Overview

- Notation for the mean, standard deviation and variance
- The Binomial Model
- Bernoulli Trials

Notation for the mean, standard deviation and variance

We use different symbols to represent samples, populations and random variables. Fill in as many as you can and we will talk about them.

	Sample Statistics	Population Parameters	Discrete Random Variables
Mean	$\bar{x}$ x-bar	μ mu	E(x)
Standard Deviation	s	σ sigma	SD(x)
Variance	s^2	σ^2	Var(x)

Example 1. Remember in the *Intro to Probability packet,* when you flipped a coin 20 times? Discuss these questions with the people sitting around you.

a. How many heads did you get out of the 20 flips? 11 15

b. How likely do you think it is to get that result (very, somewhat or unlikely)? likely less likely

c. Which values for the number of heads do you think would be most likely? 10

d. Which values for the number of heads do you think would be least likely? 0 or 20

Let X = the Number of Heads flipped in 20 trials. Is this random variable **discrete** or **continuous**?

you can only get a whole number of heads

0, 1, 2, 3, ..., 19, 20

We want to make a **probability distribution function (PDF)** for this random variable. We could find the probabilities by drawing a giant tree but it would have 20 layers of branches!

heads

X	P(X)
0	small $= .5^{20}$
1	small
2	.0002
3	.0011
4	.0046
5	.0148
6	.037
7	.0739
8	.1201
9	.1602
10	.1762
11	.1602
12	.1201
13	.0739
14	.037
15	.0148
16	.0046
17	.0011
18	.0002
19	small
20	small $.5^{20}$

20 times

$P(T)\cdot P(T)\cdot P(T)\cdot P(T)\cdots P(T) = (.5)^{20} =$ .000000954

1 Head $\rightarrow$ 1st, or 2nd, or 3rd, or 4th

$\underbrace{P(H)\cdot P(H)\cdot P(H)\cdots P(H)}_{20 \text{ times}} = (.5)^{20} =$

Fortunately, this is one of many patterns that statisticians have studied and named as the **Binomial Distribution**. Let's explore the distribution using GeoGebra.

GeoGebra: Graphing a Binomial Distribution

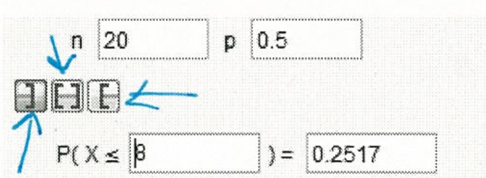
View > Probability Calculator > Select **Binomial** in the dropdown menu under the graph

Type in the values for n (number of trials) and p (probability of getting desired result)

The probability of getting each outcome is listed on the right and shown on the graph. Does the picture make sense? Fill in the probability distribution function on the first page.

Finding Binomial Probabilities

Now you can use the probability calculator by typing in values at the bottom of the screen. Select **]** for less than, **[ ]** for between two values, and **[** for greater than.



inputs (parameters)
n – number of trials
p – probability of getting the desired result

use 4 decimal places $X \sim B(\overset{n}{20}, \overset{p}{.5})$

Find the following probabilities using GeoGebra.

[ ] $P(X = 10)$ = .1762 (10 heads)

[ ] $P(X = 4)$ = .0046

] $P(X \leq 8)$ = .2517

[$P(X \geq 8)$ = .8684

] $P(X < 8)$ =
$P(X \leq 7) = .1316$

[$P(X > 8)$ =
$P(X \geq 9) = .7483$

[ ] $P(5 \leq X \leq 10)$ = .5822

[ ] $P(5 < X \leq 10)$ =
$P(6 \leq X \leq 10) = .5674$

[ ] $P(5 \leq X < 10)$ =
$P(5 \leq X \leq 9) = .406$

[ ] $P(5 < X < 10)$ =
$P(6 \leq X \leq 9) = .3912$

What is the Expected Value of the Number of Heads, E(X) or μ =
$20(.50) = 10$ heads

$\mu = n \cdot p$

What is the Standard Deviation of the Number of Heads, SD(X) or σ =

$q = 1 - p$ $\sqrt{npq} = \sqrt{20(.50)(.50)} = 2.24$ heads

Example 2. What would the distribution look like if we had a different type of coin that had a 30% chance of coming up heads? What would you expect the number of heads to be?

$20(.30) = 6$ heads

What if the coin had an 80% chance of coming up heads? What would you expect the number of heads to be? $20(.8) = 16$ heads

If there is a given probability, number of trials, and independence of the trials we can use the Binomial distribution.

