}$ watts per square meter and represents the least intense sound that a human ear can detect.

(a) Normal conversation has an loudness level of 60 dB. What is the intensity of this sound?

(b) A jet takeoff has an loudness level of 140 dB. What is the intensity of this sound?

Example 8. Solve the following logarithmic equations for x . Clearly state the solution set.

(a) $\log_6(x + 4) + \log_6(x + 3) = 1$

(b) $\log_7(x - 1) - \log_7(x + 5) = -1$

Example 9. Solve the following logarithmic equations for x . Clearly state the solution set.

(a) $\log_2(x + 1) + \log_2(x + 7) = 3$

(b) $\ln(5x + 1) - \ln(3x) = 2$

Example 10. Solve the following logarithmic equations for x . Clearly state the solution set.

(a) $\log_5(x + 3) = 1 - \log_5(x - 1)$

Example 11. The following equations cannot easily be solved algebraically. Use your graphing calculator solve each equation, rounded accurately to three decimal places.

(a) $\log_2(x - 1) - \log_6(x + 2) = 2$

(b) $e^{2x} = x + 2$