Binomial Probability Model for Bernoulli Trials	$X \sim Binomial(n, p)$ or $X \sim B(n, p)$
X = the **number of successes in n trials**	$X \sim B(20, .5)$
p = probability of "success" or desired result	
$q = 1 - p$ = probability of "failure"	
Probability of x successes in n trials:	$P(X = x)$
Expected Value:	$\boxed{E(X) = \mu = np}$ mean
Standard Deviation:	$\boxed{\sigma = \sqrt{npq}}$

Bernoulli Trials

Repeated trials of an experiment are called Bernoulli trials if the following conditions are met:

1. Each trial has only two possible outcomes (generally designated as "success" and "failure")
2. The probability of "success", p, remains the same for each trial
3. *The trials are independent. (The outcome of one trial has no influence on the next)

***The 10% condition:** Bernoulli trials must be independent. When selecting items without replacement, we know they are not independent. However, if we are selecting less than 10% of the population it is okay to assume independence and proceed with this model.

Example 3. Determine which of the following situations involve Bernoulli trials.

a. You are rolling 5 dice and need to get at least two 6's to win the game.

1. 2-outcomes : 6, not 6 ✓
2. P(six) = $\frac{1}{6}$ for each die (same for each trial) ✓
3. the dice are independent of each other ✓

are Bernoulli

b. We record the distribution of eye colors found in a group of 500 students.

1. not a yes/no or only 2 possibilities X

not Bernoulli

c. A city council of 11 Republicans and 8 Democrats picks a committee of 4 at random. What's the probability that they choose all democrats?

1. only 2 outcomes : R or D ✓
2. 3. trials are not independent because we would choose without replacement. 10% condition : we are selecting more than 10% X

not Bernoulli

Example 4. A basketball player makes 82% of her freethrows. Assume each shot is independent of the last. She's going to shoot 10 free throws.

Define the distribution and use GeoGebra to find the following.

$X \sim B(\underset{n}{10}, \underset{p}{.82})$

a. What's the probability that she makes exactly 5 free throws?

$P(X=5) = .0177$

b. What's the probability that she makes 9 or 10 free throws?

$P(X = 9 \text{ or } 10) = P(9 \leq X \leq 10) = .4392$

c. What's the probability that she makes 2 or fewer free throws?

$P(X \leq 2) = 0$

d. What's the expected number of baskets she makes? What's the standard deviation?

$\mu = 8.2$ baskets $\qquad \sigma = 1.2149$ basket

#trials -Binomial .50

Example 5. A student takes a 10-question true/false quiz, but did not study and randomly guesses each answer. Find the probability that the student passes the quiz with a grade of at least 70%.

$X \sim B(10, .50)$

$P(X \geq 7) = .1719$

Practice Problems

1. An Olympic Archer is able to hit the bull's-eye 75% of the time. For this problem he is going to shoot 6 arrows and we will assume each shot is independent of the others.

Let X=Number of Bull's-eyes.

Give the Distribution of X. X ~ Binomial(6 , 0.75)

What is the probability of each of the following results?

a. He gets exactly 4 bull's-eyes

$P(X=4) = .2966$

b. He gets at least 4 bull's-eyes.

$P(X \geq 4) = .8306$

c. He gets at most 4 bull's-eyes

$P(X \leq 4) = .4661$

d. How many bull's-eyes do you expect him to get?

$\mu = 4.5$ bull's-eyes

e. With what standard deviation?

$\sigma = 1.0607$ bull's-eyes

2. It has been estimated that only 30% of California residents have adequate earthquake supplies. Suppose you randomly survey 11 California residents. We are interested in the number who have adequate earthquake supplies.

a. In words, define the random variable X.

Let X be the number out of 11 surveyed that have adequate earthquake supplies

b. List the values X may take on.

0, 1, 2, 3, ... 11

c. Give the distribution of X. X ~ B (11, .30)

n p

d. What is the probability that at least eight have adequate earthquake supplies?

$P(X \geq 8) = .0043$

e. Is it more likely that all or none of the residents surveyed will have adequate earthquake supplies and why? It is more likely that none will have adequate supplies because the value for p is less than 50%.

f. How many residents to you expect to have adequate earthquake supplies?

$\mu = 3.3$ residents that are prepared out of 11.

3. Suppose a computer chip manufacturer rejects 3% of the chips produced overall because they fail presale testing. You select 30 chips at random from the day's production.

$X \sim B(30, .03)$

a. What is the probability of getting more than 2 bad chips?

$P(X > 2) = P(X \geq 3) = .0601$

b. What is the probability of getting no bad chips?

$P(X = 0) = .401$

c. What is the expected number of bad chips?

$E(X) = n \cdot p = 30(.03) = .9$ bad chips

d. What is the standard deviation of the number of bad chips?

$SD(X) = \sqrt{npq} = \sqrt{30(.03)(.97)} = .9343$

4. Ken Griffey Jr. has a lifetime batting average of .305. (This is the probability of getting a hit). If he batted 5 times in one game, what is the probability that he gets at least 3 hits? $X \sim B(5, .305)$

$P(X \geq 3) = .1698$